

## Index

- amplitude, 2
- angular frequency, 3
  - harmonic oscillator, 3
  - plane waves, 169
  - standing waves, 105
- antinodes, 104
- antisymmetric matrix, 81
- arcsine
  - derivative, 137
  - relation to log, 136
- arcsinh (relation to log), 136
- area element, 155, 250–251
  - cylindrical coordinates, 251
  - spherical coordinates, 163, 251
- area integral, 155–156
- average, 194
- averaging property, 165–167
- axially symmetric, 175
  
- basis, 68, 79
  - canonical, 67
- beats, 103
- Bessel function, 19, 174, 210
  - orthogonality, 180
  - recursion relation, 174
  - spherical, 19, 171
- Bessel's equation, 173
  - Frobenius method, 173–175
- Biot–Savart law, 216
- bisection, 208
  - convergence, 210
- black hole, 12
- boundary values, 3
  - numerical, 220
  
- canonical basis, 67
- capacitor, 53
- Cartesian coordinates, 242
  - four-dimensional, 252
- center of mass, 66
- change of variables, 134
- characteristic curves, 97
- charge density, 159, 182
- closed path, 154
- closed surface, 155
- complex conjugate, 22
- complex driving force, 38
- complex impedance, 56
- complex numbers, 19–23
  - addition, 20
  - Cartesian components, 20
  - complex conjugate, 22
  - imaginary axis, 20
  - imaginary part, 19
  - multiplication, 20
  - polar components, 20
  - real axis, 20
  - real part, 19
- complex time, 26, 184, 202
- conjugate transpose, 78
- conservation law, 114–115, 185, 189
  - integral form, 114–115, 159–160
  - three-dimensional, 159–160
- conservation of energy, 7–14, 130
  - relativistic, 139
- conservation of mass, 113–114
- conservative force, 5–8
  - closed curve, 154
  - path independence, 154
  - three-dimensional, 154–155
- continuity, 32
  - violation, 126
- convective derivative, 185, 186
- cosine, 18
  - angle addition, 23
  - hyperbolic, 25
- cosine series, 44, 48
- Coulomb force, 13
- critically damped, 34–35
- cross product, 147
  - of basis vectors, 247
  - right-hand rule, 148
- cross-correlation, 64
- cross-derivative equality, 94
- curl, 150–152, 164–165
  - cylindrical coordinates, 241
  - general coordinates, 245
  - right-hand rule, 151
  - spherical coordinates, 247
- curl theorem, 160–162
- current density, 182
- curvature, 192

