

Index

- L -Lipschitz continuous gradient, 91, 143
- ℓ_p -ball, 36
- L -smooth, 88, 89, 143
- σ -strongly convex, 41
- $s - t$ -flow, 2, 96
- $s - t$ -maximum flow problem, 2, 3, 96, 105, 159, 215, 248
- $s - t$ -minimum cost flow problem, 12, 215, 226, 236, 243
- $s - t$ -minimum cut problem, 3, 105, 135, 159, 248

- accelerated gradient descent, 7, 158
- affine hull, 20
- affine invariance, 170
- affinely independent, 20, 271
- affine scaling, 213
- affine span, 20
- analytic center, 189, 202, 238, 303
- arithmetic operations, 53

- barrier function, 11, 188
- binary search, 252
- bit complexity, 54, 185, 208, 268
- black box model, 61, 94, 158
- breadth-first search, 97
- Bregman divergence, 42, 89, 113, 116, 124, 133, 147

- Cauchy-Binet formula, 306
- Cauchy-Schwarz inequality, 23, 86, 92, 124, 172, 174, 199, 200, 241
 - general, 24
- central path, 189, 193, 202, 220, 229
- centroid, 302

- classification, 136
- closed, 117, 122
 - function, 25, 75
- complementary slackness, 77
- complexity class
 - #P, 67
 - BPP, 52, 63
 - NP-hard, 67
 - P, 52, 63
 - RP, 52, 63
- concave, 37
- conjugate function, 75, 109, 122
- conjugate gradient, 156
- convex combination, 36
- convex function, 36
 - strong, 154
- convex hull, 36
- convexity, 36
- convex program, 49
 - constrained, 49
 - nonsmooth, 49
 - smooth, 49
 - unconstrained, 49
- convex set
 - intersection, 46
 - membership, 53
 - union, 46
- coordinate descent, 103
- cutting plane method, 10, 252, 254

- depth-first search, 142, 228
- derivative, 17
- differentiable, 17
 - continuous, 17, 39
 - twice, 18, 39

differential, 18
 Dikin ellipsoid, 209, 235, 238, 259, 302
 directional derivative, 17, 38, 86
 downward closed, 282
 dual feasibility, 77
 duality, 3, 69
 gap, 72, 210
 strong, 72
 weak, 71
 dynamical system, 25
 continuous-time, 25, 26, 86, 93, 174
 discrete-time, 25, 26
 Euler discretization, 26, 86
 Eisenberg-Gale convex program, 68, 81
 ellipsoid, 36, 54
 Ellipsoid Method, 291
 ellipsoid method, 10, 68, 248, 260
 enclosing ellipsoid, 259
 epigraph, 44, 47, 72
 estimate sequence, 145
 exponential gradient descent, 112, 121
 first-order, 5, 158
 first-order approximation, 20, 38
 first-order convexity, 37
 first-order optimality, 162
 first-order oracle, 62, 84, 88, 295, 296
 follow the regularized leader, 138
 Ford-Fulkerson method, 3
 Frank-Wolfe method, 104
 full-dimensional, 36
 function oracle, 62
 fundamental theorem of algebra, 22
 fundamental theorem of calculus, 19, 90,
 109, 181
 Gaussian elimination, 5, 50, 167
 generalized negative entropy, 48
 generalized Pythagorean theorem, 117, 119,
 124, 148
 geodesically convex, 309
 global optimum, 45
 gradient, 17, 18
 bounded, 88, 108
 G -Lipschitz, 109
 Lipschitz, 108
 gradient descent, 5, 62, 85, 96, 108,
 144, 158
 accelerated, 95
 coordinate, 103

initial point, 88
 learning rate, 87
 projected, 95, 100, 108
 step length, 87
 stochastic, 133
 strongly convex, 102
 subgradient, 103
 gradient flow, 86, 170, 173
 graph, 27
 acyclic, 27, 282
 adjacency matrix, 28
 bipartite, 27, 67
 complete, 27
 perfect matching, 126
 complete, 27, 65
 connected, 27
 cut, 27
 cycle, 27
 degree matrix, 28
 directed, 27
 disconnected, 27
 edge, 27
 head, 27
 tail, 27
 flow, 28
 independent set, 66
 Laplacian, 28, 100
 loop, 27
 matching, 28, 250
 perfect, 28
 perfect matching, 67
 simple, 27
 spanning tree, 28, 65, 289
 $s - t$ -cut, 27
 $s - t$ -flow, 28
 subgraph, 27
 tree, 27
 undirected, 27
 vertex, 27, 56
 degree, 27
 walk, 27, 51
 weighted, 29
 graphic matroid, 282
 halfspace, 35, 53
 Hamiltonian dynamics, 80
 heavy ball method, 157
 Hessian, 18
 Hessian metric, 175
 hybrid barrier, 234
 hybrid center, 303

