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FOUNDATIONS OF STABLE HOMOTOPY THEORY

The beginning graduate student in homotopy theory is confronted with a vast literature on spectra that is scattered across books, articles and decades. There is much folklore, but very few easy entry points. This comprehensive introduction to stable homotopy theory changes that. It presents the foundations of the subject together in one place for the first time, from the motivating phenomena to the modern theory, at a level suitable for those with only a first course in algebraic topology.

Starting from stable homotopy groups and (co)homology theories, the authors study the most important categories of spectra and the stable homotopy category, before moving on to computational aspects and more advanced topics such as monoidal structures, localisations and chromatic homotopy theory. The appendix containing essential facts on model categories, the numerous examples and the suggestions for further reading make this a friendly introduction to an often daunting subject.

David Barnes is a Senior Lecturer in Mathematics at Queen's University Belfast. His work centres on stable homotopy theory, usually with either a monoidal or equivariant focus, often using algebra to describe the structures in question.

Constanze Roitzheim is a Senior Lecturer in Mathematics at the University of Kent. Her work focuses on localisations of the stable homotopy category and related questions in algebra.

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Foundations of Stable Homotopy Theory

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