ELECTROMAGNETIC OPTICS OF THIN-FILM COATINGS

Three experts in the field of thin-film optics present a detailed and self-contained theoretical study of planar multilayers and how they can be effectively exploited in both traditional and modern applications.

Starting with a discussion of the relevant electromagnetic optics, the fundamental optical properties of multilayers are introduced using an electromagnetic approach based on a direct solving of Maxwell’s equations by Fourier transforms. This powerful approach is illustrated through the comprehensive description of two of the most important phenomena in multilayers, i.e., giant field enhancement in dielectric stacks and light scattering from thin-film optical filters. The same approach is extended to the description of the operation of planar microcavities and the balance of energy between radiated and trapped light.

This book will be valuable to researchers, engineers, and graduate students with interests in nanophotonics, optical telecommunications, observational astronomy, and gravitational wave detection.

Claude Amra is Director of Research at the Centre National de la Recherche Scientifique (CNRS), Institut Fresnel, which he co-founded and led. He was previously Director of Laboratoire d’Optique des Surfaces et des Couches Minces and then Deputy Scientific Director of the Institute of Systems and Engineering Sciences within CNRS.

Michel Lequime is an emeritus professor at Ecole Centrale Marseille, Institut Fresnel. He was Head of Division at BERTIN, led the Optical Thin Films Group at the Institut Fresnel, and served as Secretary to the French Optical Society.

Myriam Zerrad is a research engineer at Aix-Marseille University, Institut Fresnel. She heads the Scattering Group and leads the Laser Interferometer Space Antenna (LISA) team at the Institut Fresnel.
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Light Scattering, Giant Field Enhancement, and Planar Microcavities

CLAUDE AMRA
Institut Fresnel, CNRS

MICHEL LEQUIME
Institut Fresnel, Centrale Marseille

MYRIAM ZERRAD
Institut Fresnel, Aix-Marseille University
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About the Authors

Claude Amra joined the Centre National de la Recherche Scientifique (CNRS) in 1986 after defending a PhD on the scattering of light by roughness in interference filters; he then worked at the laboratory for optical surfaces and thin films (Laboratoire d’Optique des Surfaces et des Couches Minces [LOSCM], CNRS, École Nationale Supérieure de Physique in Marseille). His work expanded electromagnetic modeling to heterogeneous volumes and then on to luminescent microcavities for free space and modal optics. In parallel, he studied optical metrology and addressed the inverse problems inherent in characterizing disordered media, before becoming involved with the technology of optical thin films.

From 1991, he initiated and actively contributed to the introduction of new topics at LOSCM: photo-induced thermal and laser damage, trapped light, near-field microscopy, and multiscale roughness; ellipsometry of speckle patterns, achromatic thin-film light absorbers, mid infrared spectral splitters, dense wavelength multiplexing, deposition technologies using ion beam sputtering, etc.

In 1996, Claude Amra was appointed director of LOSCM, which he then reconstructed thematically. In 2000, he co-founded the Institut Fresnel, a new mixed research unit incorporating CNRS, Aix-Marseille University, and the École Centrale Marseille; this unit draws together sciences and technologies of optics, electromagnetism, and signal and image processing on the campus of Marseille Nord. Over the course of his two terms as director from 2000 to 2008, the Institut Fresnel emerged and cemented its position on the European stage with one dedicated building housing all the staff, and a second building funded to house a technology center (the Espace Photonique).

In 2009, he was named Deputy Scientific Director of the Institute of Systems and Engineering Sciences within CNRS (INSIS, Paris); since 2016, he
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Claude Amra has chaired Section 8 of the National Committee for Scientific Research, covering photonics, electronics, micro- and nanotechnologies and micro- and nanosystems, electrical energy, electromagnetism, and antennas. Prior to that, he chaired or participated in numerous committees from the national funding or research assessment agencies and the national universities’ council, while also organizing numerous international conferences.

Within the context of the CONCEPT team that he has been running since 2012 at the Institut Fresnel, Claude Amra’s research interests cover light scattering in interference filters, giant optical field enhancements, polarization and coherence in disordered media, terahertz probing and mimetic thermal radiation for applications in the space and military fields, energy and the environment, biomedical optics, and precision agronomy.

Michel Lequime received his master’s in engineering from the Institut d’Optique Graduate School in 1974 and his PhD degree from Paris-Sud University in 1977. His doctoral research was devoted to the study of third-order nonlinear optical processes at the picosecond scale, especially in one-dimensional conjugated polymers (École Polytechnique, Laboratoire d’Optique Quantique).

