CAMBRIDGE TRACTS IN MATHEMATICS

219 Operator Analysis

CAMBRIDGE TRACTS IN MATHEMATICS

A complete list of books in the series can be found at www.cambridge.org/mathematics.

Recent titles include the following:

183. *Period Domains over Finite and p-adic Fields*. By J.-F. Dat, S. Orlik, and M. Rapoport
184. *Algebraic Theories*. By J. Adámek, J. Rosický, and E. M. Vitale
185. *Rigidity in Higher Rank Abelian Group Actions I: Introduction and Cocycle Problem*. By A. Katok and V. Niţică
186. *Dimensions, Embeddings, and Attractors*. By J. C. Robinson
187. *Convexity: An Analytic Viewpoint*. By B. Simon
188. *Modern Approaches to the Invariant Subspace Problem*. By I. Chalendar and J. R. Partington
189. *Nonlinear Perron–Frobenius Theory*. By B. Lemmens and R. Nussbaum
190. *Jordan Structures in Geometry and Analysis*. By C.-H. Chu
191. *Malliavin Calculus for Lévy Processes and Infinite-Dimensional Brownian Motion*. By H. Osswald
192. *Normal Approximations with Malliavin Calculus*. By I. Nourdin and G. Peccati
193. *Distribution Modulo One and Diophantine Approximation*. By Y. Bugeaud
194. *Mathematics of Two-Dimensional Turbulence*. By S. Kuksin and A. Shirikyan
195. *A Universal Construction for Groups Acting Freely on Real Trees*. By I. Chiswell and T. Müller
196. *The Theory of Hardy's Z-Function*. By A. Ivić
197. *Induced Representations of Locally Compact Groups*. By E. Kaniuth and K. F. Taylor
198. *Topics in Critical Point Theory*. By K. Perera and M. Schechter
199. *Combinatorics of Minuscule Representations*. By R. M. Green
200. *Singularities of the Minimal Model Program*. By J. Kollár
201. *Coherence in Three-Dimensional Category Theory*. By N. Gurski
202. *Canonical Ramsey Theory on Polish Spaces*. By V. Kanovei, M. Sabok, and J. Zapletal
203. *A Primer on the Dirichlet Space*. By O. El-Fallah, K. Kellay, J. Mashreghi, and T. Ransford
204. *Group Cohomology and Algebraic Cycles*. By B. Totaro
205. *Ridge Functions*. By A. Pinkus
206. *Probability on Real Lie Algebras*. By U. Franz and N. Privault
207. *Auxiliary Polynomials in Number Theory*. By D. Masser
208. *Representations of Elementary Abelian p-Groups and Vector Bundles*. By D. J. Benson
209. *Non-homogeneous Random Walks*. By M. Menshikov, S. Popov and A. Wade
210. *Fourier Integrals in Classical Analysis (Second Edition)*. By C. D. Sogge
211. *Eigenvalues, Multiplicities and Graphs*. By C. R. Johnson and C. M. Saiago
212. *Applications of Diophantine Approximation to Integral Points and Transcendence*. By P. Corvaja and U. Zannier
213. *Variations on a Theme of Borel*. By S. Weinberger
214. *The Mathieu Groups*. By A. A. Ivanov
215. *Slenderness I: Foundations*. By R. Dimitric
216. *Justification Logic*. By S. Artemov and M. Fitting
217. *Defocusing Nonlinear Schrödinger Equations*. By B. Dodson
218. *The Random Matrix Theory of the Classical Compact Groups*. By E. S. Meckes

Operator Analysis

Hilbert Space Methods in Complex Analysis

JIM AGLER
University of California, San Diego

JOHN EDWARD McCARTHY
Washington University in St. Louis

NICHOLAS YOUNG
Leeds University and Newcastle University

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the Universitys mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108485449
DOI: 10.1017/9781108751292

First published 2020

Printed in the United Kingdom by TJ International Ltd, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

ISBN 978-1-108-48544-9 Hardback

Cambridge University Press
978-1-108-48544-9 — Operator Analysis
Jim Agler , John Edward McCarthy , Nicholas John Young
Frontmatter
More Information

To
Sarah, Suzanne, and *Зинаида*

Contents

Preface

The philosophy of this book is that Hilbert space geometry binds function theory and operator theory together, not only allowing each to aid the other but creating a rich structure that can be used to discover new phenomena. There is a "three-way street" between operator theory and function theory: sometimes one uses function theory to prove operator theorems, sometimes one uses operator theory to prove function theorems, and sometimes the theories are so interwoven that one cannot even state the theorem without using the language of both operator theory and function theory.

The main thrust of the book is to discover and prove theorems about holomorphic functions and complex geometry with the aid of Hilbert space geometry and operator theory. The holomorphic functional calculus permits us to substitute commuting operators for the variables of a holomorphic function, a step that reveals hidden properties of the function—something every function theorist should want to do.

It is remarkable how little operator theory one needs in order to prove significant facts in function theory. There will be no call here for the detailed and subtle theories of particular classes of operators, but we shall make heavy use of the functional calculus for operators. The theories of operator dilations and spectral sets also play an important role. We explain carefully what is required from these theories. The reader may either take this material on trust or consult the references that we give.

Part I of the book is devoted to holomorphic functions on domains in complex Euclidean space, beginning with scalar-valued functions on the unit disc in the complex plane, where intuition is most easily developed. Here the central notion of a Hilbert space *model* of a function is introduced, as are several types of arguments that will recur throughout the book. Gradually we build up to various domains in higher dimensions and to operator-valued functions, the latter being important for engineering applications. We do not aim for maximum generality, which can come at the cost of a sacrifice of elegance and impact.

Part II concerns *non-commutative functions*, that is, functions of non-commuting variables. This is a topic of relatively recent study, and one that is currently undergoing rapid development. It transpires that the Hilbert space methods of Part I are well suited to this new context, and we derive many analogs of classical theorems.

Our intended audience is graduate students and mathematicians interested in complex function theory and/or operator theory. We do not require familiarity with several complex variables, but we do assume that the reader has a basic knowledge of complex analysis and functional analysis.

The moment you buy into the functional calculus, you're ready to roll!

Acknowledgments

The catalyst for this book was two series of lectures, given by the first and second authors at Newcastle University in an LMS lecture series in April 2017. The authors would like to thank the London Mathematical Society for supporting this event. In addition, the first author was partially supported by National Science Foundation Grant DMS 1665260, the second author was partially supported by National Science Foundation Grant DMS 1565243, and the third author was partially supported by UK Engineering and Physical Sciences Research Council grants EP/K50340X/1 and EP/N03242X/1, and London Mathematical Society grants 41219 and 41730. We would like to thank the National Science Foundation, the UK Engineering and Physical Sciences Research Council, and the London Mathematical Society for their support.

Parts of the book were read in draft form by several people, who caught some mistakes. We are specially grateful to Alberto Dayan, Chris Felder, Michael Hartz, Mark Mancuso, James Pascoe, and Jeet Sampat. All remaining mistakes and typographical errors are, of course, the responsibility of the authors.