- hypercube, 55
- hyperplane, 35, 42
- input length, 54
- interior, 189
- interior point method, 12, 142, 190, 219
- isolation lemma, 243
- iterative rounding, 308
- Jacobian, 165
- John ellipsoid, 237, 246, 259
- KKT conditions, 77
- Kullback-Leibler (KL) divergence, 113
- Lagrange
 - multiplier, 70
- Lagrangian, 70
 - dual, 69, 71
 - duality, 70
- Lagrangian dynamics, 80
- Laplacian system, 6, 100, 227
- law of cosines, 116, 120, 124
- learning rate, 87
- Lee-Sidford barrier, 237, 239, 250
- Legendre-Fenchel dual, 80, 122
- leverage score, 235
- linear function, 36
- linear hull, 20
- linear program, 8, 50, 60, 66, 96, 249
 - canonical form, 79, 185
 - dual, 73
 - duality, 72
 - standard form, 72, 220
- linear programming, 60, 64, 185
- linear span, 20
- linear system, 12, 50, 155
- Lipschitz
 - constant, 87
 - continuous, 87
 - gradient, 88
- local inner product, 173
- local norm, 12, 168, 173
- local optimum, 45
- logarithmic barrier, 188, 202, 220, 230, 235, 302
- Lovász extension, 286
- lower bound, 158
- manifold, 175
- matching polytope, 29, 249, 250, 277
- matrix
 - column rank, 21
 - condition number, 32
 - copositive, 66
 - eigenvalue, 22, 56
 - eigenvector, 22
 - full rank, 21
 - identity, 20
 - image, 21
 - inverse, 21
 - kernel, 21
 - norm, 23
 - nullspace, 21
 - positive definite, 21
 - positive semidefinite, 21, 56, 58
 - pseudoinverse, 21, 100
 - rank, 21
 - row rank, 21
 - spectral norm, 23, 167
 - spectrum, 22
 - square-root, 22
 - symmetric, 20
 - totally unimodular, 33
 - $2 \rightarrow 2$ norm, 23
- matrix multiplication, 167
- matroid, 282
 - base, 286
 - rank function, 282
- matroid base polytope, 286
- matroid polytope, 282–284, 286
- max-flow min-cut theorem, 3
- maximum cardinality matching problem, 248
- maximum cut, 142
- maximum entropy distribution, 15, 279, 281
- maximum entropy principle, 308
- maximum entropy problem, 15, 289
- mean value theorem, 163, 200
- membership problem, 282
- minimum spanning tree problem, 248
- Minkowski sum, 47
- min-max theorem, 135
- mirror descent, 7, 108, 109, 121, 122, 144, 158
- mirror map, 122, 123
- multiplicative weights update, 7, 125
- negative entropy, 36
- Newton-Raphson method, 160
- Newton step, 166
- Newton's method, 11, 160, 220
- NL condition, 177
- nonsmooth, 237

norm, 24
 dual, 24, 124
 Euclidean, 24
 ℓ_2 , 24
 ℓ_p , 24
 matrix, 66

odd minimum cut problem, 273
 online convex optimization, 138
 open ball, 25
 optimal assignment problem, 243
 oracle
 first-order, 109
 orthogonal projection, 21, 98, 221, 231
 orthogonal projection matrix, 204

path-following interior point method, 190
 perfect matching polytope, 249
 Physarum dynamics, 183, 244
 Pinsker's inequality, 120, 124
 polyhedron, 36, 50, 54, 185
 polynomial, 182
 polytope, 4, 36, 55, 208, 252, 257
 vertex, 55, 60, 63, 270
 preconditioning, 170, 227, 228
 primal, 70
 primal-dual interior point method, 210, 214
 primal feasibility, 77
 probability simplex, 108, 113, 288, 289
 projected gradient descent, 187
 projection, 95
 projective scaling, 213
 proximal method, 109
 Pythagoras theorem, 117

quadratic convergence, 163
 quadratic function, 36

RAM model, 52
 randomized oracle, 63
 rational convex program, 68, 82
 recentering, 193, 231, 258, 303
 regret, 138
 regularization, 110
 regularizer, 112, 154
 residual graph, 3
 Riemannian gradient, 175, 184
 Riemannian manifold, 12, 174
 Riemannian metric, 12, 170, 174

second-order approximation, 20
 second-order convexity, 39

second-order method, 160
 second-order oracle, 62, 167
 second-order, approximation, 163
 self-adjoint, 224
 self-concordance, 13, 215, 232, 233, 236,
 241, 245
 self-concordant barrier, 232, 234, 245
 semidefinite programming, 239
 separating hyperplane, 42
 separating hyperplane theorem, 43
 separation oracle, 13, 57, 251, 253, 282,
 295, 298

set
 boundary, 25, 42, 187
 bounded, 25, 50
 closed, 25, 44, 50, 75
 closure, 25
 compact, 25, 50
 convex, 35
 interior, 25, 292
 limit point, 25
 open, 25
 polar, 80
 relative interior, 25, 44
 Shannon entropy, 15
 shortest path problem, 51
 simplex method, 10, 187, 213
 simultaneous Diophantine approximation,
 273
 Slater's condition, 71, 72, 78
 soft-max function, 133
 spanning tree polytope, 15, 29, 55, 58, 249,
 277, 290
 sparsest cut, 142
 standard basis vector, 18
 steepest descent, 12, 85, 170
 stochastic gradient descent, 63
 strict convexity, 40
 strictly convex, 117, 173
 strong convexity, 41
 strong duality, 72, 77, 78, 290
 nonconvex, 73
 strongly convex, 111
 subgradient, 38, 103, 288, 296
 existence, 44
 sublevel set, 37, 47, 289, 298
 submodular function, 14, 284
 Submodular Function Minimization, 281
 submodular function minimization, 14, 279,
 281, 285
 supporting hyperplane, 42
 supporting hyperplane theorem, 43

- tangent space, 175, 221, 222
- Taylor approximation, 19, 114
- Taylor series, 19, 200
- totally unimodular, 2, 9
- traveling salesman problem, 15
- Turing machine, 52, 63
 - oracle, 52
 - randomized, 52, 63
- unbiased estimator, 62
- uniform matroid, 282
- universal barrier, 13, 234
- vertex-edge incidence matrix, 2, 28, 96, 216
- volumetric barrier, 13, 234, 245, 250
- volumetric center, 302
- weak duality, 71
- Winnow algorithm, 136
- Young-Fenchel inequality, 75
- 0-1-polytope, 281
- 0-1-polytopes, 14, 248, 251, 269, 277