- curvature vector, 192
- cylindrical basis vectors, 239
- cylindrical coordinates, 172, 238–241
  - area element, 251
  - basis vectors, 239
  - curl, 241
  - divergence, 241
  - gradient, 240
  - Laplacian, 241
  - line element, 249
  - volume element, 250
- damped driven harmonic oscillator, *see* harmonic oscillator
- damped harmonic oscillator, *see* harmonic oscillator
- damping, 32–38
- dashpot, 33
- determinant, 72–73, 76, 80
- diagonalized matrix, 77
- differential, 28
- dimension, 66
- dimensionless equation, 212–214, 225, 227
  - restoring dimension, 214
- Dirac delta function, 59–60, 62–63
  - and Heaviside step function, 62
  - and Kronecker delta, 60
  - three-dimensional, 163–165
- discrete Fourier transform, 229–233
- divergence, 150–152, 163–164
  - of basis vectors, 251
  - cylindrical coordinates, 241
  - general coordinates, 245
  - spherical coordinates, 247
- divergence theorem, 156–159
- dot product, 67, 69, 71, 146
- drag, 33
- driven harmonic oscillator, *see* harmonic oscillator
- eigenfunction, 81, 224
- eigenvalue problem, 76–79, 224
  - continuous, 81, 199, 224
  - numerical solution, 227–228
- eigenvalues, 76–81, 224
- eigenvectors, 75–81
- electric field, 182, 207
- electrical circuit, 51, 53–57
  - complex impedance, 56
  - gain, 54
  - power, 54
  - Q factor, 54–55
  - resonance, 54
- electromagnetic waves, 181–184
- electrostatic force, 13
- elliptic integral
  - approximation, 129
  - first kind ( $F$ ), 133
  - second kind ( $E$ ), 129
- elliptical cylindrical coordinates, 251
- elliptical motion, 127–129
  - perimeter, 127–129
  - semi-major axis, 127
  - semi-minor axis, 127
- energy density, 95, 185, 186
- energy quantization, 200, 204
- equilibrium, 5
  - minimum, 6
  - stable, 9
  - unstable, 6, 9
- escape speed, 10–11
- ether, 182
- Euler equations, 185–187
  - approximate, 187–189
  - three-dimensional, 186–187
- Euler's formula, 20, 22
  - hyperbolic, 25
- even function, 42–44, 47–48, 177, 204
- event horizon, 12
- expectation value, 194
- exponential, 15–16, 23–26
  - characteristic time, 36
  - complex argument, 20
  - derivatives, 23
  - Euler's formula, 20
  - infinite sum, 15–16
  - integrals, 25, 40
  - matrix, 121
- exponential ansatz, 234
- fast Fourier transform, 232
- filter, 233
- finite difference, 220–224
  - matrix form, 222
- fluids, 185–189
- Fourier series, 40–51, 229
  - alternative definition, 44
  - cosine series, 44
  - discretized, 229
  - even and odd functions, 42–44
  - Gibb's phenomenon, 44–46
  - sine series, 44, 101
  - square wave, 41–44
  - triangle wave, 47
- Fourier transform, 57–64
  - cross-correlation, 64
  - Dirac delta function, 59–60
  - discrete, 229–233
  - and Fourier series, 57–58
  - Parseval's relation, 61
  - power spectrum, 61
  - real signal, 62
  - wave equation, 102–103
- frequency, 2
  - angular, 3
  - and eigenvalues, 84

- frequency (cont.)  
 beats, 103  
 filter, 233  
 harmonic oscillator, 2  
 negative, 61  
 Nyquist, 231  
 plane waves, 107, 169  
 standing waves, 105  
 Frobenius method, 14–19  
 Bessel's equation, 173–175  
 cosine and sine, 17–18  
 exponential, 15–16  
 Hermite's equation, 204–205  
 Legendre's equation, 176–177  
 recursion relation, 15  
 fundamental theorem of calculus, 136, 152, 156, 160
- gain, 54  
 Gauss's law, 14, 162  
 Gaussian, 60  
 density, 195  
 general basis vectors, 242–243  
 general coordinates, 242  
 basis vectors, 242–243  
 curl, 245  
 divergence, 245  
 gradient, 244–245  
 Laplacian, 245  
 Gibb's phenomenon, 44–46  
 gradient, 148–149, 238  
 cylindrical coordinates, 240  
 general coordinates, 244–245  
 spherical coordinates, 246  
 gravitational field, 14  
 gravity  
 force, 10  
 near surface of the earth, 186  
 potential energy, 10  
 grid, 211, 221, 224  
 ground state, 206
- harmonic function, 165–168  
 averaging property, 165–167  
 no local max/min, 167  
 uniqueness, 167–168  
 harmonic oscillator, 1–3  
 amplitude, 2  
 angular frequency, 3  
 boundary values, 3  
 critically damped, 34–35  
 damped driven, 51–55, 64, 123–126  
 damping, 32–38, 63–64, 122  
 delays, 110–111  
 dimensionless, 213–214  
 driven, 38–39, 50–51, 122–123  
 first-order form, 120–126  
 Fourier series, 50–51  
 Fourier transform, 63–64  
 frequency, 2  
 initial values, 2  
 integral solution, 123  
 kinetic friction, 31–32  
 numerical solution, 215, 223–224  
 overdamped, 34–36  
 period, 2, 35, 131–132, 140–142  
 potential energy, 130, 149  
 probability density, 197–198  
 Q factor, 36  
 quantum mechanical, 202–205  
 relativistic, 139–142, 215  
 resonance, 50, 52  
 series solution, 17–18  
 three-dimensional, 149–150  
 two-dimensional, 126–129  
 underdamped, 34–35  
 work, 155  
 heat equation, 102  
 Heaviside step function, 62  
 Hermite polynomials, 204  
 orthogonality, 206  
 Rodrigues formula, 206  
 Hermite's equation, 203–205  
 Frobenius method, 204–205  
 homogeneous problem, 29  
 hydrogen, 226–227  
 numerical solution, 229  
 hyperbolic cosine, 25  
 hyperbolic sine, 25
- identity matrix, 71  
 image compression, 79  
 impedance, 56  
 incident wave, 108  
 inductor, 53  
 infinite square well  
 classical, 195–196  
 numerical solution, 229  
 quantum mechanical, 200–202, 225–226  
 initial values, 2  
 numerical, 212  
 integration  
 area element, 155, 250–251  
 area integral, 155–156  
 change of variables, 134  
 curl theorem, 160–162  
 divergence theorem, 156–159  
 fundamental theorem of calculus, 136  
 line element, 153, 248, 249  
 line integral, 153–154  
 numerical, 215–220  
 by parts, 136–137  
 product rule, 136  
 Simpson's rule, 218  
 trapezoidal approximation, 218

