

Topographic Effects in Stratified Flows

Covering both theory and experiment, this text describes the behaviour of homogeneous and density-stratified fluids over and around topography. Its presentation is suitable for advanced undergraduate and graduate students in fluid mechanics, as well as for practising scientists, engineers and researchers.

Using laboratory experiments and illustrations to further understanding, the author explores topics ranging from the classical hydraulics of single-layer flow to more complex situations involving stratified flows over two- and three-dimensional topography, including complex terrain. A particular focus is placed on applications to the atmosphere and ocean, including discussions of downslope windstorms, and of oceanic flow over continental shelves and slopes.

This new edition has been restructured to make it more digestible, and updated to cover significant developments in areas such as exchange flows, gravity currents, waves in stratified fluids, stability and applications to the atmosphere and ocean.

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Topographic Effects in Stratified Flows

SECOND EDITION

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In memory of Adrian Gill and Angus McEwan.

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Some books are to be tasted, others to be swallowed,
and some few to be chewed and digested.
FRANCIS BACON, 1561–1626; *Essay 1, Of Studies*.

Contents

	<i>Preface to the Second Edition</i>	page xi
	<i>Preface to the First Edition</i>	xiii
1	Background	1
	1.1 Equations for fluid motion	3
	1.2 Boundary conditions	8
	1.3 Conservation relations	10
	1.4 Terminology	13
2	Non-linear single-layer flow: classical hydraulics	16
	2.1 Basic equations	16
	2.2 Flows with small obstacle height	17
	2.3 One-dimensional non-linear hydrostatic flow	31
	2.4 Downslope flows with frictional drag	48
	2.5 Granular flows	54
3	Non-linear single-layer flow past obstacles: jumps, bores and wave dispersion	56
	3.1 Non-linear waves	56
	3.2 The QRS framework	61
	3.3 Application to hydraulic jumps and undular bores	66
	3.4 Single-layer flow over topography with non-linearity and dispersion	71
	3.5 Non-linear flow past three-dimensional obstacles	78
4	Two-layer flow with jumps and topography	95
	4.1 Basic equations	95
	4.2 Linear waves	97
	4.3 Equations for one-dimensional non-linear hydrostatic flow	98
	4.4 Two-layer hydraulic jumps	102
	4.5 Hydrostatic flow over topography	109

viii	<i>Contents</i>	
	4.6 Non-linear waves and internal bores	120
	4.7 Topographic forcing with non-linearity and dispersion	124
	4.8 Downstream effects	126
5	Two-layer and stratified flow through contractions	129
	5.1 Two-layered flow through contractions with a free upper surface	130
	5.2 Two-layered flow through contractions with a rigid upper boundary	132
	5.3 Non-linearity with dispersion in contractions	141
	5.4 Multi-layered flow through contractions	142
	5.5 Continuously stratified flow through contractions	145
6	Exchange flows	148
	6.1 Two-layer exchange flow in a uniform channel over topography	149
	6.2 Two-layer exchange flow through contractions	149
	6.3 Exchange flows through doorways and windows	163
	6.4 Multi-layer and continuously stratified exchange flows	164
7	Gravity currents, downslope and anabatic flows, and stratified hydraulic jumps	172
	7.1 Gravity currents over horizontal terrain in uniform environments	172
	7.2 Gravity currents in density-stratified environments	176
	7.3 Gravity currents down slopes	179
	7.4 Hydraulic jumps in stratified flow	189
	7.5 Anabatic flows	192
8	Waves in stratified fluids	196
	8.1 Waves in multi-layered models	196
	8.2 Continuously stratified fluids: equations	202
	8.3 Waves in finite-depth systems	205
	8.4 Waves in infinitely deep stratified fluids	211
	8.5 Trapped and leaky modes	217
	8.6 The effects of molecular viscosity and diffusion on internal waves	220
	8.7 Energy and momentum transport in a non-uniformly moving fluid	221
	8.8 The “slowly varying” or WKB approximation	226
	8.9 Critical layers	229
	8.10 Wave-overturning and saturation	238
	8.11 Wave propagation in three dimensions	239
9	The stability of stratified flows	243
	9.1 Stability of stratified shear flow: a general criterion	244

