FIELD THEORY OF NON-EQUILIBRIUM SYSTEMS

The physics of non-equilibrium many-body systems is a rapidly expanding area of theoretical physics. Traditionally employed in laser physics and superconducting kinetics, these techniques have more recently found applications in the dynamics of cold atomic gases, mesoscopic, and nano-mechanical systems, and quantum computation. This book provides a detailed presentation of modern non-equilibrium field-theoretical methods, applied to examples ranging from biophysics to the kinetics of superfluids and superconductors. A highly pedagogical and self-contained approach is adopted within the text, making it ideal as a reference for graduate students and researchers in condensed matter physics.

In this second edition, the text has been substantially updated to include recent developments in the field such as driven-dissipative quantum systems, kinetics of fermions with Berry curvature, and Floquet kinetics of periodically driven systems, among many other important new topics. Problems have been added throughout, structured as compact guided research projects that encourage independent exploration.

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Second Edition

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To Julia and Andrei

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Preface to the Second Edition

Since publication of the first edition more than 10 years ago, there have been significant developments in both experimental realization and theoretical understanding of non-equilibrium matter. The second edition is an attempt to catch up with some of them. It is also an opportunity to rectify certain omissions in the scope of the first book. Major new subjects that found their way into this edition include Lindblad dynamics of driven-dissipative quantum systems, kinetics of fermions with Berry curvature, Floquet kinetics of periodically driven systems, butterfly effect and out-of-time-order correlation functions, macroscopic fluctuation theory and counting statistics in classical stochastic models, and Boltzmann–Langevin theory. The book also received new chapters on hydrodynamics of Fermi liquids and electron–phonon interactions in disordered metals and superconductors. Needless to say, all these topics are presented within the unified framework of the functional treatment of evolution along the closed time contour. A notable addition to the book's structure is a set of problems at the end of each chapter. They are structured as compact guided research projects, which may be suggested to students for independent exploration. Most of them contain useful supplemental information, which is scattered around the periodic literature but has not yet been presented in textbooks. I hope you'll find it useful.

I am deeply indebted to numerous colleagues and current and former students and post-docs, who helped to shape my views and saved me from many pitfalls and omissions. Last but not least, I can't express enough appreciation to my family, whose love, support, and patience made this book possible. During my work on the second edition I was partially supported by National Science Foundation grant DMR-2037654.

Preface to the First Edition

The quantum field theory (QFT) is the universal common language of the condensed matter community. Like any living language it keeps evolving and changing. The change comes as a response to new problems and developments, trends from other branches of physics, and from internal pressure to optimize its own vocabulary to make it more flexible and powerful. There are many excellent books that document this evolution and give snapshots of “modern” QFT in condensed matter theory for almost half a century. In the beginning QFT was developed in the second quantization operator language. It produced such monumental books as Kadanoff and Baym [1], Abrikosov, Gor’kov, and Dzyaloshinski (AGD) [2], Fetter and Walecka [3], and Mahan [4]. The advent of renormalization group and Grassmann integrals stimulated development of functional methods of QFT. They were reflected in the next generation of books such as Itzykson and Zuber [5], Negele and Orland [6], and Fradkin [7]. The latest generation (e.g. Tsvelik [8], Altland and Simons [9]) is not only fully based on functional methods, but also deeply incorporates ideas of symmetry-based universality, geometry, and topology. (I do not mention here some excellent specialized texts devoted to applications of QFT in superconductivity, magnetism, phase transitions, mesoscopics, one-dimensional physics, etc.)

Following AGD authority, most of these books (with the notable exception of Kadanoff and Baym) employ imaginary time Matsubara formalism [11] of finite temperature equilibrium QFT. The irony is that a much more powerful non-equilibrium QFT pioneered by Schwinger [12], Konstantinov and Perel [13], Kadanoff and Baym [1], and Keldysh [14] was developed almost at the same time as the Matsubara technique. Being widely scattered across the periodic scientific literature, it has barely penetrated into the mainstream pedagogical texts. The very few books I am aware of are Kadanoff and Baym [1], Lifshitz and Pitaevskii [15], Smith and Jensen [16], Haug and Jauho [17], and Rammer [18]. (There are also a number of useful reviews [19–23].) In my personal opinion, the reasons for such a disparity are twofold: (i) There may be a perception that all subtle and interesting effects take place only in equilibrium; non-equilibrium systems are too "violent" to be treated by QFT. Instead, the kinetic equation approach is the best one can hope for; the latter may be obtained with the Golden rule and thus does not need QFT. (ii) The formalism is too involved, too complicated, to be a part of the "common" knowledge.

