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Cambridge University Press 978-1-108-48689-7 — The Bellman Function Technique in Harmonic Analysis Vasily Vasyunin , Alexander Volberg Table of Contents <u>More Information</u>

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