- trigonometric substitution, 134–136
- volume element, 155, 250
- volume integral, 155
- integration by parts, 136–137
- inverse trigonometric functions, 135
  - derivatives, 137
- Jacobian, 242–244, 246
  - inverse, 244
- kinetic energy, 8
- kinetic friction, 31–33
- Kirchoff's voltage law, 53, 56
- Korteweg–de Vries (KdV) equation, 189
- Kronecker delta, 41
  - and Dirac delta function, 60
- Laplace equation, 102
- Laplace's equation, 165, 172–179, 184
  - cylindrical coordinates, 172–173
  - separation of variables, 173–179
- Laplacian, 165–168
  - of basis vectors, 168, 170, 251
  - cylindrical coordinates, 172–173, 241
  - general coordinates, 245
  - spherical coordinates, 175, 247
- Legendre polynomials, 177
  - orthogonality, 177–178
  - Rodrigues formula, 179
- Legendre's equation, 176
  - Frobenius method, 176–177
- light, 184
- line element, 153, 248
  - cylindrical coordinates, 249
  - spherical coordinates, 249
- line integral, 153–154
- linear combination, 67
- linear transformation, 73
- $\ln$ , 24
- local maximum, 6, 9
- local minimum, 6
- logarithm, 23–26
  - derivatives, 24
  - integrals, 25
  - natural, 24
- longitudinal, 191
- longitudinal waves, 169
- Mach angle, 112
- Mach cone, 113
- magnetic field, 182, 207, 216
  - numerical approximation, 220
- matrix, 69–73
  - addition, 70
  - antisymmetric, 81
  - determinant, 72–73, 80
  - diagonalized, 77
  - eigenvalue problem, 76–79, 224
  - eigenvalues, 76–81, 224
  - eigenvectors, 75–81
  - exponential, 121
  - identity, 71
  - inverse, 71
  - Jacobian, 242–244, 246
  - matrix multiplication, 71
  - orthogonal, 74
  - scalar multiplication, 70
  - symmetric, 77–79
  - trace, 80
  - transpose, 71, 74
  - and vectors, 69–70, 74
  - vector multiplication, 70–71
- Maxwell's equations, 162, 182
  - in vacuum, 182
- method of characteristics, 97–98
- traffic flow, 115–116
- wave equation, 114
- method of Frobenius, 14–19
  - Bessel's equation, 173–175
  - cosine and sine, 17–18
  - exponential, 15–16
  - Hermite's equation, 204–205
  - Legendre's equation, 176–177
  - recursion relation, 15
- Minkowski length, 142–143
- negative mass motion, 12
- Neumann function, 175
- Newton's second law, 211
  - relativistic, 145
- nodes, 104
- nonlinear wave equation, 190–192
  - inextensible limit, 191–192
- normal modes, 84–87
  - achieving, 85
- Nyquist frequency, 231
- O notation, 92
- odd function, 42–44, 47–48, 177, 204
- ODE
  - asymptotic solution, 203
  - continuity, 26–27, 32
  - exponential ansatz, 234
  - Fourier series, 48–51
  - Fourier transform, 63–64
  - homogeneous solution, 29–30, 38
  - numerical solution, 211–215, 220–224
  - orthogonality, 177–178, 202, 206
  - peel off, 35, 203, 205
  - plan of attack, 234–236
  - polynomial ansatz, 235
  - separation of variables, 27–28
  - sourced solution, 29–30, 38
  - superposition, 28–29

