

Essential Statistical Physics

This clear and pedagogical text delivers a concise overview of classical and quantum statistical physics. *Essential Statistical Physics* shows students how to relate the macroscopic properties of physical systems to their microscopic degrees of freedom, preparing them for graduate courses in areas such as biophysics, condensed matter physics, atomic physics and statistical mechanics. Topics covered include the microcanonical, canonical, and grand canonical ensembles, Liouville's theorem, kinetic theory, non-interacting Fermi and Bose systems and phase transitions, and the Ising model. Detailed steps are given in mathematical derivations, allowing students to quickly develop a deep understanding of statistical techniques. End-of-chapter problems reinforce key concepts and introduce more advanced applications, and appendices provide a detailed review of thermodynamics and related mathematical results. This succinct book offers a fresh and intuitive approach to one of the most challenging topics in the core physics curriculum, and provides students with a solid foundation for tackling advanced topics in statistical mechanics.

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Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
1.1 What is Statistical Mechanics?	1
1.2 Probabilistic Behaviour	2
1.2.1 Axioms of Probability	3
1.2.2 Example: Coin Toss Experiment	4
1.2.3 Probability Distributions	5
1.2.4 Example: Random Walk	7
1.2.5 Large- N Limit of the Binomial Distribution	11
1.2.6 Central Limit Theorem	15
1.3 Microstates and Macrostates	16
1.3.1 Example: Non-interacting Spins in a Solid	17
1.4 Information, Ignorance and Entropy	19
1.5 Summary	21
Problems	21
2 The Microcanonical Ensemble	25
2.1 Thermal Contact	26
2.1.1 Heat Flow in the Approach to Equilibrium	28
2.1.2 Principle of Maximum Entropy	29
2.1.3 Energy Resolution and Entropy	30
2.2 Gibbs Entropy	31
2.3 Shannon Entropy	32
2.4 Example: Non-interacting Spins in a Solid	34
2.5 Summary	36
Problems	37
3 Liouville's Theorem	41
3.1 Phase Space and Hamiltonian Dynamics	41
3.2 Ergodic Hypothesis	45
3.2.1 Non-ergodic Systems	46
3.3 Summary	46
Problems	46

4	The Canonical Ensemble	48
4.1	Partition Function	48
4.2	Bridge Equation in the Canonical Ensemble	50
4.2.1	Boltzmann Distribution	52
4.2.2	Derivatives of the Partition Function	52
4.2.3	Equivalence of the Canonical and Microcanonical Ensembles	54
4.3	Connections to Thermodynamics	55
4.4	Examples	56
4.4.1	Two-Level System	56
4.4.2	Quantum Simple Harmonic Oscillator	59
4.4.3	Classical Partition Function and Classical Harmonic Oscillator	61
4.4.4	Rigid Rotor	62
4.4.5	Particle in a Box	64
4.5	Ideal Gas	65
4.5.1	Uncoupled Subsystems	65
4.5.2	Distinguishable and Indistinguishable Particles	67
4.5.3	Ideal Gas	70
4.5.4	Example: Entropy of Mixing	73
4.6	Non-ideal Gases	75
4.7	The Equipartition Theorem	78
4.7.1	Example: The Ideal Gas	79
4.7.2	Dulong and Petit Law	79
4.8	Summary	80
	Problems	81
5	Kinetic Theory	86
5.1	Maxwell–Boltzmann Velocity Distribution	86
5.1.1	Density of States	87
5.1.2	Maxwell–Boltzmann Velocity Distribution	87
5.2	Properties of the Maxwell–Boltzmann Velocity Distribution	89
5.2.1	Distribution of Speeds	90
5.3	Kinetic Theory for an Ideal Gas	91
5.3.1	Pressure in an Ideal Gas	92
5.3.2	Effusion	94
5.4	Brownian Motion	95
5.5	Diffusion	98
5.5.1	Mean Free Path and Collision Time	98
5.5.2	Fick’s Law	100
5.6	Transport	102
5.7	Summary	103
	Problems	104

