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**The following Supplementary Appendices, which are referenced in this text, are available online at <http://www.cambridge.org/millerandblair>**

- Appendix SA2.1 The Relationship between Approaches I and II*  
*Appendix SA3.1 Basic Relationships in the Multiregional Input–Output Model*  
*Appendix SA4.1 Supplemental Discussion of Aggregation Bias*  
*Appendix SA5.3 Left and Right Inverses in Nonsquare Input–Output Systems*  
*Appendix SA7.2 Hypothetical Extractions with Partitioned Matrices*  
*Appendix SA8.1 Alternative Additive Decompositions of  $\mathbf{x} = \mathbf{LBf}$*

<i>Appendix SA8.2</i>	<i>Additional Early Additive Structural Decomposition Studies</i>
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