- ODE (cont.)  
 variation of parameters, 29, 30  
 vector form, 120–126  
 orthogonal matrix, 74  
 overdamped, 34–36
- Parseval's relation, 61  
 partial derivatives, 92–95  
 of basis vectors, 246  
 cross-derivative equality, 94  
 path, 153, 207  
 closed, 154  
 PDE, 96  
 method of characteristics, 97–98, 114  
 Riemann problem, 116–119  
 separation of variables, 98–101, 171, 173–179,  
 198–199  
 series solution, 101–102  
 soliton, 189  
 weak solution, 117  
 pendulum, 11–12  
 approximation, 11  
 comparison, 13  
 numerical solution, 215  
 period, 132–133  
 simple, 13  
 period, 2, 35  
 harmonic oscillator, 2, 131–132  
 pendulum, 132–133  
 plane waves, 107  
 relativistic oscillator, 140–142  
 standing waves, 104  
 periodic function, 40  
 phase, 3  
 piecewise solution, 32  
 plane waves, 107–109  
 angular frequency, 169  
 electromagnetic, 183–184  
 frequency, 107, 169  
 incident, 108  
 period, 107  
 polarization, 183, 184  
 reflected, 108  
 three-dimensional, 169  
 transmitted, 108  
 transverse, 183  
 wave number, 169  
 wave vector, 169, 183  
 wavelength, 107, 169  
 Poisson equation, 168  
 uniqueness, 168  
 polarization, 183, 184  
 polynomial ansatz, 235  
 potential, 209  
 of moving charge, 209  
 potential energy, 5  
 Coulomb, 226  
 equilibrium, 5  
 gravitational, 10  
 maximum, 9  
 spherically symmetric, 130  
 Yukawa, 12  
 power method, 227–229  
 power spectrum, 61, 86  
 pressure, 186  
 probability density, 193–198  
 constant, 195–196  
 Gaussian, 195  
 harmonic oscillator, 197–198  
 infinite square well, 195–196  
 normalization, 194  
 quantum mechanical harmonic oscillator, 205  
 quantum mechanical infinite square well, 201  
 statistical interpretation, 196  
 time average, 197–198  
 time independent, 199  
 product log, 111, 210  
 product rule, 136  
 prolate spheroidal coordinates, 251  
 proper time, 143  
 Q factor, 36, 54–55  
 quadratic interpolation, 218  
 radius of curvature, 192  
 recursion relation, 15, 17  
 asymptotic, 204, 206  
 reflected wave, 108  
 reflection coefficient, 109  
 relativistic length, 142–143  
 relativistic oscillator, 139–142  
 numerical solution, 215  
 period, 140–142  
 residuals, 215  
 resistor, 53  
 resonance, 50, 52, 54  
 resonance curve, 54  
 rest frame, 143  
 retarded time, 207  
 implicit equation, 208  
 Riemann problem, 116–119  
 right-hand rule  
 cross product, 148  
 curl, 151  
 Rodrigues formula  
 Hermite polynomials, 206  
 Legendre polynomials, 179  
 root finding problem, 207  
 rotation, 74, 80  
 Schrödinger's equation, 103–104, 193, 198–205  
 allowed energies, 200  
 boundary conditions, 199  
 dimensionless, 227

