CAMBRIDGE

Cambridge University Press
978-1-108-48138-0 — A Short Introduction to String Theory
Thomas Mohaupt
Table of Contents
More Information

Contents

Preface *page* xiii
Acknowledgements xv
Introduction xvi

Part I From Particles to Strings 1

1 Classical Relativistic Point Particles **3**
1.1 Minkowski Space 3
1.2 Particles 5
1.3 A Non-covariant Action Principle for Relativistic Particles 6
1.4 Canonical Momenta and Hamiltonian 9
1.5 Length, Proper Time, and Reparametrisations 9
1.6 A Covariant Action for Massive Relativistic Particles 11
1.7 Particle Interactions 12
1.8 Canonical Momenta and Hamiltonian for the Covariant Action 14
1.9 A Covariant Action for Massless and Massive Particles 15
1.10 Literature 17

2 Classical Relativistic Strings **18**
2.1 The Nambu–Goto Action 18
2.1.1 Action, Equations of Motion, and Bounday Conditions 18
2.1.2 D-branes 21
2.1.3 Constraints 23
2.2 The Polyakov Action 24
2.2.1 Action, Symmetries, Equations of Motion 24
2.2.2 Interpretation as a Two-Dimensional Field Theory 26
2.2.3 The Conformal Gauge 27
2.2.4 Light-Cone Coordinates 28
2.2.5 From Symmetries to Conservation Laws 29
2.2.6 Explicit Solutions – Periodic Boundary Conditions 32
2.2.7 Explicit Solutions – Neumann Boundary Conditions 34
2.2.8 Explicit Solutions – Dirichlet Boundary Conditions 35
2.2.9 Non-oriented Strings 36
2.2.10 Literature 37

3 Quantised Relativistic Particles and Strings **38**
3.1 Quantised Relativistic Particles 38
3.2 Field Quantisation and Quantum Field Theory 41

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-48138-0 — A Short Introduction to String Theory
Thomas Mohaupt
Table of Contents
More Information

3.3 Quantised Relativistic Strings 43
3.4 Literature on Quantum Field Theory 46

Part II The World-Sheet Perspective 47

4 The Free Massless Scalar Field on the Complex Plane 49
4.1 The Cylinder and the Plane 49
4.2 Infinitesimal and Finite Conformal Transformations 51
4.3 From Commutators to Operator Product Expansions 53
4.4 From Operators to States 59

5 Two-Dimensional Conformal Field Theories 63
5.1 Some Remarks on Conformal Field Theories in General Dimension 63
5.2 Conformal Primaries 65
5.2.1 Definition of Conformal Primaries, Holomorphic, and Chiral Fields 65
5.2.2 Hermitian Conjugation 67
5.2.3 Operator Products and Commutators 68
5.3 Energy-Momentum Tensor and Virasoro Algebra 69
5.4 The State – Operator Correspondence 72
5.4.1 The $SL(2,\mathbb{C})$ Vacuum – Primary States 72
5.4.2 Highest Weight States 73
5.4.3 Descendant Fields 74
5.5 General Aspects of Two-Dimensional Conformal Field Theories 75
5.5.1 General Discussion of OPEs 75
5.5.2 Unitary Conformal Field Theories 75
5.5.3 Null Vectors 76
5.5.4 Results on Classification 76
5.6 Literature 77

6 Partition Functions I 78
6.1 Partition Functions for Particles and Strings 78
6.2 The Chiral Partition Function of a Free Boson 81

Part III The Space-Time Perspective 85

7 Covariant Quantisation I 87
7.1 Outline of Covariant Quantisation 87
7.2 The Fock Space 87
7.3 Implementation of the Constraints 88
7.4 Mass Eigenstates 90
7.5 Physical States of the Open String 91
7.6 The Photon 92
7.7 The Tachyon 95
7.8 Literature 98

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-48138-0 — A Short Introduction to String Theory
Thomas Mohaupt
Table of Contents
More Information

8 Intermezzo – Representations of the Poincaré Group 99
8.1 Review of Representations of the Poincaré Group 99
8.2 Group Theoretical Interpretation of the Photon State 101
8.3 Virasoro Constraints, Poincaré Representations, and Effective Field Theory 102
8.4 Literature 103

9 Covariant Quantisation II 104
9.1 The Graviton 104
9.2 The Kalb–Ramond Field (B-Field) 108
9.2.1 Physical States 108
9.2.2 Dualisation of Antisymmetric Tensor Fields 109
9.3 Vertex Operators 111
9.4 The Dilaton 112
9.5 The No-Ghost Theorem 114
9.6 Further Remarks and Literature 116

