A Short Introduction to String Theory

Suitable for graduate students in physics and mathematics, this book presents a concise and pedagogical introduction to string theory. It focuses on explaining the key concepts of string theory, such as bosonic strings, D-branes, supersymmetry, and superstrings and on clarifying the relationship between particles, fields, and strings without assuming an advanced background in particle theory or quantum field theory, thus making it widely accessible to interested readers from a range of backgrounds. Important ideas underpinning current research, such as partition functions, compactification, gauge symmetries, and T-duality are analysed both from the world-sheet (conformal field theory) and the space-time (effective field theory) perspectives. Ideal for either self-study or a one semester graduate course, *A Short Introduction to String Theory* is an essential resource for students studying string theory, containing examples and homework problems to develop understanding, with fully worked solutions available to instructors.

Thomas Mohaupt obtained his PhD in Theoretical Physics from the University of Münster and is Reader in Theoretical Physics at the University of Liverpool. He has published extensively on string theory, specialising in black hole entropy and supergravity, and has taught various courses on related topics in the UK and Germany. He has been a visiting scholar at Stanford University, a visiting professor at the University of Nancy, and a Senior Fellow at the Erwin Schrödinger International Institute for Mathematics and Physics in Vienna.

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This book is dedicated to my parents, Dr Helga and Manfred Mohaupt.

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Preface

This book gives a short introduction into the basic ideas and concepts of string theory. It is aimed, primarily, at PhD students who are commencing their studies and for other readers who would like to approach the topic at a gentler pace than in other textbooks, and with a minimum of prerequisites. The level of this book is in between Zwiebach (2009), which is a pedagogical introduction into string theory for advanced undergraduates, and textbooks such as Polchinski (1998a), Blumenhagen et al. (2013), Becker et al. (2007), and Schomerus (2017), which go quite steeply into the subject and also are quite comprehensive with regard to the topics they cover.

The approach and selection of topics reflect my experience with teaching fledgling PhD students in the United Kingdom for the past 15 years, including a short course for the MAGIC consortium since 2013. I have expanded the treatment of topics where, in my experience, beginners tend to struggle or to get confused. Part I covers the classical theory and quantisation of particles, strings, and, to some extent, fields. Some material that is typically found in introductory quantum field theory texts has been included: first, because it helps to see this material in context, and second, to start developing the effective field theory perspective, which is one of the threads of this book. Parts II and III unfold the world-sheet and the space-time perspectives of string theory. Part II introduces the minimal background in conformal field theory needed to underpin covariant quantisation, and later toroidal compactification. Part III analyses the physical spectrum in the covariant and light-cone approach and includes a mini-review of the representation theory of the Poincaré group in order to provide a solid understanding of why string states correspond to elementary particles. The critical dimension $D = 26$ of the bosonic string and its shift $a = 1$ in ground state energy are derived in the light-cone formulation as a guided exercise. Parts II and III also contain chapters on partition functions which, in particular, introduce the concept of modular invariance. By the end of Part III, the reader should have a solid understanding of the free quantum bosonic string with periodic, Neumann, and Dirichlet boundary conditions. Part IV goes deeper (and steeper) into three topics: interactions, compactifications, and matter (fermions). Besides a general discussion of amplitudes, the closed string four-scalar amplitude is computed and its structure is analysed and compared to point particle amplitudes. Curved backgrounds are discussed briefly from the sigma model and effective field theory perspective, putting emphasis on the fact that scattering amplitudes and curved backgrounds are complementary ways to describe string interactions. A comprehensive chapter on compactification starts with a detailed treatment of the simplest case, compactification on a circle, as this already allows one to uncover most of the special features of string compactifications. Orbifolds are exemplified using the orbicircle and general toroidal compactifications are treated in some detail, including their generic massless spectra, their effective actions, symmetry enhancement, and the Higgs effect.

T-duality is developed for both closed and open strings, including the Buscher rules. We close this chapter with some remarks on more general compactifications, the vacuum selection problem, and the swampland programme. In the last chapter, we explain how matter (fermions) arises through world-sheet supersymmetry and how modular invariances leads one to the existence of 5 superstring theories defined on 10-dimensional Minkowski space. The chapter concludes with a brief overview of the dualities that relates these theories to each other and to 11-dimensional M-theory.