In 1979, he joined BERTIN, a French contract research organization, to participate in the creation of an optical group at the company: he was initially appointed a study engineer, then head of department, and finally head of division. The division’s activities dealt mainly with space optics (multicolor camera for the observation of Halley’s Comet – European Space Agency [ESA] Giotto probe, optical beacon for the ESA Semi-Conductor Inter Satellite Link EXperiment [SILEX]) and optical fiber sensors (inline color measurement of refined fuels, pressure and temperature measurements for down-hole applications).

In 2000, he joined the École Centrale Marseille as Professor of Optics and Project Management and the Institut Fresnel as Senior Scientist. From 2002 to 2014, he was in charge of the Optical Thin Films Group at the Institut Fresnel. He is currently an emeritus professor at Centrale Marseille and continues to pursue his research activities at the Institut Fresnel on the use of metamaterials in optical thin-film coatings and on the development of instrumentation for the comprehensive characterization of the scattering properties of multilayer components.

Michel Lequime served as Secretary to the French Optical Society from 2009 to 2012 and is the author of more than 280 publications and communications in the field of optics.
About the Authors

Myriam Zerrad is an engineer who graduated in 2003 from the École Centrale Marseille and the École Nationale Supérieure de Physique in Marseille, obtaining a PhD in physics from Aix-Marseille University in 2007.

As a research engineer at Aix-Marseille University and a member of the CONCEPT team at the Institut Fresnel, she is a recognized expert on the phenomena of light scattering in optical interference coatings, involving modeling, instrumentation, and reconstruction. Since 2010, she has widened her area of expertise to include the optical probing of disordered media using statistical analysis of speckle patterns and has moved on to the synthesis of resonant planar components and the numerical modeling of thermal metamaterials.

She now heads the Scattering Group within the Institut Fresnel, with a dozen people working on her different areas of expertise. As part of her research activities, she developed the DIFFUSIF instrumentation platform optimized for an accurate metrology of scattered light and/or using scattered light in metrological applications. Today, this platform is a reference for various space agencies and for workers in the field of space optics, both academic and industrial.

In the context of developing the new generation of gravitational wave detector (Laser Interferometer Space Antenna [LISA]), Myriam Zerrad is a member of the international consortium and leads the LISA team at the Institut Fresnel; the institute has overall responsibility for the metrology and modeling of light scattered by coatings at the component level. She has authored more than 300 publications and communications in the fields of optics and photonics.
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David Lynch’s Dune starts with the striking statement that “A beginning is a very delicate time,” and this introduction is no different. However, this fragility clearly must not present an obstacle to any rational and concise answer to the two questions that inevitably arise from the publication of a new piece of work, namely Why? and How?

Let us begin by explaining the Why?

Interference optical filters represent a rather special scientific domain – in more ways than one. To start with, there are as many scientists and engineers involved in the industry as there are in the universities (indeed, the commercial market is highly lucrative, with the global optical coatings market projected to exceed USD 12 billion by 2023 at a compound annual growth rate of 7%\(^1\)), and for decades these two communities have been accustomed to collaborating freely and with the greatest mutual respect. This is also a case in which perfection is required, and the impossible a necessity: we must not forget that optical filtering involves finding the structure of a stack consisting of several hundred layers at nanometer thicknesses that, once created, gives rise to myriad elementary waves whose interference effects will shape with unequalled precision the spectrum of light transmitted and/or reflected by the planar component thus created. Finally, this specialist area does not, in our view, receive the academic recognition it deserves, not least for the complexity of approach required to control, via the solutions to a multitude of inverse problems, the phenomena of spatial and spectral dispersion that are consequent to solutions of Maxwell’s equations in a nonperiodic stratified medium.

Therefore, a book that might shed some light on these profound concepts and this elegance of treatment seemed to us to be highly desirable.

Clearly, much of the work dedicated to this topic is dominated by Angus Macleod’s book *Thin Film Optical Filters*, already in its fifth edition, the first being published in 1986. This book, intended for fabricators and users, aims to give as complete a description as possible of thin-film optical coatings (design, fabrication, characterization, and applications). Our own purpose is very different, since the book you have before you is *purely theoretical*; we do not mean by this that subjects resulting in applications are not considered, but that the book is restricted to providing an explanatory mathematical description. Here *mathematical* does not mean divorced from physical meaning; we shall see that one of the fundamental notions on which this book is based is that of *complex admittance*, an idea that generalizes to stationary waves the concept of effective refractive index, a concept usually associated with propagating waves. Over the course of the chapters of this book, the reader will gradually discover that the matrix formalism often prominent in the description of stacks of optical thin films is barely touched on, being replaced (much more usefully, in our opinion) by that of admittance, since this is indeed imbued with a physical meaning on which the designer’s intuition can draw.

