1 INTRODUCTION

1.1 MULTIBODY SYSTEMS

The primary purpose of this book is to develop methods for the dynamic analysis of *multibody systems* (MBS) that consist of interconnected *rigid* and *deformable* components. In that sense, the objective may be considered as a generalization of methods of structural and rigid body analysis. Many mechanical and structural systems such as vehicles, space structures, robotics, mechanisms, and aircraft consist of interconnected components that undergo large translational and rotational displacements. Figure 1.1 shows examples of such systems that can be modeled as multibody systems. In general, a multibody system is defined to be a collection of subsystems called *bodies*, *components*, or *substructures*. The motion of the subsystems is kinematically constrained because of different types of joints, and each subsystem or component may undergo large translations and rotational displacements.

Basic to any presentation of MBS mechanics is the understanding of the motion of subsystems (bodies or components). The motion of material bodies formed the subject of some of the earliest researches pursued in three different fields, namely, *rigid body mechanics*, *structural mechanics*, and *continuum mechanics*. The term *rigid body* implies that the deformation of the body under consideration is assumed small such that the body deformation has no effect on the gross body motion. Hence, for a rigid body, the distance between any two of its particles remains constant at all times and all configurations. The motion of a rigid body in space can be completely described by using six generalized coordinates. However, the resulting mathematical model in general is highly nonlinear because of the large body rotation. On the other hand, the term *structural mechanics* has come into wide use to denote the branch of study in which the deformation is the main concern. Large body rotations are not allowed, thus resulting in inertia-invariant structures. In many applications, however, a large number of elastic coordinates have to be included in the mathematical model in order to accurately describe the body deformation. From the study of these two

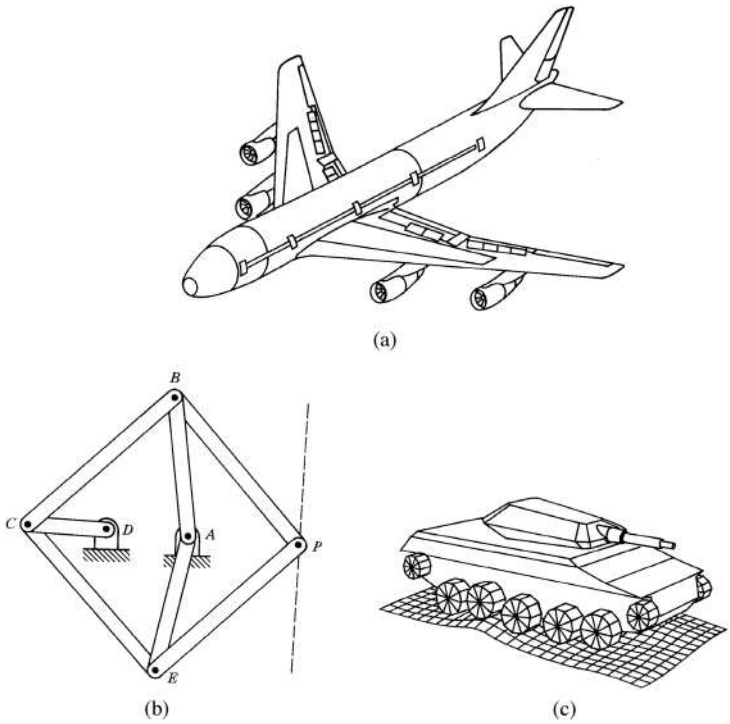

Figure 1.1 Mechanical and structural systems.

subjects, rigid body and structural mechanics, there has evolved the vast field known as *continuum mechanics*, wherein the general body motion is considered, resulting in a mathematical model that has the disadvantages of the previous cases, mainly nonlinearity and large dimensionality. This constitutes many computational problems that will be addressed in subsequent chapters.

