

## FORMULATIONS OF GENERAL RELATIVITY

This monograph describes the different formulations of Einstein's General Theory of Relativity. Unlike traditional treatments, Cartan's geometry of fibre bundles and differential forms is placed at the forefront, and a detailed review of the relevant differential geometry is presented. Particular emphasis is given to General Relativity in 4D space-time, in which the concepts of chirality and self-duality begin to play a key role. Associated chiral formulations are catalogued, and shown to lead to many practical simplifications. The book develops the chiral gravitational perturbation theory, in which the spinor formalism plays a central role. The book also presents in detail the twistor description of gravity, as well as its generalisation based on geometry of 3-forms in seven dimensions. Giving valuable insight into the very nature of gravity, this book joins our highly prestigious Cambridge Monographs in Mathematical Physics series. It will interest graduate students and researchers in the fields of Theoretical Physics and Differential Geometry.

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Cambridge University Press  
978-1-108-48164-9 — Formulations of General Relativity  
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## Gravity, Spinors and Differential Forms

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Excerpt  
[More Information](#)

**CAMBRIDGE**  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
79 Anson Road, #06–04/06, Singapore 079906

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[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781108481649](http://www.cambridge.org/9781108481649)  
DOI: 10.1017/9781108674652

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First published 2020

*A catalogue record for this publication is available from the British Library.*

ISBN 978-1-108-48164-9 Hardback

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To the memory of the two Peter Ivanovichs in my life:  
Peter Ivanovich Fomin, who got me interested in gravity,  
and Peter Ivanovich Holod, who formed my taste for mathematics.