

## Index

- $G_2$  structure, 331, 337
  - nearly parallel, 341
  - on  $S^7$ , 343
  - parallel, 340
- 2-form field, 114, 123, 174, 177, 192, 224, 277, 281, 295, 361
  - self-dual, 183
  - spinor expression, 260
- affine connection, 71
- Affine formalism, 87
- almost complex structure, xv, 40, 93, 143, 183, 322, 327, 343
  - on twistor space, non-integrable, 328, 346
- anti-Hermitian 2x2 matrices, 45, 49, 50, 100, 127, 152, 257, 315, 326
- Ashtekar's Hamiltonian formulation of GR, 132, 139, 183
- BF theory, 114, 129, 172, 180, 183, 186
- bi-metric gravity theory, 120
- Bianchi I, 224, 253
- Bianchi identities, 73, 79, 134
- Bianchi IX, 237
- bivector, 182, 306
- boundary terms, 110
- branch of the square root problem, 118, 203, 208
- bundle
  - fibre, 51
  - frame, 94
  - nontrivial, 52
  - parallelisable, 94
  - principal, 3, 55
  - tangent, 3, 52, 71, 92
  - vector, 6, 65
- Cartan's magic formula, 31
- Cauchy–Riemann equations, 337, 362
- Chern–Simons formalism, 109, 128
- Chern–Simons theory, 341
- chiral Cartan formalism, 137
- chiral modifications of GR, 184, 193, 224, 227, 237, 241, 249, 250
- chiral pure connection formalism, 194
- chiral soldering form, 164, 166
- Christoffel connection, 6, 78, 97
- Christoffel symbols, 72, 78, 80, 83, 92
- Clifford algebra, 73
  - as exterior algebra, 74
- Clifford product, 74
- cocycle, 56, 65
- coframe, 93
- cohomology, 266, 272
- complex
  - de Rham, 15
- complexification, 75, 99, 133, 138, 156, 164, 306
- conformal invariance, 304
- conformal metric from Hodge star, 141
- connection, xvi, 3, 51
  - affine, 71
  - as a 1-form, 54
  - as Lie algebra valued 1-form, 57, 59
  - Cartan, 109
  - coefficients of, 69
  - Ehresmann, 53
  - from metric in total space, 55, 63
  - Levi–Civita, 72
  - principal, 57, 92
  - spin, 96
  - Weitzenböck, 107
- cosmological constant, 87, 111, 127, 134, 173, 194, 224
- covariant derivative, 68, 70, 71
  - curvature of, 70
  - exterior, 69
- curvature, 1, 51
  - as 2-form, 60, 70
  - as measure of non-integrability, 54, 61
  - Bianchi identities, 73
  - chiral part of, 163
  - computation of, chiral method, 168
  - of affine connection, 71
  - of Ehresmann connection, 53
  - of metric on 2-sphere, 99
  - of spherically symmetric metric, 101, 169
  - Reimann, 72
  - Ricci, 72, 98
  - Ricci, linearisation of, 85

- curvature (cont.)  
 Riemann, xiv, 78, 98, 133, 182  
 Riemann, from spin connection, 98  
 Riemann, linearisation of, 83  
 scalar, xiii, 72, 79, 134  
 Weyl, 125, 133, 173, 176, 179, 180, 182,  
 218, 238, 335, 360
- curve  
 horizontal lift of, 53  
 length of, 35
- de Donder gauge, 274  
 De Sitter metric, 228  
 definite connection, 130, 201  
 deformed GR, 184, 193, 220, 224, 227, 237,  
 241, 249, 250  
 diffeomorphism, 6, 10, 269, 295  
 action on chiral connection, 197  
 linearised, 82  
 one-parameter group of, 28, 46  
 differential form, xiv, xvi, 2, 12, 90  
 as linear map, 25  
 as tensor, 27  
 integration of, 19  
 dilatation, 309  
 dimensional reduction, 88, 112  
 Dirac operator, 263  
 distribution, 33  
 integrable, 33
- Eddington–Schödinger action, 87, 108  
 Eguchi–Hanson metric, 244  
 Einstein equations, 80, 183  
 on  $S^4$ , Plebanski version of, 334  
 Einstein–Cartan formalism, 104  
 Einstein–Hilbert action, 79  
 linearisation of, 85  
 Euler angles, 220  
 exterior algebra, 73
- fibre bundle, xiv, xvi, 2, 51  
 cross-section of, 52  
 Hopf, 52, 55, 62, 325  
 pull back of, 52  
 tangent bundle, 52
- flag manifold, 310  
 foliation, 33, 51  
 frame bundle, 55, 67  
 Fubini–Study metric, 220, 324
- gauge field, 5, 255  
 gauge transformation, 60  
 gauge-fixing, 179, 274, 275, 287, 290, 297, 300  
 geometric structure, 92  
 as reduction of the structure group, 94
- geometry, 1  
 2-manifold, xv, 1  
 7-manifold, 7  
 complex manifold, xv  
 Riemannian, xvi, 1, 3, 78, 91  
 Grassmanian, 12  
 of 2-planes, as twistor space, 307  
 of 3-planes, 143  
 of lines, as projective twistor space, 310  
 gravitational instanton, 176, 178, 247,  
 250, 335  
 pure connection description of, 218  
 gravitational waves, 362  
 graviton, 255  
 group action, 42  
 on itself, 42  
 orbit of, 43  
 group coset, 43  
 as orbit of group action, 44  
 group homomorphism, 43
- Hamiltonian analysis, 105  
 Hodge star, xiv, 132, 183, 304, 339, 361  
 commutativity with Riemann, 135  
 determining conformal metric, 141  
 holonomy, 340  
 Holst term, 115, 137  
 Hopf coordinates, 212, 351
- incidence relation, in twistor space, 312  
 index-free notation, 127, 334, 360  
 interior product, 24  
 inverse densitised metric, 274  
 isometry, 36
- Kähler form, 348  
 Kähler potential, 220  
 Kaluza–Klein, 55, 88, 112, 304
- Lagrange multiplier field, 115, 172,  
 184, 192  
 Leibnitz rule, 69, 71  
 Levi–Civita connection, 72, 95, 97,  
 103, 134  
 Lichnerowicz Laplacian, 84  
 Lie bracket, 24, 31  
 Lie derivative, 31  
 commutativity with exterior derivative, 32  
 computation of, 32  
 Lie group, 38  
 light cone, 305, 307  
 linearisation  
 of chiral pure connection action, 177, 180,  
 246, 247  
 of Einstein–Cartan action, 108