10 Light-Cone Quantisation 117
10.1 Light-Cone Gauge and Light-Cone Quantisation for Particles 117
10.2 The Light-Cone Gauge for Open Strings 118
10.3 Light-Cone Quantisation for Open Strings 119
10.4 Ground State Energy via ζ-Function Method 120
10.5 Open String Spectrum in the Light-Cone Gauge 121
10.6 Lorentz Covariance in the Light-Cone Gauge 122
10.7 Literature 124

11 Partition Functions II 125
11.1 Partition Functions for Massive Particles 125
11.2 Partition Functions for Open Strings 127
11.3 Partition Functions for Closed Strings 131
11.4 Further Remarks and Literature 132

Part IV Outlook 135

12 Interactions 137
12.1 Amplitudes 137
12.1.1 General Discussion of Amplitudes 137
12.1.2 The Four Scalar Amplitude 140
12.1.3 The Four Graviton Amplitude 143
12.1.4 Effective Actions from Amplitudes 144
12.2 Curved Backgrounds 146
12.2.1 String Action for Curved Backgrounds and Background Fields 146
12.2.2 Conformal Invariance and the Equations of Motion 148
12.2.3 Consistent Backgrounds and Background Independence 149
12.2.4 Marginal Deformations 152
12.3 Further Remarks and Literature 153

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-48138-0 — A Short Introduction to String Theory
Thomas Mohaupt
Table of Contents
More Information

13 Dimensional Reduction and T-Duality **155**
13.1 Overview 155
13.2 Closed Strings on S^1 156
13.2.1 Generic Massless States and Their Effective Field Theory 158
13.2.2 U(1) Gauge Symmetry from the World-Sheet Point of View 161
13.2.3 Enhanced Symmetries and Extra Massless States 164
13.2.4 SU(2) Gauge Symmetry from the World-Sheet Point of View 165
13.2.5 U(1) Gauge Symmetry, Higgs Effect, and Massive Vector Fields 168
13.2.6 $SU(2)_L \times SU(2)_R$ Gauge Symmetry from the Space-Time Point of View 170
13.2.7 The Higgs Effect from the World-Sheet Point of View 172
13.2.8 Partition Functions for Winding States 174
13.3 Closed Strings on $S^1/\mathbb{Z}_2$ and Orbifolds 177
13.4 T-Duality for Closed Strings 182
13.5 T-duality and Target Space Gauge Symmetries 184
13.6 T-duality for Open Strings 187
13.7 Toroidal Compactification of Closed Strings 191
13.7.1 The Narain Lattice 191
13.7.2 T-duality for Toroidal Compactifications 194
13.7.3 Symmetry Enhancement 196
13.7.4 Effective Actions for Toroidal Compactifications 200
13.7.5 Compactification on a Two-Torus 202
13.7.6 The Buscher Rules 208
13.8 The Vacuum Selection Problem 209
13.8.1 Beyond Toroidal Compactifications 209
13.8.2 From Moduli Spaces to the Landscape 210
13.8.3 The Landscape and the Swampland 212
13.9 Literature 214

14 Fermions and Supersymmetry **215**
14.1 The Supersymmetric Harmonic Oscillator 216
14.1.1 The Simple Supersymmetric Harmonic Oscillator 216
14.1.2 Supersymmetrisation of the Light-Cone Hamiltonian 218
14.2 General Discussion of Supersymmetry 219
14.3 Supersymmetric Field Theories in Two Dimensions 223
14.4 The RNS String 226
14.5 Type-II Superstrings 231
14.6 Type-I Superstrings 235
14.7 Heterotic Strings 236
14.8 Looking for the Big Picture 236
14.9 Final Remarks and Literature 237

Appendix A Notation and Conventions **239**

Appendix B Units, Constants, and Scales **240**

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-48138-0 — A Short Introduction to String Theory
Thomas Mohaupt
Table of Contents
More Information

Appendix C Fourier Series and Fourier Integrals **242**

Appendix D Modular Forms and Special Functions **244**

Appendix E Young Tableaux **247**

Appendix F Gaussian Integrals and Integral Exponential Function **250**

Appendix G Lie Algebras, Lie Groups, and Symmetric Spaces **253**

References 257
Index 263

© in this web service Cambridge University Press www.cambridge.org