In recent years, greater emphasis has been placed on the design of high-speed, lightweight, precision systems. Generally these systems incorporate various types of driving, sensing, and controlling devices working together to achieve specified performance requirements under different loading conditions. The design and performance analysis of such systems can be greatly enhanced through transient dynamic simulations, provided all significant effects can be incorporated into the mathematical model. The need for a better design, in addition to the fact that many mechanical and structural systems operate in hostile environments, has made necessary the inclusion of many factors that have been ignored in the past. Systems such as engines, robotics, machine tools, and space structures may operate at high speeds and in high-temperature environments. The neglect of the deformation effect, for example, when these systems are analyzed leads to a mathematical model that poorly represents the actual system.

Consider, for instance, the Peaucellier mechanism shown in Fig. 1.1(b), which is designed to generate a straight-line path. The geometry of this mechanism is such that $BC = BP = EC = EP$ and $AB = AE$. Points A, C, and P should always lie on a

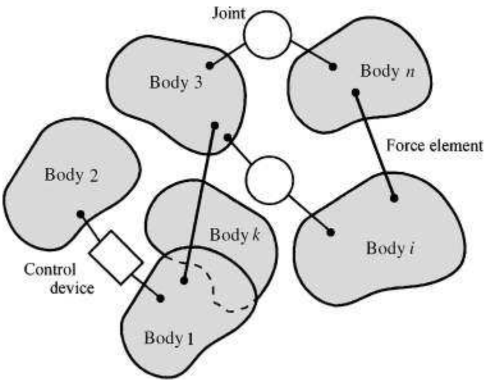

Figure 1.2 Multibody systems.

straight line passing through A. The mechanism always satisfies the condition $AC \times AP = c$, where c is a constant called the *inversion constant*. In case $AD = CD$, point C must trace a circular arc and point P should follow an exact straight line. However, this will not be the case when the deformation of the links is considered. If the flexibility of links has to be considered in this specific example, the mechanism can be modeled as a multibody system consisting of interconnected rigid and deformable components, each of which may undergo finite rotations. The connectivity between different components of this mechanism can be described by using revolute joints (turning pairs). This mechanism and other examples shown in Fig. 1.1, which have different numbers of bodies and different types of mechanical joints, are examples of mechanical and structural systems that can be viewed as a multibody system shown in the abstract drawing in Fig. 1.2. In this book, computer-based techniques for the dynamic analysis of general multibody systems containing interconnected sets of rigid and deformable bodies will be developed. To this end, methods for the kinematics and dynamics of rigid and deformable bodies that experience large translational and rotational displacements will be presented in the following chapters. In the following sections of this chapter, however, some of the basic concepts that will be subject of detailed analysis in the chapters that follow are briefly discussed.

1.2 REFERENCE FRAMES

The configuration of a multibody system can be described using measurable quantities such as displacements, velocities, and accelerations. These are vector quantities that have to be measured with respect to a proper *frame of reference* or *coordinate system*. In this text, the term *frame of reference*, which can be represented by three orthogonal axes that are rigidly connected at a point called the *origin* of this reference, will be frequently used. Figure 1.3 shows a frame of reference that consists of the three orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$. A vector $\mathbf{u}$ in this coordinate system can be defined by three components u_1, u_2, and u_3, along the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively. The vector $\mathbf{u}$ can then be written in terms of its components as $\mathbf{u} = [u_1 \ u_2 \ u_3]^{\mathrm{T}}$, or as $\mathbf{u} = u_1\mathbf{i}_1 + u_2\mathbf{i}_2 + u_3\mathbf{i}_3$, where $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along

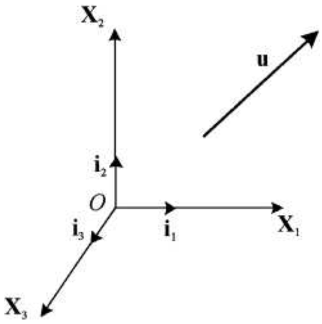

Figure 1.3 Reference frame.

