

Contents

<i>Preface</i>	<i>page</i> xiii
Introduction	1
1 Aspects of Differential Geometry	8
1.1 Manifolds	8
1.2 Differential Forms	12
1.3 Integration of Differential Forms	18
1.4 Vector Fields	20
1.5 Tensors	25
1.6 Lie Derivative	28
1.7 Integrability Conditions	33
1.8 The Metric	34
1.9 Lie Groups and Lie Algebras	38
1.10 Cartan's Isomorphisms	49
1.11 Fibre Bundles	51
1.12 Principal Bundles	55
1.13 Hopf Fibration	62
1.14 Vector Bundles	65
1.15 Riemannian Geometry	70
1.16 Spinors and Differential Forms	73
2 Metric and Related Formulations	78
2.1 Einstein–Hilbert Metric Formulation	78
2.2 Gamma–Gamma Formulation	80
2.3 Linearisation	83
2.4 First-Order Palatini Formulation	86
2.5 Eddington–Schrödinger Affine Formulation	87
2.6 Unification: Kaluza–Klein Theory	88
3 Cartan's Tetrad Formulation	89
3.1 Tetrad, Spin Connection	91
3.2 Einstein–Cartan First-Order Formulation	104
3.3 Teleparallel Formulation	105
3.4 Pure Connection Formulation	107
3.5 MacDowell–Mansouri Formulation	109

x	<i>Contents</i>	
3.6	Dimensional Reduction	112
3.7	BF Formulation	114
4	General Relativity in 2+1 Dimensions	125
4.1	Einstein–Cartan and Chern–Simons Formulations	125
4.2	The Pure Connection Formulation	129
5	The ‘Chiral’ Formulation of General Relativity	132
5.1	Hodge Star and Self-Duality in Four Dimensions	133
5.2	Decomposition of the Riemann Curvature	133
5.3	Chiral Version of Cartan’s Theory	137
5.4	Hodge Star and the Metric	140
5.5	The ‘Lorentz’ Groups in Four Dimensions	151
5.6	The Self-Dual Part of the Spin Connection	160
5.7	The Chiral Soldering Form	163
5.8	Plebański Formulation of GR	171
5.9	Linearisation of the Plebański Action	174
5.10	Coupling to Matter	180
5.11	Historical Remarks	182
5.12	Alternative Descriptions Related to Plebański Formalism	183
5.13	A Second-Order Formulation Based on the 2-Form Field	187
6	Chiral Pure Connection Formulation	192
6.1	Chiral Pure Connection Formalism for GR	192
6.2	Example: Page Metric	211
6.3	Pure Connection Description of Gravitational Instantons	218
6.4	First-Order Chiral Connection Formalism	223
6.5	Example: Bianchi I Connections	224
6.6	Spherically Symmetric Case	232
6.7	Bianchi IX and Reality Conditions	237
6.8	Connection Description of Ricci Flat Metrics	241
6.9	Chiral Pure Connection Perturbation Theory	247
7	Deformations of General Relativity	250
7.1	A Natural Modified Theory	250
8	Perturbative Descriptions of Gravity	255
8.1	Spinor Formalism	256
8.2	Spinors and Differential Operators	262
8.3	Minkowski Space Metric Perturbation Theory	274
8.4	Chiral Yang–Mills Perturbation Theory	275
8.5	Minkowski Space Chiral First-Order Perturbation Theory	280
8.6	Chiral Connection Perturbation Theory	295

<i>Contents</i>		xi
9	Higher-Dimensional Descriptions	304
9.1	Twistor Space	306
9.2	Euclidean Twistors	318
9.3	Quaternionic Hopf Fibration	329
9.4	Twistor Description of Gravitational Instantons	335
9.5	Geometry of 3-Forms in Seven Dimensions	337
9.6	G_2 -Structures on S^7	343
9.7	3-Form Version of the Twistor Construction	355
10	Concluding Remarks	360
	<i>References</i>	365
	<i>Index</i>	369