- of Einstein–Hilbert action, 82, 83, 85
- of MacDowell–Mansouri action, 110
- of Plebanski action, 175
- of pure spin connection action, 118
- Lorentz group, 49, 50, 92, 103, 115, 132, 133, 138, 151, 183, 255, 305, 309, 330
- Möbius strip, 18
- MacDowell–Mansouri formalism, 109, 110, 114, 175, 193
- magic formula, 31
- manifold, 8
  - complex, 324
  - integral, 33
  - orientation of, 18
  - smooth, 9
  - topological, 9
- Maurer–Cartan 1-form, 100
- Maxwell theory, 114, 304
- metric, 34
  - Einstein, xv, 80, 133
  - from a 3-form in 7 dimensions, 338
  - Hermitian, 41, 324
  - Kähler, 220
  - pull back of, 35
  - why nonzero question, 90
- metricity constraints, 176, 188
- Minkowski space, 152
- Newmann–Penrose formalism, 183
- non-metricity, 105
- normal subgroup, 43
- operator
  - covariant derivative, 68
  - covariant exterior derivative, 69
  - creation-annihilation, 75
  - Dirac, xv, 263, 287
  - exterior derivative, xv, 14
  - exterior derivative as Dirac, 268
  - Hodge, 132
  - Lichnerowicz, 84, 85
- Page metric, 211
- Palatini formalism, 86
- parabolic subgroup, 310
- perfect connection, 218, 221
- perfect fluid, 6, 182
- perturbation theory, 361
  - chiral, 280
  - chiral connection, 295
  - metric, 274
  - Yang–Mills, 275
- Petrov classification, 182
- Pfaffian, 34
- Plücker coordinates, 307, 313
- Plebanski formalism, 140, 163, 171, 183, 224, 247, 270, 280, 295, 334, 336, 339, 361
  - matter coupling, 180
- Pontryagin term, 139, 175, 219
- principal bundle, 55
  - cross-section of, 60
  - Hopf, 64
  - trivialisation of, 58, 59
- principal connection
  - curvature of, 60
- pullback
  - bundle, 52
  - differential form, 15
  - function, 15
- pure connection action for GR, 198
- push forward, 21
- quantum group, 129
- quaternionic Hopf fibration, 329, 337
- quaternions, 154, 309, 318, 329
- Ray–Singer torsion, 129
- reality conditions, 138, 164, 172, 197, 237, 309
  - compatibility with dynamics, 240
- renormalisation of the action, 110
- Ricci tensor, 72
- Schwarzschild solution, 171
  - second-class constraints, 105, 139
- semi-definite connection, 201
- sign of a connection, 201
- singularity, 231, 254
- soldering form, xv, 93, 95, 103, 114, 360
  - chiral, 164
  - spinor expression, 260
- spherically symmetric problem, 101, 169, 232, 242, 303
- spin connection, 6, 96, 104
  - chiral part of, 160, 171
  - chiral part of, on  $S^4$ , 326, 333
  - formalism for 3D gravity, 129
  - formalism for GR, 107, 116, 361
- spin foam models, 116
- spin group, 74
- spinor, xv, 3, 8, 49, 51, 62, 73, 140, 183, 255, 278, 299, 305, 309, 320, 342, 343
  - as differential form, 75, 77
  - primed, unprimed, 257
  - raising-lowering of indices, 258
- spinor helicity formalism, 269

372

*Index*

- stable forms, 337, 339
- Stelle–West formalism, 111
- stress-energy tensor, 180
- symmetry, 42
- symplectic form, 42
  
- tangent space, 20
- Taub–NUT metric, 223
- Teleparallel formalism, 105
- tensor, 26
  - contraction of, 27
  - geometric structure, model of, 93
- tensor calculus, xvi, 1, 7
- tensor product, 26
- tetrad, xv, 6, 91, 163, 165, 255
- theorem
  - Frobenius, 34
  - Newlander–Nirenberg, 335
  - Newlander–Nirenberg, 359
  - Stokes, 18, 20
- torsion, 71, 92, 96, 104, 105, 127, 167, 188
- Turaev–Viro model, 129
- twistor, xv, 306
  - as a pair of spinors, 309
  - double fibration, 310
  - Euclidean, fibre bundle structure of, 318
  - Hermitian form on twistor space, 317
  - isomorphism, 143, 156
- Urbantke metric, 144, 150, 165, 172, 195, 201, 251, 338
  
- vector bundle, 65
  - associated, 67, 92
  - cross-section of, 65
  - transition function, 65
- vector field, 20
  - as derivation, 23
  - horizontal, 53, 59, 64
  - integral curve of, 29
  - involution of, 33
  - left-invariant on a group, 46
  - Lie algebra of, 44
  - Lie bracket, 24, 31
  - projectable, 21
  - velocity, 29, 48
  - vertical, 53
- wedge product, 13
- wedge product metric, 142, 143
  
- Yang–Mills, 5, 7, 90, 95, 139, 218, 256, 275, 295, 302