This book argues that the meaning of negation, perhaps the most important logical constant, cannot be defined within the framework of the most comprehensive theory of proof-theoretic semantics, as formulated in the influential work of Michael Dummett and Dag Prawitz. Nils Kürbis examines three approaches that have attempted to solve the problem – defining negation in terms of metaphysical incompatibility, treating negation as an undefinable primitive, and defining negation in terms of a speech act of denial – and concludes that they cannot adequately do so. He argues that whereas proof-theoretic semantics usually only appeals to a notion of truth, it also needs to appeal to a notion of falsity, and proposes a system of natural deduction in which both are incorporated. Offering new perspectives on negation, denial and falsity, this book will be important for readers working on logic, metaphysics and the philosophy of language.

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Proof and Falsity
A Logical Investigation

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