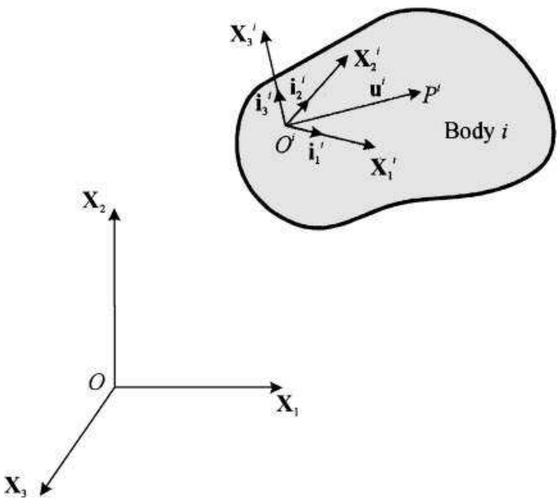

Figure 1.4 Body coordinate system.

the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively. Without loss of generality, one can assume $\mathbf{i}_1 = [1\ \ 0\ \ 0]^{\mathrm{T}}$, $\mathbf{i}_2 = [0\ \ \ 1\ \ \ 0]^{\mathrm{T}}$, and $\mathbf{i}_3 = [0\ \ \ 0\ \ \ 1]^{\mathrm{T}}$.

Generally, in dealing with multibody systems two types of coordinate systems are required. The first is a coordinate system that is fixed in time and represents a unique standard for all bodies in the system. This coordinate system will be referred to as *global*, or *inertial frame* of reference. In addition to this inertial frame of reference, a *body reference* is assigned to each component in the system. This body reference translates and rotates with the body; therefore, its location and orientation with respect to the inertial frame change with time. Figure 1.4 shows a typical body, denoted as body i in the multibody system. The coordinate system $\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3$ is the global inertial frame of reference, and the coordinate system $\mathbf{X}_1^i\mathbf{X}_2^i\mathbf{X}_3^i$ is the body coordinate system. Let $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ be unit vectors along the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively, and let $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ be unit vectors along the body axes $\mathbf{X}_1^i$, $\mathbf{X}_2^i$, and $\mathbf{X}_3^i$, respectively. The unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are fixed in time; that is, they have constant magnitude and direction, while the unit vectors $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ have changeable orientations. A vector $\mathbf{u}^i$ can be defined in the body coordinate system or the global coordinate system as

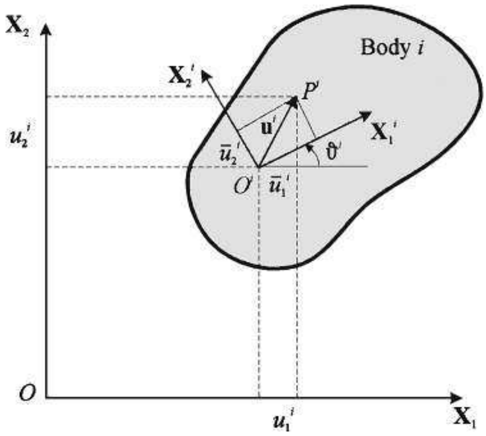

Figure 1.5 Planar motion.

$$\mathbf{u}^i = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i + \bar{u}_3^i \mathbf{i}_3^i = u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2 + u_3^i \mathbf{i}_3 \tag{1.1}$$