6	The Grand Canonical Ensemble	107
6.1	Chemical Potential	107
6.1.1	Example: Ideal Gas	109
6.2	Grand Canonical Partition Function	111
6.2.1	Bridge Equation	112
6.2.2	Derivatives of the Grand Potential	114
6.3	Examples	115
6.3.1	Fermions in a Two-Level System	115
6.3.2	The Langmuir Adsorption Isotherm	117
6.4	Chemical Equilibrium and the Law of Mass Action	118
6.4.1	The Law of Mass Action	119
6.5	Summary	121
	Problems	122
7	Quantum Statistical Mechanics	125
7.1	Quantum Statistics	125
7.2	Distinguishable Particles and Maxwell–Boltzmann Statistics	127
7.2.1	Maxwell–Boltzmann Statistics	128
7.3	Quantum Particles in the Grand Canonical Ensemble	129
7.3.1	Fermi–Dirac Distribution	131
7.3.2	Bose–Einstein Distribution	132
7.4	Density of States and Thermal Averages	133
7.4.1	Thermal Averages Using the Density of States	135
7.5	Summary	136
	Problems	137
8	Fermions	139
8.1	Chemical Potential for Fermions	139
8.1.1	Zero Temperature: The Fermi Energy	139
8.1.2	Non-zero Temperature: Sommerfeld Expansion	140
8.1.3	Temperature Dependence of the Chemical Potential	143
8.2	Thermodynamic Properties of a Fermi Gas	144
8.2.1	Energy and Heat Capacity	144
8.2.2	Pressure of a Fermi Gas	145
8.2.3	Entropy of a Fermi Gas	146
8.2.4	Number Fluctuations	147
8.2.5	Another View of Temperature Dependence of Thermodynamic Properties	148
8.3	Applications	148
8.3.1	Metals and the Fermi Sea	148
8.3.2	White Dwarf Stars	151
8.3.3	Neutron Stars	157
8.4	Summary	157
	Problems	157

9	Bosons	161
9.1	Photons and Blackbody Radiation	161
9.1.1	Blackbody Radiation	162
9.1.2	Density of States	162
9.1.3	Number Density	163
9.1.4	Energy Density	164
9.1.5	Example: Cosmic Microwave Background Radiation	167
9.1.6	Radiation Pressure	168
9.1.7	Stefan–Boltzmann Law	169
9.2	Bose–Einstein Condensation	170
9.2.1	Superfluidity	174
9.3	Low-Temperature Properties of a Bose Gas	176
9.3.1	Chemical Potential	176
9.3.2	Internal Energy and Heat Capacity	177
9.4	Bosonic Excitations: Phonons and Magnons	180
9.4.1	Phonons	180
9.4.2	The Debye Model	181
9.4.3	Magnons	184
9.5	Summary	185
	Problems	186
10	Phase Transitions and Order	190
10.1	Introduction to the Ising Model	190
10.2	Solution of the Ising Model	193
10.2.1	Order Parameters and Broken Symmetry	193
10.2.2	General Strategy for Solution of the Ising Model	194
10.2.3	Mean Field Theory	195
10.3	Role of Dimensionality	202
10.3.1	One Dimension	202
10.3.2	Two Dimensions	203
10.4	Exact Solutions of the Ising Model	205
10.4.1	Exact Solution in One Dimension	205
10.4.2	Exact Solution in Two Dimensions	208
10.5	Monte Carlo Simulation of the Ising Model	209
10.5.1	Importance Sampling	209
10.5.2	Metropolis Algorithm	210
10.5.3	Initial Conditions and Equilibration	210
10.6	Connection between the Ising Model and the Liquid–Gas Transition	212
10.7	Landau Theory	213
10.7.1	Symmetry-Breaking Fields	214
10.7.2	Landau Theory and First-Order Phase Transitions	216
10.8	Summary	216
	Problems	217

Appendix A	Gaussian Integrals and Stirling's Formula	222
A.1	Gaussian Integrals	222
A.2	Gamma Function	223
A.3	Stirling's Formula	224
Appendix B	Primer on Thermal Physics	227
B.1	Thermodynamic Equilibrium	227
B.1.1	Reversible and Irreversible Processes	227
B.1.2	State Functions	228
B.1.3	Work and Heat	230
B.2	The Laws of Thermodynamics	231
B.3	Thermodynamic Potentials	233
B.3.1	Legendre Transforms and Free Energies	235
B.4	Maxwell Relations	236
B.4.1	Useful Partial Derivative Relations	237
B.4.2	Example: Relationship between C_V and C_P	238
Appendix C	Heat Capacity Cusp in Bose Systems	239
C.1	Heat Capacity	241
	<i>References</i>	243
	<i>Index</i>	244

Preface

The ideas of statistical physics allow one to relate macroscopic properties of systems to their microscopic degrees of freedom and play important roles in many branches of physics and other sciences, and disciplines further afield such as economics. In particular, the concept of entropy as a measure of ignorance has found very wide application, especially recently in its relation to information theory. Statistical physics is hence one of the core courses in many undergraduate degree programs, sometimes taught as a combined course with thermal physics, but often as a course in its own right.