As far as the first reason is concerned, it was realized decades ago that even the "simple" kinetic equation may not actually be so simple. Time and again it was shown that the kinetic equation for superfluids, superconductors, fermion–boson mixtures, disordered normal metals, and so on can't be deduced from the Golden rule and has to be derived using the non-equilibrium QFT methods. Yet, most of the traditional experimental systems, such as liquid helium, bulk magnets, superconductors, or disordered normal metals, can hardly be driven substantially away from the equilibrium. This created the comforting impression that studying the equilibrium plus linear response properties is largely sufficient to describe experiment. The last two decades have changed this perception dramatically. First, mesoscopic normal metals and superconductors have demonstrated that non-equilibrium conditions may be achieved in controlled and reproducible ways and a number of unusual specifically non-equilibrium phenomena do emerge. Then came cold atomic gases in magnetic and optical traps. These systems are rarely truly at equilibrium, yet they exhibit a rich phenomenology, which calls for a theoretical description. Lately, the flourishing fields of nanomechanics and nanomagnetics emerged, which deal with stochastic mechanical and magnetic systems driven far away from the equilibrium. All these developments call for the systematic non-equilibrium theory to be a part of the "standard package" of a theoretical physicist.

As for technical complexity and lack of "aesthetic" appeal, there is some truth to this, especially when the story is told in the old operator formalism. (This is exactly how most of the currently existing books approach the subject.) I can see how one can be overwhelmed by the number of different Green functions, rules to follow, and the length of the calculations. Fortunately, the structure of the theory becomes much more transparent when it is presented in the functional formalism. Instead of keeping track of matrix Green functions and tensor vertices, one has to follow the scalar action, which is a functional of two fields. Yes, one still has to double the number of degrees of freedom. However, when taken in appropriate linear combinations (Keldysh rotation), they acquire a transparent physical meaning. Then the causality principle emerges as a simple and natural way to navigate through the calculations. The main goal of this book is to present a thorough and self-contained exposition of the non-equilibrium QFT *entirely in the functional formalism.*

I tried to pay special attention to specific peculiarities of non-equilibrium (i.e. closed time contour) QFT, which do not show up in the imaginary time or $T = 0$

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formalism. In particular, presentation starts from the simplest possible systems and develops all minute technical details, exposes pitfalls, and explains the internal structure for such "trivial" situations. Then the systems are gradually taken to be more and more complex. I still tried my best to emphasize peculiarities of non-equilibrium calculations in comparison with the probably more familiar equilibrium ones. Although the book is meant to be entirely self-contained, some common subjects between equilibrium and non-equilibrium techniques (e.g. diagrammatic expansion, Dyson equation, renormalization group) are introduced in a rather compact way. In such cases I mostly focused on differences between the two approaches, possibly at the expense of the common themes. Therefore some prior familiarity with the imaginary time QFT is beneficial (although not compulsory) for a reader.

What are the benefits of learning non-equilibrium QFT? (i) It naturally provides a way to go beyond the linear response and derive consistent kinetic theory (e.g. quasiparticle kinetics coupled to the dynamics of the order parameter). As we have mentioned, there is a rapidly growing list of fields where such an approach is unavoidable. (ii) Even for linear response problems it allows one to circumvent analytic continuation to real time, which may be quite cumbersome. (iii) In its functional form it provides a natural and seamless connection to the huge field of classical stochastic systems, their universality classes, and phase transitions. In fact, a big part of this book is devoted to such classical problems. For this reason the word "quantum" does not appear in the title. Yet from the point of view of the formalism, non-equilibrium QFT and the theory of classical stochastic systems are virtually indistinguishable. (iv) Some subjects of great current interest (e.g. full counting statistics, or fluctuation relations) cannot even be approached without the formalism, presented here. (v) Non-equilibrium QFT (again in its functional form) appears to be extremely effective in dealing with systems with quenched disorder. Even if purely equilibrium or linear response properties are in question, closed time contour QFT is much more natural and efficient than the imaginary time one. All these items are the subject of the present book. I hope you'll find it useful.

This book is intended for advanced graduate students, post-docs, and faculty who want to enrich their understanding of non-equilibrium physics. It may be used as a guide for an upper-division graduate class on QFT methods in condensed matter physics. There is practically no discussion of relevant experimental results in the book. This omission is intentional, since the scope is rather broad and inclusion of experiments could easily increase the volume by a factor. Finally, the bibliography is not meant to be exhaustive or complete. In most cases the references are to the original works where the presented results were obtained, to their immediate extensions, and to review articles. I sincerely apologize to the many authors whose works I was not able to cover.

Finally, this is an opportunity to express my deep appreciation to all of my coauthors, colleagues, and students from whom I learned a great deal about the subjects of this book. During my work on the first edition I was partially supported by National Science Foundation grant DMR-0804266.