

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
1.1 What is Statistical Mechanics?	1
1.2 Probabilistic Behaviour	2
1.2.1 Axioms of Probability	3
1.2.2 Example: Coin Toss Experiment	4
1.2.3 Probability Distributions	5
1.2.4 Example: Random Walk	7
1.2.5 Large- N Limit of the Binomial Distribution	11
1.2.6 Central Limit Theorem	15
1.3 Microstates and Macrostates	16
1.3.1 Example: Non-interacting Spins in a Solid	17
1.4 Information, Ignorance and Entropy	19
1.5 Summary	21
Problems	21
2 The Microcanonical Ensemble	25
2.1 Thermal Contact	26
2.1.1 Heat Flow in the Approach to Equilibrium	28
2.1.2 Principle of Maximum Entropy	29
2.1.3 Energy Resolution and Entropy	30
2.2 Gibbs Entropy	31
2.3 Shannon Entropy	32
2.4 Example: Non-interacting Spins in a Solid	34
2.5 Summary	36
Problems	37
3 Liouville's Theorem	41
3.1 Phase Space and Hamiltonian Dynamics	41
3.2 Ergodic Hypothesis	45
3.2.1 Non-ergodic Systems	46
3.3 Summary	46
Problems	46

4 The Canonical Ensemble	48
4.1 Partition Function	48
4.2 Bridge Equation in the Canonical Ensemble	50
4.2.1 Boltzmann Distribution	52
4.2.2 Derivatives of the Partition Function	52
4.2.3 Equivalence of the Canonical and Microcanonical Ensembles	54
4.3 Connections to Thermodynamics	55
4.4 Examples	56
4.4.1 Two-Level System	56
4.4.2 Quantum Simple Harmonic Oscillator	59
4.4.3 Classical Partition Function and Classical Harmonic Oscillator	61
4.4.4 Rigid Rotor	62
4.4.5 Particle in a Box	64
4.5 Ideal Gas	65
4.5.1 Uncoupled Subsystems	65
4.5.2 Distinguishable and Indistinguishable Particles	67
4.5.3 Ideal Gas	70
4.5.4 Example: Entropy of Mixing	73
4.6 Non-ideal Gases	75
4.7 The Equipartition Theorem	78
4.7.1 Example: The Ideal Gas	79
4.7.2 Dulong and Petit Law	79
4.8 Summary	80
Problems	81
5 Kinetic Theory	86
5.1 Maxwell–Boltzmann Velocity Distribution	86
5.1.1 Density of States	87
5.1.2 Maxwell–Boltzmann Velocity Distribution	87
5.2 Properties of the Maxwell–Boltzmann Velocity Distribution	89
5.2.1 Distribution of Speeds	90
5.3 Kinetic Theory for an Ideal Gas	91
5.3.1 Pressure in an Ideal Gas	92
5.3.2 Effusion	94
5.4 Brownian Motion	95
5.5 Diffusion	98
5.5.1 Mean Free Path and Collision Time	98
5.5.2 Fick’s Law	100
5.6 Transport	102
5.7 Summary	103
Problems	104