Reading these last sentences, one could be forgiven for thinking that we have already unconsciously started on the *How?* We are far from it, however, since the illumination of this idea of admittance and the engineering with which it is associated clearly constitutes one of the aims of this book, and to finally convince the reader of this, we refer him or her to the chapter on the scattering of light by multilayer stacks (for example), or to the sections dealing with microcavities and giant enhancements of the electromagnetic field.

In publishing this book, another of our objectives was to show how the massive use of the *Fourier transformation* and the concomitant panoply of properties enabled the uniform solution of Maxwell’s equations in a medium invariant in translation in a plane. All the elementary waves considered are therefore the result of a double *spectral decomposition*, first over time and then over the two dimensions of this plane of invariance. We leave it to the reader to discover, in particular in the chapter on frequential wave packets, the consequences of this temporal Fourier transformation for describing interference phenomena in the harmonic regime, a regime where time is excluded and in which space dominates.

Now for the *How?*

This book is effectively divided into three parts: first there are five intro-
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Introductory chapters dedicated to the sometimes quite detailed presentation of generic ideas related to Maxwell’s equations. The use of both frequential and spatial wave packets, drawing on the formalism of distributions and their Fourier transforms, provides an anchor for a uniquely macroscopic description of all the classical concepts of electromagnetism (polarization of matter, inertia, dispersion and absorption, causality, spatial and temporal coherence linked with detection, resolution in time and space, polarization of light, etc.). These first five chapters are treated with the utmost care, since they provide the key to unambiguously tackling the detailed notions in the pivotal chapter that follows them, being entirely dedicated to a concise description of optical interference filters. Finally, we get to three chapters that are more thematic and that delve more deeply into the use of all the ideas tackled in the previous parts via three a priori unconnected questions (reading these chapters will demonstrate that these notions reveal a certain amount of overlap); these chapters deal first with the use in total reflection of multilayer stacks and the giant electromagnetic field enhancements that might result (Chapter 7), then with light scattering by stacks containing slightly rough interfaces (Chapter 8), and finally planar multilayer microcavities (Chapter 9).

Chapter 7 covers the often poorly understood notions of total reflection and evanescent waves and highlights the presence of absorption lines in the spectrum reflected by a planar multilayer component in this particular operational regime. The chapter also identifies the connection between these resonances in absorption and the guided modes of the multilayer structure considered in isolation. In this total reflection regime, in which admittances are purely imaginary, it is possible to make the substrate admittance zero by inserting a thin layer; for a long time, we have hesitated to call this a magic layer, though the effects of its presence have huge consequences for the local intensity of the electromagnetic field. Exploiting these field enhancements finds numerous applications in the domains of sensors and microsources.

The ideas set out in detail in Chapter 8 show how an extremely reliable prediction of the spatial and spectral distribution of the light scattered by a stack of thin layers may be obtained, however complex it may be. Experience shows that highly intense and very narrow angular or spectral lobes may appear in the scattering distributions of these complex filters (such as those used in Earth observation satellites), and it is therefore necessary to understand their precise characteristics to avoid the sometimes serious problems of ghosting between several detection spectral bands. This time in the monochromatic regime, however, these same notions can also help in assessing the restrictions imposed by the problems of scattered light inter-
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Chapter 9 is dedicated to studying a configuration somewhat different from those in the other chapters, one in which the source of light, here an emissive surface, is located within the multilayer stack. The electromagnetic approach we develop shows that the emission into free space of the microcavity thus created is governed by satisfying two resonance conditions, one being related to the structure of the multilayer stack being considered, the other to the position of the emissive surface within the stack. Furthermore, this same approach allows the energy balance of this microcavity to be set out in detail, making it possible to make a comparative assessment of the amount of light emitted into free space and the light trapped in the component, either by total reflection within the substrate or by coupling in the guided modes associated with the multilayer structure.

Is the view that we propose in this book exhaustive? Clearly not, but that was never our intention even though, during the slow gestation of this book, we sometimes envisaged the addition of other chapters, especially in relation to metamaterials or the phase properties of stratified media. But in the next edition, who knows?

Finally, the reader may quite legitimately be surprised by the absence of bibliographic references within the body of this book, although it does end with a list of selected books for the reader to pursue his or her further interests. This was a deliberate choice on our part, as our aim was to emphasize the internal coherence of the argument at the expense of its context. Note that this rule applies equally to our own publications, particularly to those relating to the approach detailed in this book. Obviously, such an absence of references should not be seen as a lack of willingness on our part to include them.

In conclusion, we sincerely hope this book provides as much interest to you, the reader, as the satisfaction we derived from writing it.
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