where $\bar{u}_1^i$, $\bar{u}_2^i$, and $\bar{u}_3^i$ are the components of the vector $\mathbf{u}^i$ in the local body coordinate system; and u_1^i, u_2^i, and u_3^i are the components of the same vector $\mathbf{u}^i$ in the global coordinate system. One, therefore, can have the two different representations of Eq. 1 for the same vector $\mathbf{u}^i$, one in terms of the body coordinates and the other in terms of global coordinates. Since it is easier to define the vector in terms of the local body coordinates, it is useful to have relationships between the local and global components. Such relationships can be obtained by developing the transformation between the local and global coordinate systems. For instance, consider the *planar motion* of the body shown in Fig. 1.5. The coordinate system $\mathbf{X}_1\mathbf{X}_2$ represents the inertial frame and $\mathbf{X}_1^i\mathbf{X}_2^i$ is the body coordinate system. Let $\mathbf{i}_1$ and $\mathbf{i}_2$ be unit vectors along the $\mathbf{X}_1$ and $\mathbf{X}_2$ axes, respectively, and let $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$ be unit vectors along the body axes $\mathbf{X}_1^i$ and $\mathbf{X}_2^i$, respectively. The orientation of the body coordinate system with respect to the global frame of reference is defined by the angle θ^i. Since $\mathbf{i}_1^i$ is a unit vector, its component along the $\mathbf{X}_1$ axis is $\cos\theta^i$, while its component along the $\mathbf{X}_2$ axis is $\sin\theta^i$. Assuming $\mathbf{i}_1 = [1 \quad 0]^{\mathrm{T}}$ and $\mathbf{i}_2 = [0 \quad 1]^{\mathrm{T}}$, one can then write the unit vectors $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$ in the global coordinate system as

$$\left.\begin{aligned} \mathbf{i}_1^i &= \cos\theta^i \mathbf{i}_1 + \sin\theta^i \mathbf{i}_2 = \begin{bmatrix}\cos\theta^i & \sin\theta^i\end{bmatrix}^{\mathrm{T}} \\ \mathbf{i}_2^i &= -\sin\theta^i \mathbf{i}_1 + \cos\theta^i \mathbf{i}_2 = \begin{bmatrix}-\sin\theta^i & \cos\theta^i\end{bmatrix}^{\mathrm{T}} \end{aligned}\right\} \tag{1.2}$$

The vector $\mathbf{u}^i$ is defined in the body coordinate system as $\mathbf{u}^i = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i$, where $\bar{u}_1^i$ and $\bar{u}_2^i$ are the components of the vector $\mathbf{u}^i$ in the body coordinate system. Using the expressions for $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$, one can write

$$\begin{aligned} \mathbf{u}^i = u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2 &= \begin{bmatrix} u_1^i \\ u_2^i \end{bmatrix} = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i = \begin{bmatrix} \mathbf{i}_1^i & \mathbf{i}_2^i \end{bmatrix} \begin{bmatrix} \bar{u}_1^i \\ \bar{u}_2^i \end{bmatrix} \\ &= \begin{bmatrix} \cos\theta^i & -\sin\theta^i \\ \sin\theta^i & \cos\theta^i \end{bmatrix} \begin{bmatrix} \bar{u}_1^i \\ \bar{u}_2^i \end{bmatrix} = \mathbf{A}^i \bar{\mathbf{u}}^i \end{aligned} \tag{1.3}$$

where u_1^i and u_2^i are the components of the vector $\mathbf{u}^i$ defined in the global coordinate system and given, respectively, by $u_1^i = \bar{u}_1^i \cos\theta^i - \bar{u}_2^i \sin\theta^i$, and $u_2^i = \bar{u}_1^i \sin\theta^i + \bar{u}_2^i \cos\theta^i$. Equation 3 shows the algebraic relationships between the local and global components in the planar analysis expressed in a matrix form as $\mathbf{u}^i = \mathbf{A}^i \bar{\mathbf{u}}^i$, where $\mathbf{u}^i = \left[u_1^i \ u_2^i\right]^{\mathrm{T}}$, $\bar{\mathbf{u}}^i = \left[\bar{u}_1^i \ \bar{u}_2^i\right]^{\mathrm{T}}$, and $\mathbf{A}^i$ is the planar transformation matrix defined as

$$\mathbf{A}^i = \begin{bmatrix} \cos\theta^i & -\sin\theta^i \\ \sin\theta^i & \cos\theta^i \end{bmatrix} \tag{1.4}$$

It is clear from this equation that the columns of the transformation matrix $\mathbf{A}^i$ are unit vectors ($\mathbf{i}_1^i$ and $\mathbf{i}_2^i$) along the axes of the body coordinate system. The planar transformation matrix $\mathbf{A}^i$ is an orthogonal matrix, that is, $\mathbf{A}^i\mathbf{A}^{i\mathrm{T}} = \mathbf{A}^{i\mathrm{T}}\mathbf{A}^i = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. In Chapter 2, the spatial kinematics will be discussed, and the spatial transformation matrix and its important properties will be examined.

