

Contents

<i>Introduction</i>	<i>page</i>	<i>x</i>
I.1 Preface		x
I.2 Acknowledgments		xii
I.3 The Short History of the Bellman Function		xii
I.4 The Plan of the Book		xv
I.5 Notation		xvi
1 Examples of Exact Bellman Functions		1
1.1 A Toy Problem		1
1.2 Buckley Inequality		7
1.3 John–Nirenberg Inequality		14
1.4 Homogeneous Monge–Ampère Equation		28
1.5 Bellman Function for General Integral Functionals on BMO		30
1.6 Dyadic Maximal Operator		43
1.7 Weak Estimate of the Martingale Transform		62
1.8 Burkholder’s Bellman Function		80
1.9 On the Bellman Function Generated by a Weak Type Estimate of a Square Function		113
1.10 More about Buckley’s Inequality		137
1.11 Hints and Answers		144
2 What You Always Wanted to Know about Stochastic Optimal Control, but Were Afraid to Ask		150
2.1 Disclaimer		150
2.2 Stochastic Integrals Are Not That Simple		150
2.3 Itô’s Definition of Stochastic Integral		153
2.4 Stochastic Differential and Itô’s Formula		155

viii	<i>Contents</i>	
2.5	Bellman Functions of Stochastic Optimal Control Problems and Bellman PDEs	156
2.6	Almost Perfect Analogy between Stochastic Optimal Control and Harmonic Analysis: Functions of One Variable	164
2.7	Almost Perfect Analogy between Stochastic Optimal Control and Harmonic Analysis: Functions on the Complex Plane	182
2.8	A Problem of Gohberg–Krupnik from the Point of View of Stochastic Optimal Control	195
3	Conformal Martingale Models: Stochastic and Classical Ahlfors–Beurling Operators	206
3.1	Estimates of Subordinated Martingales	206
3.2	Conformal Martingales and the Ahlfors–Beurling Transform	208
3.3	Proof of Theorem 3.2.3: Right-Hand Side Conformality, $2 < p < \infty$	210
3.4	Proof of Theorem 3.2.3: Left-Hand Side Conformality, $1 < p < 2$	226
3.5	Burkholder, Bellman, and Ahlfors–Beurling Operator in L^p for Large p	232
4	Dyadic Models: Application of Bellman Technique to Upper Estimates of Singular Integrals	247
4.1	Dyadic Shifts and Calderón–Zygmund Operators	247
4.2	Sharp Weighted Estimate of Dyadic Shifts	266
4.3	Universal Sufficient Condition: Boundedness of All Calderón–Zygmund Operators in Two Different Weighted Spaces	292
5	Application of Bellman Technique to the Endpoint Estimates of Singular Integrals	334
5.1	Endpoint Estimate	334
5.2	The Bellman Function of Weak Weighted Estimate of the Martingale Transform and Its Logarithmic Blow-Up	337
5.3	Sharp Weak Weighted Estimate for the Martingale Transform	353
5.4	Obstacle Problems for Unweighted Square Function Operator: Burkholder–Gundy–Davis Function	367
5.5	Bollobás Function	383
5.6	The Weak Norm of the Square Function	395
5.7	Saturation of Estimates by Extremal Sequences	398
5.8	An Obstacle Problem Associated with the Chang–Wilson–Wolff Theorem	400
5.9	Strong Weighted Estimate of the Square Function	402

	<i>Contents</i>	ix
5.10	Weak Weighted Estimates for the Square Function	411
5.11	Restricted Weak Weighted Estimate of the Square Function	432
	<i>References</i>	434
	<i>Index</i>	443