As an instructor for an upper-level undergraduate course in statistical physics, I have found it challenging to find a text for the course that contains the important ideas of statistical physics at a level the students can relate to, and at an affordable price. My goal in writing this book has been to cover key concepts and examples in statistical physics in a pedagogical and clear manner, to include mathematical detail in a way that is useful to upper-level undergraduates and beginning graduate students, while also being concise and affordable. This necessarily leads to a limited scope. Unlike many existing texts on statistical physics, I do not try to cover thermal physics and statistical mechanics in the same book, nor do I try to be comprehensive in covering both undergraduate and graduate statistical mechanics in the same volume. My goal is that this book is one that will provide students and instructors with the essential ideas of statistical physics in a way that will prove useful to all students. For students continuing to graduate studies in physics and related topics I aim to give a comprehensive coverage of ideas that are needed in graduate school, and for students who do not continue to graduate studies I aim to give a solid background in statistical physics that will allow them to apply these ideas in other contexts.

The book is based on lecture notes for a one-semester (13 weeks, 3 hours of lectures/week) undergraduate course on statistical physics that I have delivered three times at Simon Fraser University (SFU). I have assumed that the reader of the book has already taken (or is taking at the same time) a course in thermal physics, so I do not elaborate greatly on thermodynamic quantities, although I have included a brief primer on thermal physics in Appendix B, so that the reader can look up ideas as needed. I also assume that the reader has taken at least one quantum mechanics course, so they are familiar with the solution of quantum mechanical problems like the particle in a box and the simple harmonic oscillator. My target audience for this book is primarily undergraduate students learning statistical mechanics for the first time. However, I also believe that it will be of interest to graduate students wanting a clear reference for statistical mechanics that provides details of calculations and clarification of concepts, and to instructors looking for an affordable, well-structured text or reference for statistical physics courses.

My choice of content is what I consider to be the main ideas and examples of statistical physics that will allow a starting graduate student in physics to have a solid foundation in the subject, but is not intended to be exhaustive. Hence, after an introduction to probability, microstates and macrostates (Chapter 1), I cover the microcanonical (Chapter 2), canonical (Chapter 4) and grand canonical (Chapter 6) ensembles. I take the opportunity to discuss Liouville's theorem and ergodicity after discussing the microcanonical ensemble (Chapter 3) and give an introduction to kinetic theory after the canonical ensemble (Chapter 5). After discussing the grand canonical ensemble, I introduce quantum statistical mechanics (Chapter 7) and devote a chapter each to fermions (Chapter 8) and bosons (Chapter 9), touching on Bose–Einstein condensation and superfluidity and applications of Fermi statistics to metals and compact stars. Finally, I finish with a brief introduction to phase transitions, broken symmetry and ordering, using the Ising model as the main example, and introduce Landau's theory of phase transitions (Chapter 10). There are problems at the end of all the chapters to help reinforce the concepts discussed in the bodies of the chapters.

I had had a vague notion that at some point I would like to write the book here for a number of years; almost certainly nothing would have come of these ideas but for a meeting with Nicholas Gibbons of Cambridge University Press in 2015. Nicholas encouraged me to flesh out the lecture notes that I had prepared for the course into a text. It has taken several years for that to happen, but I hope you find the end result useful.

In addition to Nicholas Gibbons, I would like to thank everyone at Cambridge University Press who I have interacted with on this project, particularly Liso Pinto, Maggie Jeffers and Rachel Norridge who have helped keep me on track and provided much helpful advice. I would also like to thank Jeff McGuirk and Michael Plischke from SFU, who have shared their experience of teaching statistical mechanics with me, the students at SFU who have provided me with considerable feedback on early drafts of this book, specifically Florian Baer, Matt Wiens, Aidan Wright, Frank Wu and Adrian Yeung, and my father Brian Kennett for detailed feedback on all aspects of the book. Finally, I would like to thank my wife Kaila and my children Heath and Eily for their love, support and patience during the writing of this book, especially in the later stages of writing.