1.3 PARTICLE MECHANICS

Dynamics in general is the science of studying the motion of particles or bodies. The subject of dynamics can be divided into two major branches, *kinematics* and *kinetics*. In the kinematic analysis, we study the motion regardless of the forces that cause this motion, while kinetics deals with the motion and forces that produce it. Therefore, in kinematics attention is focused on the geometric aspects of motion. The objective is, then, to determine the positions, velocities, and accelerations of the system under investigation. In order to understand the dynamics of multibody systems containing rigid and deformable bodies, it is important to understand first the body dynamics. To this end, a brief discussion on the dynamics of particles that form the rigid and deformable bodies is first presented.

Particle Kinematics A *particle* is assumed to have no dimensions and accordingly can be treated as a point in a three-dimensional space. Therefore, in studying the kinematics of particles, one is concerned primarily with the translation of a point with respect to a selected frame of reference. The position of the particle can then be defined using three coordinates. Figure 1.6 shows a particle p in a

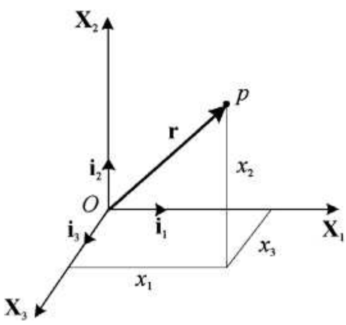

Figure 1.6 Position vector of the particle p.

three-dimensional space. The position, velocity, and acceleration vectors **r**, **v**, and **a** of this particle can be written, respectively, as

$$\left.\begin{aligned} \mathbf{r} &= x_1\mathbf{i}_1 + x_2\mathbf{i}_2 + x_3\mathbf{i}_3 = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}} \\ \mathbf{v} &= \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \dot{x}_1\mathbf{i}_1 + \dot{x}_2\mathbf{i}_2 + \dot{x}_3\mathbf{i}_3 = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3]^{\mathrm{T}} \\ \mathbf{a} &= \frac{d\mathbf{v}}{dt} = \ddot{x}_1\mathbf{i}_1 + \ddot{x}_2\mathbf{i}_2 + \ddot{x}_3\mathbf{i}_3 = [\ddot{x}_1 \quad \ddot{x}_2 \quad \ddot{x}_3]^{\mathrm{T}} \end{aligned}\right\} \tag{1.5}$$

where $(\dot{\ })$ denotes differentiation with respect to time; $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along the $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ axes; x_1, x_2, and x_3 are the Cartesian coordinates of the particle; $\dot{x}_1, \dot{x}_2$, and $\dot{x}_3$ are the Cartesian components of the velocity vector; and $\ddot{x}_1, \ddot{x}_2$, and $\ddot{x}_3$ are the Cartesian components of the acceleration vector. In the above equation, the velocity of the particle is defined to be the time derivative of the position vector with the assumption that the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ are fixed in time such that the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ have a constant magnitude and direction.

Choice of Coordinates The set of coordinates that can be used to define the particle position is not unique. In addition to the Cartesian representation, other sets of coordinates can be used for the same purpose. In Fig. 1.7, the position of particle p can be defined using the three *cylindrical coordinates*, r, ϕ, and z, while in Fig. 1.8, the particle position is identified using the *spherical coordinates* r, θ, and ϕ. In many situations, however, it is useful to obtain kinematic relationships between different

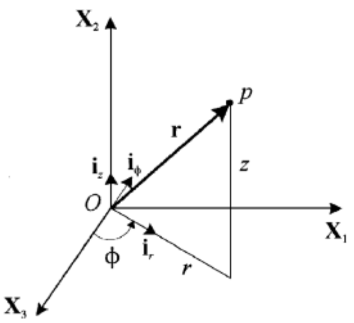

Figure 1.7 Cylindrical coordinates.