Contents

ix

9.2	The process and products of the instability of shear flows	245
9.3	Instability in laminar boundary-layers: Tollmien–Schlichting waves	254
9.4	The stability of internal waves	259
10	Stratified flow over two-dimensional obstacles: linear and near-linear theory	261
10.1	Observations of flows of infinite depth	263
10.2	Infinite-depth flows: theory for small Nh/U	275
10.3	Infinite-depth flows: finite-amplitude topography and “Long’s model”	286
10.4	Infinite-depth flows with $Nh/U > (Nh/U)_c$: numerical studies	294
10.5	Linear theory for small Nh_m/U : finite depth	297
10.6	Comparison between linear theory, and observations and numerical results for finite depth and small Nh/U	304
10.7	Long-model solutions for finite depth	313
11	Stratified flow over two-dimensional obstacles: non-linear hydraulic models with applications	321
11.1	Models with non-linearity and dispersion	321
11.2	Non-linear hydraulic flow theory for finite depth	325
11.3	Applications of the hydraulic theory	336
11.4	The approach to continuous stratification	344
11.5	Observations and numerical results for finite Nh/U in finite depth: short obstacles	358
11.6	Application of the hydraulic model to infinite-depth flows	365
11.7	Flows with large Nh/U : deep blocked flow, topographic drag and clear-air turbulence	368
11.8	Details of the dynamics of downslope windstorms	372
11.9	Flow across valleys	378
12	Stratified flow over three-dimensional topography: linear theory	390
12.1	Linear theory for small-amplitude topography, with the lower boundary as a stream surface	391
12.2	Linear theory for trapped lee waves	414
12.3	Atmospheric lee waves	419
12.4	Limitations and extensions of linear theory	424
13	Three-dimensional stratified flow over finite obstacles	433
13.1	The topology of the flow field on the surface of an obstacle	433
13.2	Observations of the flow past three-dimensional obstacles	439
13.3	Flow properties for finite Nh/U : theoretical aspects	473
13.4	The drag force on isolated obstacles: $Nh/U > 1$	484

14	Flow over complex and realistic terrain in the atmosphere and ocean	486
14.1	Flow over complex terrain	486
14.2	Some atmospheric examples in the troposphere	489
14.3	Internal waves in the upper atmosphere	499
14.4	Internal waves in the deep ocean	502
14.5	Topographic effects in coastal oceanography	502
14.6	Oscillating ocean flows and tides	504
15	Applications to practical modelling of flow over complex terrain	505
15.1	Laboratory modelling	505
15.2	The natural ventilation of buildings	510
15.3	Parametrisation of the effects of sub-grid-scale orography in large-scale numerical models	510
	<i>References</i>	520
	<i>Index</i>	542

Preface to the Second Edition

The main objective of this second edition is to bring the topic up to date at time of writing (~2019 to early 2020). The material covered is somewhat broader: every topic described in the first edition is included here, and a number of new topics such as downslope flows and waves in the upper atmosphere have also been added. In addition, some corrections have been made, and more emphasis has been given to applications. These are becoming more apparent as observations in the ocean and atmosphere improve. The effects of rotation have still largely been omitted, though the Coriolis force/frequency does get an occasional mention. There is a finite limit to everything.

To some extent, the subject is a closed book (or at least, more so than for the first edition), but the type of analysis described here may (or does) have application to other fields. Two under-developed topics that have been proposed are non-linear optics and Bose–Einstein condensates. The latter is probably relevant to the missing mass of the universe.

I would also like to express my appreciation of the work of the staff of Cambridge University Press, particularly David Tranah, for his support and professionalism, and to Leon Chan and Jimmy Philip for assistance with some simulations and figures.

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Preface to the First Edition

This project was conceived about 10 years ago, but the incentive to pursue and complete it was hampered until recently because of several fundamental unresolved questions about the nature of stratified flow around topography. Within the last few years it has become possible to answer these questions, as a result of the efforts of several people, and the answers are embodied in the synthesis presented here.

Who will benefit from purchasing, reading or thumbing through this book? It is primarily addressed to fluid dynamicists, meteorologists, oceanographers, engineers, physicists and mathematicians who wish to learn more about the dynamics of stratified fluids. Some background in fluid dynamics is probably necessary, but the subject is treated from first principles and is developed from simple situations toward more complex ones. Overall, the order of presentation is based on logic rather than the historical development. There is balance between theory and experiment, where the comparison is made whenever possible, and a consistent attempt has been made to provide a physical understanding of the phenomena involved.

I have gone to some length to make the material easily assimilable, as the number of figures testifies. As I see it, a book such as this is the next step in the scientific process of the documentation of a subject, following the initial “source” material in journals. It is a documented attempt to digest such material, and should therefore be easier to read. However, much of the material presented here is new, as part of the process of filling gaps and providing a (more) complete picture. A number of new experiments have been carried out at Aspendale specifically for this volume.

I have attempted to give an adequate list of references so that readers can delve deeper into the subject, but it is not exhaustive, and some relevant work may have been omitted. I apologise in advance to any colleagues to whom due reference has not been given.

In its present form, this book has been made possible by the dedicated and professional efforts of several people at Aspendale: most notably David Murray, who has played a major part in most of my experimental studies over the past 10

years; David Whillas, whose talents are evident in several photographs; and Sean Higgins, who has skillfully adapted and created many of the line drawings. Thanks are also due to others who have provided continual background support.

I am grateful to several colleagues who have contributed photographs or figures, and to various copyright holders for permission to use some figures, and these are acknowledged in the captions. I am also grateful to others who have taken the time to read and comment on drafts of chapters in varying degrees of imperfection, and specifically these include Jim Rottman in particular, and Ian Castro, Terry Clark, Jack Katzfey, Peter Killworth, Greg Lawrence, Mike Sewell, Bill Snyder, Larry Armi and Sharan Majumdar. Thanks are also due to numerous colleagues for informative discussion on the material of this book over many years, to George Batchelor for his advice and support, and to Alan Harvey, Brian Watts and the staff of Cambridge University Press for their cooperation and attention to detail.