I hope that this book will be useful for self-study and as a companion for an introductory string theory course at postgraduate level. It includes a number of exercises which, besides encouraging active reading, can be used in classes. The price for a detailed and pedagogical treatment in a short volume is the reduction in the number of topics. D-branes, superstrings, and gravitational aspects of string theory are treated relatively briefly, but the basic ideas are introduced and illustrated. Unfortunately, black hole entropy and the AdS/CFT correspondence had to be omitted, as this would have required to abandon the very idea of a 'short introduction'. Also, we have restricted ourselves to old covariant quantisation and light-cone quantisation, which are sufficient for the topics we cover in this short introduction, while leaving out the path integral and BRST formalisms, which are well covered in the more advanced and comprehensive textbooks mentioned above and in the Introduction.

Large parts of this book should be accessible to any reader with a solid background in quantum mechanics and special relativity. I imagine that the typical reader will already have some background in quantum field theory, general relativity, and group theory and will hopefully develop their understanding of these topics in parallel to studying string theory. I have tried throughout to minimise assumptions about the reader's background, thus making the book as accessible and self-contained as possible. Some relevant background material is provided through appendices or 'narrative summaries' within the main text, together with pointers to the literature.

Acknowledgements

This book is based on various long and short courses I gave on string theory over the past 20 years, in particular a full-year course for advanced undergraduates at the University of Jena, introductory courses for fledgling PhD students at the University of Liverpool, and a short course I have been contributing to the programme of the MAGIC consortium since 2013. While this book does not directly follow any of these lectures, it has grown out of this experience, and the questions and comments I have received from the students attending my courses were essential in shaping it. I would like to thank my colleagues and the PhD students in the String and Beyond the Standard Model Phenomenology research cluster at the Department of Mathematical Sciences, University of Liverpool, for providing a stimulating environment, in particular through our weekly group meetings. I am particularly indebted to Gabriel Lopes Cardoso, Harold Erbin, Maxime Médevielle, and Flavio Tonioni for their detailed feedback.

Introduction

String theory is an extension of quantum field theory which includes gravity and provides a framework for constructing models in particle physics and cosmology. It aims to be a complete fundamental theory of space, time, and matter. Given that the particle concept becomes subtle once special relativity and quantum theory merge into quantum field theory, not to speak of curved backgrounds or even full-fledged quantum gravity, the idea ‘to replace point particles by one-dimensional extended objects, called strings’ may sound somewhat naive. And historically, string theory did not start with this idea, but with the observation that certain amplitudes with desirable properties could be interpreted as scattering amplitudes of strings. The history of string theory has many twists and turns, which tell us a lot about the erratic nature of theory evolution. In the author’s view, string theory is an example of a theory which has not been constructed according to a master plan or programme, but has been discovered – or, if you prefer, developed – in continuous reaction to often unexpected new insights and observations. This said, we will avoid historical and philosophical reflections and develop string theory systematically on the basis of quantum theory and special relativity as taught in an undergraduate course. In time, we will make quite a few intriguing observations, which hopefully will convince the reader that this pursuit is worth our time and effort.

While we will present explicit computations, and ask the reader to perform several more as exercises, equal emphasis will be given to ideas and concepts. Some themes are recurrent throughout this book. One is the complementarity between the *space-time* and the *world-sheet perspectives*. Essentially, this is the observation that the mechanics of a one-dimensional continuum can be formulated as a field theory in 1 + 1 dimensions. In the currently dominant ‘first quantised’ formulation of string theory, the world-sheet (trajectory, generalised world-line) plays a central role, in contrast to quantum field theory which, in its standard formulation, takes the space-time perspective. As the particle spectrum of a quantised string contains an infinite tower of massive states – at least as long as we neglect the backreaction on space-time which may lead to the generation of black holes – it is natural to restrict oneself to the dynamics of the most relevant, that is the lightest modes, and often just the massless modes. This naturally leads us to consider the *effective field theory* of the light modes, a concept that universally applies whenever we distinguish between a microscopic and potentially fundamental, and a macroscosopic and effective level of description. Another thread is the comparison between particles and strings, that is the question what makes strings special. Here, we will see that world-sheet *modular invariance* acts like a built-in UV cut-off, while *T-duality* (also called space-time modular invariance) introduces a fundamental length scale and forces us to re-consider how we think about space-time geometry. We will also encounter the power of anomalies, which makes string theory highly restricted and potentially unique. In particular, we will see how the Lorentz covariance of string theory in Minkowski space fixes the number of space-time dimensions. While this makes the theory unique in theory (if you pardon the pun), in practice non-uniqueness bites back in the form of the non-uniqueness of solutions: the predicted number of space-time dimensions is larger than the observed one, thus forcing us to compactify the extra dimensions. At some points in the book we will pause to bring in some additional mathematical tools, or to summarise results from quantum field theory to provide background. This is done in a minimalistic but, hopefully, sufficiently self-contained way and references for further study are provided.