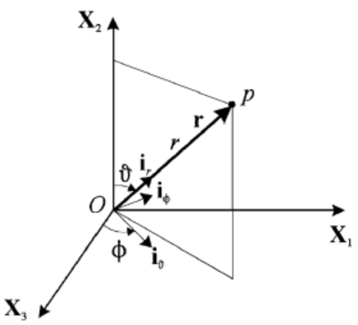

Figure 1.8 Spherical coordinates.

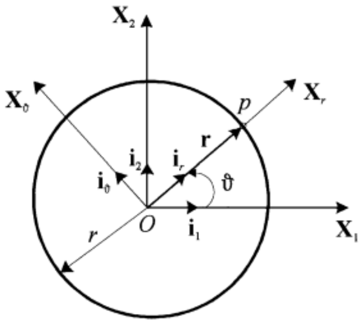

Figure 1.9 Circular motion of a particle.

sets of coordinates. For instance, if one considers the planar motion of a particle p in a circular path as shown in Fig. 1.9, the position vector of the particle can be written in the fixed coordinate system $\mathbf{X}_1\mathbf{X}_2$ as $\mathbf{r} = [x_1 \ x_2]^{\mathrm{T}} = x_1\mathbf{i}_1 + x_2\mathbf{i}_2$, where x_1 and x_2 are the coordinates of the particle, and $\mathbf{i}_1$ and $\mathbf{i}_2$ are unit vectors along the fixed axes $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. In terms of the polar coordinates r and θ, the components x_1 and x_2 are given by $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, and the position, velocity, and acceleration vectors $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$ can be written, respectively, as

$$\left.\begin{aligned}
\mathbf{r} &= r\cos\theta\,\mathbf{i}_1 + r\sin\theta\,\mathbf{i}_2 = r[\cos\theta \quad \sin\theta]^{\mathrm{T}} \\
\mathbf{v} &= \frac{d\mathbf{r}}{dt} = r\dot{\theta}(-\sin\theta\mathbf{i}_1 + \cos\theta\mathbf{i}_2) = r\dot{\theta}[-\sin\theta \quad \cos\theta]^{\mathrm{T}} \\
\mathbf{a} &= \frac{d\mathbf{v}}{dt} = r\ddot{\theta}(-\sin\theta\mathbf{i}_1 + \cos\theta\mathbf{i}_2) + r\left(\dot{\theta}\right)^2(-\cos\theta\mathbf{i}_1 - \sin\theta\mathbf{i}_2) \\
&= r\left[-\left(\ddot{\theta}\sin\theta + \left(\dot{\theta}\right)^2\cos\theta\right) \quad \left(\ddot{\theta}\cos\theta - \left(\dot{\theta}\right)^2\sin\theta\right)\right]^{\mathrm{T}}
\end{aligned}\right\} \tag{1.6}$$

These kinematic equations are obtained with the assumption that r in this example is constant, and $\mathbf{i}_1$ and $\mathbf{i}_2$ are fixed vectors. One can verify that the acceleration vector $\mathbf{a}$ in this equation can be written as $\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v}$, where $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are, respectively, the vectors $\boldsymbol{\omega} = \dot{\theta}\mathbf{i}_3$, and $\boldsymbol{\alpha} = \ddot{\theta}\mathbf{i}_3$.

One may also define the position vector of p in the moving coordinate system $\mathbf{X}_r\mathbf{X}_\theta$. Let, as shown in Fig. 1.9, $\mathbf{i}_r$ and $\mathbf{i}_\theta$ be unit vectors along the axes $\mathbf{X}_r$ and $\mathbf{X}_\theta$, respectively. It can be verified that these two unit vectors can be written in terms of the unit vectors along the fixed axes as $\mathbf{i}_r = \cos\theta\mathbf{i}_1 + \sin\theta\mathbf{i}_2$, $\mathbf{i}_\theta = -\sin\theta\mathbf{i}_1 + \cos\theta\mathbf{i}_2$ and their time derivatives can be written as

