

Computational Bayesian Statistics

An Introduction

Meaningful use of advanced Bayesian methods requires a good understanding of the fundamentals. This engaging book explains the ideas that underpin the construction and analysis of Bayesian models, with particular focus on computational methods and schemes. The unique features of the text are the extensive discussion of available software packages combined with a brief but complete and mathematically rigorous introduction to Bayesian inference. The text introduces Monte Carlo methods, Markov chain Monte Carlo methods, and Bayesian software, with additional material on model validation and comparison, transdimensional MCMC, and conditionally Gaussian models. The inclusion of problems makes the book suitable as a textbook for a first graduate-level course in Bayesian computation with a focus on Monte Carlo methods. The extensive discussion of Bayesian software – R/R-INLA, OpenBUGS, JAGS, STAN, and BayesX – makes it useful also for researchers and graduate students from beyond statistics.

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Computational Bayesian Statistics

An Introduction

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Preface to the English Version

This book is based on lecture notes for a short course that was given at the *XXII Congresso da Sociedade Portuguesa de Estatística*. In the translation from the original Portuguese text we have added some additional material on sequential Monte Carlo, Hamiltonian Monte Carlo, transdimensional Markov chain Monte Carlo (MCMC), and variational Bayes, and we have introduced problem sets. The inclusion of problems makes the book suitable as a textbook for a first graduate-level class in Bayesian computation with a focus on Monte Carlo methods. The extensive discussion of Bayesian software makes it useful also for researchers and graduate students from beyond statistics.

The core of the text lies in Chapters 4, 6, and 9 on Monte Carlo methods, MCMC methods, and Bayesian software. Chapters 5, 7, and 8 include additional material on model validation and comparison, transdimensional MCMC, and conditionally Gaussian models. Chapters 1 through 3 introduce the basics of Bayesian inference, and could be covered fairly quickly by way of introduction; these chapters are intended primarily for review and to introduce notation and terminology. For a more in-depth introduction we recommend the textbooks by Carlin and Louis (2009), Christensen et al (2011), Gelman et al (2014a) or Hoff (2009).

Preface

In 1975, Dennis Lindley wrote an article in *Advances in Applied Probability* titled “The future of statistics: a Bayesian 21st century,” predicting for the twenty-first century the predominance of the Bayesian approach to inference in statistics. Today one can certainly say that Dennis Lindley was right in his prediction, but not exactly in the reasons he gave. He did not foresee that the critical ingredient would be great advances in computational Bayesian statistics made in the last decade of the twentieth century. The “Bayesian solution” for inference problems is highly attractive, especially with respect to interpretability of the inference results. However, in practice, the derivation of such solutions involves in particular the evaluation of integrals, in most cases multi-dimensional, that are difficult or impossible to tackle without simulation. The development of more or less sophisticated computational methods has completely changed the outlook. Today, Bayesian methods are used to solve problems in practically all areas of science, especially when the processes being modeled are extremely complex. However, Bayesian methods can not be applied blindly. Despite the existence of many software packages for Bayesian analysis, it is critical that investigators understand what these programs output and why.

The aim of this text, associated with a minicourse given at the *XXII Congresso da Sociedade Portuguesa de Estatística*, is to present the fundamental ideas that underlie the construction and analysis of Bayesian models, with particular focus on computational methods and schemes.

We start in Chapter 1 with a brief summary of the foundations of Bayesian inference with an emphasis on the principal differences between the classical and Bayesian paradigms. One of the main pillars of Bayesian inference, the specification of prior information, is unfortunately often ignored in applications. We review its essential aspects in Chapter 2. In Chapter 3, analytically solveable examples are used to illustrate the Bayesian solution to statistical inference problems. The “great idea” behind the development of computational Bayesian statistics is the recognition that Bayesian infer-

ence can be implemented by way of simulation from the posterior distribution. Classical Monte Carlo methods are presented in Chapter 4 as a first solution for computational problems. Model validation is a very important question, with its own set of concepts and issues in the Bayesian context. The most widely used methods to assess, select, and compare models are briefly reviewed in Chapter 5.

Problems that are more complex than the basic ones in Chapter 4 require the use of more sophisticated simulation methods, in particular Markov chain Monte Carlo (MCMC) methods. These are introduced in Chapter 6, starting as simply as possible. Another alternative to simulation is the use of posterior approximations, which is reviewed in Chapter 8. The chapter describes, in a generic fashion, the use of integrated nested Laplace approximation (INLA), which allows for substantial improvements in both computation times (by several factors), and in the precision of the reported inference summaries. Although applicable in a large class of problems, the method is more restrictive than stochastic simulation. Finally, Chapter 9 is dedicated to Bayesian software. The possibility of resorting to MCMC methods for posterior simulation underpins the development of the software BUGS, which allows the use of Bayesian inference in a large variety of problems across many areas of science. Rapid advances in technology in general have changed the paradigm of statistics, with the increasing need to deal with massive data sets (“Big Data”), often of spatial and temporal types. As a consequence, posterior simulation in problems with complex and high-dimensional data has become a new challenge, which gives rise to new and better computational methods and the development of software that can overcome the earlier limitations of BUGS and its successors, WinBUGS and OpenBUGS. In Chapter 9 we review other statistics packages that implement MCMC methods and variations, such as JAGS, Stan, and BayesX. This chapter also includes a brief description of the R package R-INLA, which implements INLA.

For the compilation of this text we heavily relied on the book *Estatística Bayesiana* by Paulino, A. Turkman, and Murteira, published by Fundação Calouste Gulbenkian in 2003. As all copies of this book were sold a long while ago, we also extensively used preliminary work for an upcoming second edition, as well as material that we published in the October 2013 edition of the bulletin of the Sociedade Portuguesa de Estatística (SPE).

This text would not have been completed in its current form without the valuable and unfailing support of our dear friend and colleague Giovanni Silva. We owe him sincere thanks. We are also thankful to the Sociedade Portuguesa de Estatística for having proposed the wider theme of Bayesian

Preface

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statistics and for the opportunity to give a minicourse at the 22nd conference of the society. We also acknowledge the institutional support from the Universidade de Lisboa through the Centro de Estatística e Aplicações (PEst-OE/MAT/UI0006/2014, UID/MAT/00006/2013), in the Department of Statistics and Operations Research in the Faculdade de Ciências and of the Department of Mathematics in the Instituto Superior Técnico. We would like to acknowledge that the partial support by the Fundação para a Ciência e Tecnologia through various projects over many years enabled us to build up this expertise in Bayesian statistics.

Finally, we would like to dedicate this book to Professor Bento Murteira to whom the development of Bayesian statistics in Portugal owes a lot. In fact, Chapter 1 in this book reflects in many ways the flavor of his writings.

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