The book is divided into four parts. Part I compares the classical theories of relativistic particles and strings and outlines their quantisation. It also reviews how particles – and, by extension, individual excitations of strings – are described using quantum field theory. We explain the role of constraints and show that for strings we have the choice between *periodic, Neumann*, and *Dirichlet boundary conditions*, the latter implying the existence of *D-branes*. At the end of Part I, we formulate the problem of Lorentz covariant quantisation of strings. Part II develops the world-sheet perspective of string theory, that is, two-dimensional *conformal field theory*. This provides us with the tools for formulating the quantum theory in Part III. Part III takes the space-time point of view. We review the representation theory of the *Poincaré group* in order to understand precisely how string states correspond to elementary particles with a specific mass and spin (or the higher-dimensional analogue of spin, conceptualised as the *little group*). We also discuss how string states can be described from an effective field theory perspective, which helps us to make contact with the usual framework of particle theory based on quantum field theory. Besides Lorentz covariant quantisation we discuss *light-cone quantisation*, which allows us to give a complete treatment of how the number of space-time dimensions is fixed. At the end of Part III, we will have a solid understanding of the quantum theory of free bosonic strings. In Part IV, we provide an overview of three further developments of the theory: *interactions, compactifications* and *matter* (fermions). Interactions can be desribed in two ways: as scattering amplitudes in a fixed background and as the interaction of a string with a non-trivial on-shell background of coherent string states. We explain how these viewpoints are related and in which sense the theory is background independent. The next topic is how to reduce the number of visible space-time dimensions by compactification. We focus on flat backgrounds, that is circles, tori, and orbifolds thereof. The spectrum, symmetry enhancement, and symmetry breaking is discussed from the world-sheet and space-time perspectives. The existence of winding modes distinguishes strings from particles, and leads to a new symmetry, T-duality, which relates large and small compactification scales and introduces a fundamental length scale. The plethora of possible compactifications poses a serious challenge to identifying a string theory model of our universe. We briefly discuss the concepts of *landscape* and *swampland*. Compactifications on two-tori are used to introduce *Calabi – Yau manifolds* and *mirror symmetry*. While we use the bosonic string to explain the concepts and properties of string theory, in the last chapter we show how the theory can be generalised to include *fermions*. As this is intimately related to *supersymmetry*, we start by introducing supersymmetry, first using the harmonic oscillator, then as an extension of Poincaré symmetry. The RNS model serves as a simple example of a two-dimensional supersymmetric field theory, explains how space-time fermions arise through world-sheet supersymmetry, and serves as a stepping-stone to Type-II superstring theories. We sketch how modular invariance imposes restrictions which lead one to 5, 10-dimensional supersymmetric string theories. Type-I and heterotic strings are briefly introduced and we conclude with the network of dualities that relates the 5 superstring theories to each other and to 11-dimensional M-theory.

This book will show the influence of the textbooks that I have used over the years to prepare my lectures and I have not tried to be original for originality's sake. When I entered the field, Green et al. (1987) was the only textbook and, while it does not include the new developments since the later 1980s, it is still unsurpassed in what it does. My other formative text was the predecessor of Blumenhagen et al. (2013) which, in its current incarnation, is one of my standard references, together with Polchinski (1998a), Polchinski (1998b), and Becker et al. (2007). There are of course, other excellent textbooks, including Kaku (1988), Kaku (1991), Kiritsis (2007), West (2012), and Schomerus (2017) and readers who have grown out of this book will not have difficulties in finding one that fits their preferences in style and choice of topics and perspective. There also are books which provide in-depth coverage of topics that we treat relatively briefly or have been forced to omit, in particular, Johnson (2003) for D-branes, Ortin (2004) for a space-time and (super-)gravity based approach to string theory, Ibáñez and Uranga (2012) for string phenomenology, and Ammon and Erdmenger (2015) for the AdS/CFT correspondence. Mathematically inclined readers will enjoy the two volumes Deligne et al. (1999) which cover various topics in string theory and quantum field theory.