$$\left.\begin{aligned}
\dot{\mathbf{i}}_r &= \frac{d\mathbf{i}_r}{dt} = -\dot{\theta}\sin\theta\mathbf{i}_1 + \dot{\theta}\cos\theta\mathbf{i}_2 = \dot{\theta}\mathbf{i}_\theta \\
\dot{\mathbf{i}}_\theta &= \frac{d\mathbf{i}_\theta}{dt} = -\dot{\theta}\cos\theta\mathbf{i}_1 - \dot{\theta}\sin\theta\mathbf{i}_2 = -\dot{\theta}\mathbf{i}_r
\end{aligned}\right\} \tag{1.7}$$

The position vector of the particle in the moving coordinate system can be defined as $\mathbf{r} = r\mathbf{i}_r$. Using this equation, the velocity vector of particle p is given by $\mathbf{v} = d\mathbf{r}/dt = (dr/dt)\mathbf{i}_r + r(d\mathbf{i}_r/dt)$. Since the motion of point p is in a circular path, $dr/dt = 0$, and the velocity and acceleration vectors $\mathbf{v}$ and $\mathbf{a}$ reduce to

$$\left.\begin{aligned}\mathbf{v} &= r\frac{d\mathbf{i}_r}{dt} = r\dot{\theta}\mathbf{i}_\theta \\ \mathbf{a} &= \frac{d\mathbf{v}}{dt} = r\ddot{\theta}\mathbf{i}_\theta + r\dot{\theta}\frac{d\mathbf{i}_\theta}{dt} = r\ddot{\theta}\mathbf{i}_\theta - r\left(\dot{\theta}\right)^2\mathbf{i}_r\end{aligned}\right\} \tag{1.8}$$

which shows that the velocity vector of the particle is always tangent to the circular path. The first term, $r\ddot{\theta}$, in the acceleration vector **a** is called the *tangential component* of the acceleration, while the second term, $-r\left(\dot{\theta}\right)^2$, is called the *normal component*.

Particle Dynamics The study of *Newtonian mechanics* is based on Newton's three laws, which are used to study particle mechanics. *Newton's first law* states that a particle remains in its state of rest, or of uniform motion in a straight line if there are no forces acting on the particle. This means that the particle can be accelerated if and only if there is a force acting on the particle. *Newton's third law*, which is sometimes called the *law of action and reaction*, states that to every action there is an equal and opposite reaction; that is, when two particles exert forces on one another, these forces will be equal in magnitude and opposite in direction. *Newton's second law*, which is called the *law of motion*, states that the force that acts on a particle and causes its motion is equal to the rate of change of momentum of the particle, that is, $\mathbf{F} = \dot{\mathbf{P}}$ where **F** is the vector of forces acting on the particle, and **P** is the linear momentum of the particle, which can be written as $\mathbf{P} = m\mathbf{v}$, where m is the mass, and **v** is the velocity vector of the particle. It follows that $\mathbf{F} = d(m\mathbf{v})/dt$. In nonrelativistic mechanics, the mass m is constant and as a consequence, one has

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \tag{1.9}$$

where **a** is the acceleration vector of the particle. Equation 9 is a vector equation that has the three scalar components $F_1 = ma_1$, $F_2 = ma_2$, $F_3 = ma_3$, where F_1, F_2, and F_3 and a_1, a_2, and a_3 are, respectively, the components of the vectors **F** and **a** defined in the global coordinate system. The vector $m\mathbf{a}$ is sometimes called the *inertia* or the *effective* force vector.