- for hydrogen, 227
- harmonic oscillator, 202–205
- infinite square well, 200–202
- numerical solution, 224
- orthogonality, 202
- separation of variables, 198–199
- superposition, 201
- time independent, 199
- self force, 37
- semi-major axis, 127
- semi-minor axis, 127
- separation of variables, 27–28, 98–101
  - additive, 98–99
  - Laplace's equation, 173–179
  - logic, 99
  - multiplicative, 99–101
  - Schrödinger's equation, 198–199
- series expansion, 14–19
  - Bessel's equation, 173–175
  - cosine and sine, 17–18
  - exponential, 15–16
  - Hermite's equation, 204–205
  - Legendre's equation, 176–177
  - recursion relation, 15
- shallow water equations, 187–189
  - linearized, 189
  - one-dimensional, 187–189
  - two-dimensional, 189
- shocks
  - Riemann problem, 116–119
  - traffic flow, 116
  - wave equation, 112
- simple pendulum, 13
- Simpson's rule, 218
- sine, 18
  - angle addition, 23
  - hyperbolic, 25
  - integral, 46
- sine integral, 58
  - numerical approximation, 219
- sine series, 44, 48, 101
- Slinky®, 92
- small angle approximation, 96
- soliton, 189
- sonic boom, 112
- spacetime coordinates, 143
- speed of light ( $c$ ), 182
- spherical basis vectors, 246
- spherical Bessel function, 19, 171
- spherical coordinates, 130, 166, 245–248
  - area element, 251
  - basis vectors, 246
  - curl, 247
  - divergence, 247
  - gradient, 246
  - Laplacian, 247
  - line element, 249
  - volume element, 250
- springs in series, 91, 94
- square wave, 41–44, 48, 124
  - Gibb's phenomenon, 45
- stable equilibrium, 9
- standard deviation, 194
- standing waves, 104–106
  - angular frequency, 105
  - antinode, 104
  - frequency, 105
  - nodes, 104
  - period, 104
  - wavelength, 104
- stationary states, 199
- steady state solution, 52
- superposition, 28–29, 94–95, 201
- surface area, 138
- surface of revolution, 138
- symmetric matrix, 77–79
- Taylor expansion, 3–5, 40
  - vector, 157
- time independent Schrödinger's equation, 199
- trace, 80
- traffic flow, 115–119
  - Riemann problem, 116–119
  - shocks, 116
- transient solution, 52, 124
- transmission coefficient, 109
- transmitted wave, 108
- transpose, 71
- transverse, 183, 191
- transverse waves, 169
- trapezoidal approximation, 218
- triangle wave, 47
- trigonometric substitution, 134–136
- twin paradox, 143–144
- underdamped, 34–35
- uniform circular motion, 111, 126
- uniqueness, 167–168
- unit normal, 155
- unit tangent vector, 190
- unstable equilibrium, 6, 9
- variance, 194
- variation of parameters, 29, 30
- vector, 66–69, 146–148
  - addition, 66
  - area element, 155
  - area for a sphere, 156
  - basis, 68, 79
  - canonical basis, 67
  - column, 71
  - complete set, 67
  - conjugate transpose, 78
  - cross product, 147

- vector (cont.)
  - curl, 150–152, 241, 245, 247
  - curl theorem, 160–162
  - dimension, 66
  - divergence, 150–152, 241, 245, 247
  - divergence theorem, 156–159
  - dot product, 67, 69, 71, 146
  - eigenvectors, 75–81
  - gradient, 148–149, 238, 240, 244–246
  - length, 67, 147
  - matrix multiplication, 70–71
  - normalized, 67
  - orthogonal, 67
  - path, 153, 207
  - projection, 68
  - row, 71
  - scalar multiplication, 66
  - tangent, 153
  - Taylor expansion, 157
  - transpose, 71
  - unit normal, 155
  - wave equation, 170
- Verlet method, 212
- volume element, 155, 250
  - cylindrical coordinates, 250
  - spherical coordinates, 250
- volume integral, 155
  
- wave equation
  - from balls and springs, 91–92
  - changing medium, 108
  - continuity, 109
  - electromagnetic, 181–182
  - energy density, 95
  - Euler equations, 185–187
  - extensible string, 190–192
  - finite propagation speed, 110
  - Fourier transform, 102–103
  - general solution, 97–98, 184
  - inextensible string, 95–96
  - from mass conservation, 113–114
  - method of characteristics, 97–98
  - plane waves, 107–109
  - Schrödinger's equation, 193, 198–199
  - separation of variables, 98–101, 171
  - series solution, 101–102
  - shallow water, 187–189
  - shocks, 112
  - spherically symmetric, 170–171
  - standing waves, 104–106
  - superposition, 94–95, 107
  - three-dimensional, 168–171
  - varying speed, 113–114
  - vector, 170
  - wave number, 169
  - wave vector, 169, 183
  - wavelength
    - plane waves, 107, 169
    - standing waves, 104
  - weak solution, 117
  - work, 153
    - closed curve, 154
- Young's modulus, 92
- Yukawa potential, 12