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A Gentle Introduction to Homological Mirror Symmetry

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Preface

This book grew out of an advanced masters course that I teach biannually at the University of Amsterdam. The course is aimed at students who are doing a masters in algebra and geometry or mathematical physics. In this course I try to give them a feeling of what homological mirror symmetry is and how it ties together many different areas of mathematics. The focus of the course is to explain the main concepts and results and to illustrate them with examples, without getting too technical. In this way the students will be better prepared to delve into the primary literature if they want to understand the theory at a deeper and more detailed level.

As there are many different topics to cover, it is not an easy task to decide what to include and what to omit. Both symplectic geometry and algebraic geometry come with a powerful toolbox that enables researchers to deal with many different situations and levels of generality, but make the fields quite hard to penetrate for outsiders and newcomers. My own background is in representations of quivers and I am an expert in neither symplectic geometry nor algebraic geometry. Therefore, I decided to approach the book more from a representation-theoretic perspective instead of geometrical, and to use mainly examples in complex or symplectic dimension 1, in other words surfaces. This is not perfect because the one-dimensional perspective misses some important features that are crucial in understanding the higher-dimensional cases, but the main advantage is that these examples can easily be visualized. Moreover, they retain some of the key aspects of homological mirror symmetry while from a complex and symplectic view avoid many extra difficulties. Finally, these examples are often of tame type, which means that their representation theory is very well understood. Some of these examples, such as the affine and projective line or the linear quiver, are familiar to many mathematicians and therefore they offer a good hook to enter the field of homological mirror symmetry.

The book is split into three parts offering different looks at the subject.

- The first part sets up the A_∞-formalism. We first have a brief look at the representation theory of categories, then we study complexes and homology and merge the two subjects into the theory of A_∞-algebras and categories. In the final chapter of this part we apply these ideas to the representation theory of some quivers and show how we can interpret the results in two different geometrical ways: intersection theory of curves on certain surfaces, and sheaves on certain one-dimensional algebraic geometrical objects. These two interpretations offer a first glimpse at homological mirror symmetry.
- The second part takes a more classical approach. We start with a chapter of motivation from theoretical physics that explains the origins of homological mirror symmetry by seeing quantum physics as representation theory. After that we have a closer look at the A-model and the B-model and introduce various A_∞-categories that describe these models. The last chapter of this part looks at mirror symmetry for the complex torus and how to extend it using techniques from toric and tropical geometry.
- The third part of the book focuses on surfaces. First we give an explicit construction of the Fukaya category of a surface using gentle algebras and look at how these categories can be constructed by gluing simpler categories together. This perspective also allows us to construct many different mirrors in the $\mathbb{Z}_2$-graded case. Next we move to the $\mathbb{Z}$-graded setting using line fields. The final two chapters are devoted to two concepts that are important in the study of mirror symmetry but also have many applications in other fields: stability and deformation theory.

While the first part introduces some machinery concerning A_∞-categories that is needed in the other two parts, the latter two stand on their own and can be read independently.

Writing a book about a fast evolving and rapidly expanding field of research is not an easy task and keeping up with all the exciting developments is nigh impossible, let alone giving a concise overview of them. Therefore, these notes form only an incomplete and personal view on the subject and I hope that my limited understanding has not resulted in grave omissions and mistakes or that I have cut too many corners in presenting the material. Nevertheless, I hope that these notes may serve as an entrance guide for mathematicians interested in the subject and prepare them for their own forays into homological mirror symmetry.

Naturally I want to thank some people for their support while writing this book. First of all my thanks go to the students at the University of Amsterdam who took my mirror symmetry class and whose questions and comments were very helpful for organizing the material. In particular I want to thank Edward Berengoltz, Sam van den Brink, Niek Lamoree, Peter Spacek, Okke van Garderen, Ronen Brilleslijper and my PhD student Jasper van de Kreeke for the illuminating and puzzling moments in front of the blackboard when we worked out examples, stumbled on difficulties and tried to clear them up.

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