1.4 RIGID BODY MECHANICS

Unlike particles, *rigid bodies* have distributed masses. The configuration of a rigid body in space can be identified by using six coordinates. Three coordinates describe the body translation, and three coordinates define the orientation of the body. Figure 1.10 shows a rigid body denoted as body i in a three-dimensional space. Let $\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3$ be a coordinate system that is fixed in time, and let $\mathbf{X}_1^i\mathbf{X}_2^i\mathbf{X}_3^i$ be a body coordinate system or body reference whose origin is rigidly attached to a point on the rigid body. The global position of an arbitrary point P^i on the body can be defined as

$$\mathbf{r}^i = \mathbf{R}^i + \mathbf{u}^i = \mathbf{R}^i + \mathbf{A}^i\bar{\mathbf{u}}^i \tag{1.10}$$

where $\mathbf{r}^i = \left[r_1^i \; r_2^i \; r_3^i\right]^{\mathrm{T}}$ is the global position of point P^i, $\mathbf{R}^i = \left[R_1^i \; R_2^i \; R_3^i\right]^{\mathrm{T}}$ is the global position vector of the origin O^i of the body reference, $\mathbf{A}^i$ is the transformation matrix that defines the body orientation, $\bar{\mathbf{u}}^i = \left[\bar{u}_1^i \; \bar{u}_2^i \; \bar{u}_3^i\right]^{\mathrm{T}}$ is the position vector of

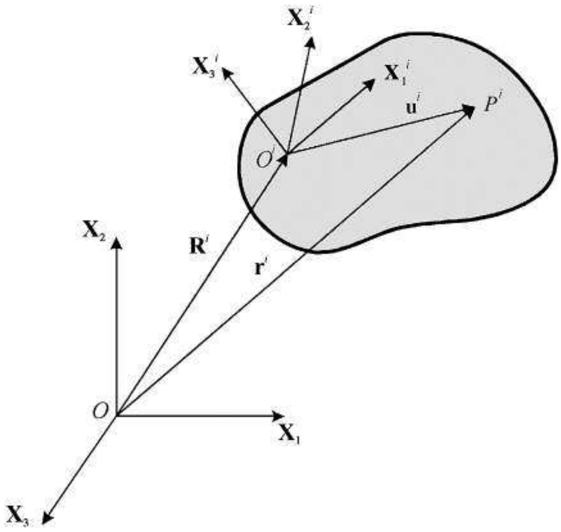

Figure 1.10 Rigid body mechanics.

the point defined in the body coordinate system, and $\mathbf{u}^i = \mathbf{A}^i\bar{\mathbf{u}}^i = \begin{bmatrix} u_1^i & u_2^i & u_3^i \end{bmatrix}^{\mathrm{T}}$ is the position vector of point P^i with respect to O^i. Since an assumption is made that the body is rigid, the distance between points P^i and O^i remains constant during the motion of the body; that is, the components of the vector $\mathbf{u}^i$ in the body coordinate system are known and constant. The vectors $\mathbf{r}^i$ and $\mathbf{R}^i$, however, are defined in the global coordinate system; therefore, it is important to be able to express the vector $\mathbf{u}^i$ in terms of its components along the fixed global axes. To this end, one needs to define the orientation of the body coordinate system with respect to the global frame of reference. A transformation between these two coordinate systems can be developed in terms of a set of rotational coordinates. However, this set of rotational coordinates is not unique, and many representations can be found in the literature. In Chapter 2, the transformation matrix that can be used to transform vectors defined in the body coordinate systems to vectors defined in the global coordinate system and vice versa is developed. Also some of the most commonly used orientation coordinates such as *Euler angles*, *Euler parameters*, *Rodriguez parameters*, and the *direction cosines* are introduced. In some of these representations, more than three orientational coordinates are used. In such cases, the orientation coordinates are not totally independent, since they are related by a set of algebraic equations.

Since Eq. 10 describes the global position of an arbitrary point on the body, the body configuration can be completely defined, provided the components of the vectors in the right-hand side of this equation are known. This equation implies that the general motion of a rigid body is equivalent to a translation of one point, say, O^i, plus a rotation. A rigid body is said to experience pure *translation* if the displacements of any two points on the body are the same. A rigid body is said to experience a pure *rotation* about an axis called the *axis of rotation*, if the particles forming the rigid body move in parallel planes along circles centered on the same axis. Figure 1.11 shows the translational and rotational motion of a rigid body. It is clear from