6 The Grand Canonical Ensemble	107
6.1 Chemical Potential	107
6.1.1 Example: Ideal Gas	109
6.2 Grand Canonical Partition Function	111
6.2.1 Bridge Equation	112
6.2.2 Derivatives of the Grand Potential	114
6.3 Examples	115
6.3.1 Fermions in a Two-Level System	115
6.3.2 The Langmuir Adsorption Isotherm	117
6.4 Chemical Equilibrium and the Law of Mass Action	118
6.4.1 The Law of Mass Action	119
6.5 Summary	121
Problems	122
7 Quantum Statistical Mechanics	125
7.1 Quantum Statistics	125
7.2 Distinguishable Particles and Maxwell–Boltzmann Statistics	127
7.2.1 Maxwell–Boltzmann Statistics	128
7.3 Quantum Particles in the Grand Canonical Ensemble	129
7.3.1 Fermi–Dirac Distribution	131
7.3.2 Bose–Einstein Distribution	132
7.4 Density of States and Thermal Averages	133
7.4.1 Thermal Averages Using the Density of States	135
7.5 Summary	136
Problems	137
8 Fermions	139
8.1 Chemical Potential for Fermions	139
8.1.1 Zero Temperature: The Fermi Energy	139
8.1.2 Non-zero Temperature: Sommerfeld Expansion	140
8.1.3 Temperature Dependence of the Chemical Potential	143
8.2 Thermodynamic Properties of a Fermi Gas	144
8.2.1 Energy and Heat Capacity	144
8.2.2 Pressure of a Fermi Gas	145
8.2.3 Entropy of a Fermi Gas	146
8.2.4 Number Fluctuations	147
8.2.5 Another View of Temperature Dependence of Thermodynamic Properties	148
8.3 Applications	148
8.3.1 Metals and the Fermi Sea	148
8.3.2 White Dwarf Stars	151
8.3.3 Neutron Stars	157
8.4 Summary	157
Problems	157

9 Bosons	161
9.1 Photons and Blackbody Radiation	161
9.1.1 Blackbody Radiation	162
9.1.2 Density of States	162
9.1.3 Number Density	163
9.1.4 Energy Density	164
9.1.5 Example: Cosmic Microwave Background Radiation	167
9.1.6 Radiation Pressure	168
9.1.7 Stefan–Boltzmann Law	169
9.2 Bose–Einstein Condensation	170
9.2.1 Superfluidity	174
9.3 Low-Temperature Properties of a Bose Gas	176
9.3.1 Chemical Potential	176
9.3.2 Internal Energy and Heat Capacity	177
9.4 Bosonic Excitations: Phonons and Magnons	180
9.4.1 Phonons	180
9.4.2 The Debye Model	181
9.4.3 Magnons	184
9.5 Summary	185
Problems	186
 10 Phase Transitions and Order	190
10.1 Introduction to the Ising Model	190
10.2 Solution of the Ising Model	193
10.2.1 Order Parameters and Broken Symmetry	193
10.2.2 General Strategy for Solution of the Ising Model	194
10.2.3 Mean Field Theory	195
10.3 Role of Dimensionality	202
10.3.1 One Dimension	202
10.3.2 Two Dimensions	203
10.4 Exact Solutions of the Ising Model	205
10.4.1 Exact Solution in One Dimension	205
10.4.2 Exact Solution in Two Dimensions	208
10.5 Monte Carlo Simulation of the Ising Model	209
10.5.1 Importance Sampling	209
10.5.2 Metropolis Algorithm	210
10.5.3 Initial Conditions and Equilibration	210
10.6 Connection between the Ising Model and the Liquid–Gas Transition	212
10.7 Landau Theory	213
10.7.1 Symmetry-Breaking Fields	214
10.7.2 Landau Theory and First-Order Phase Transitions	216
10.8 Summary	216
Problems	217

Appendix A Gaussian Integrals and Stirling's Formula	222
A.1 Gaussian Integrals	222
A.2 Gamma Function	223
A.3 Stirling's Formula	224
Appendix B Primer on Thermal Physics	227
B.1 Thermodynamic Equilibrium	227
B.1.1 Reversible and Irreversible Processes	227
B.1.2 State Functions	228
B.1.3 Work and Heat	230
B.2 The Laws of Thermodynamics	231
B.3 Thermodynamic Potentials	233
B.3.1 Legendre Transforms and Free Energies	235
B.4 Maxwell Relations	236
B.4.1 Useful Partial Derivative Relations	237
B.4.2 Example: Relationship between C_V and C_P	238
Appendix C Heat Capacity Cusp in Bose Systems	239
C.1 Heat Capacity	241
<i>References</i>	243
<i>Index</i>	244