# Foundations of Classical Mechanics

Classical mechanics is an old subject. It is being taught differently in modern times using numerical examples and exploiting the power of computers that are readily available to modern students. The demand of the twenty-first century is challenging. The previous century saw revolutionary developments, and the present seems poised for further exciting insights and illuminations. Students need to quickly grasp the foundations and move on to an advanced level of maturity in science and engineering. Freshman students who have just finished high school need a short and rapid, but also thorough and comprehensive, exposure to major advances, not only since Copernicus, Kepler, Galileo, and Newton, but also since Maxwell, Einstein, and Schrodinger. Through a careful selection of topics, this book endeavors to induct freshman students into this excitement.

This text does not merely teach the laws of physics; it attempts to show how they were unraveled. Thus, it brings out how empirical data led to Kepler's laws, to Galileo's law of inertia, how Newton's insights led to the principle of causality and determinism. It illustrates how symmetry considerations lead to conservation laws, and further, how the laws of nature can be extracted from these connections. The intimacy between mathematics and physics is revealed throughout the book, with an emphasis on beauty, elegance, and rigor. The role of mathematics in the study of nature is further highlighted in the discussions on fractals and Madelbrot sets.

In its formal structure the text discusses essential topics of classical mechanics such as the laws of Newtonian mechanics, conservation laws, symmetry principles, Euler–Lagrange equation, wave motion, superposition principle, and Fourier analysis. It covers a substantive introduction to fluid mechanics, electrodynamics, special theory of relativity, and to the general theory of relativity. A chapter on chaos explains the concept of exploring laws of nature using Fibonacci sequence, Lyapunov exponent, and fractal dimensions. A large number of solved and unsolved exercises are included in the book for a better understanding of concepts.

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*To my teachers, with much gratitude*

*and*

*to my students, with best wishes*

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# FOREWORD

I am a theoretical astrophysicist, and my professional work requires a foundation in classical mechanics and fluid dynamics, and it then draws on statistical mechanics, relativity, and quantum mechanics. How is a student to see the underlying connections between these vast subjects? Professor Pranawa Deshmukh's book '*Foundations of Classical Mechanics*' (FoCM) provides an excellent exposition to the underlying unity of physics, and is a valuable resource for students and professionals who specialize in any area of physics.

It was 2011 and I was Chair of the Department of Physics and Astronomy at the University of Western Ontario (UWO). An important global trend in education is internationalization: the effort to increase the international mobility of students and faculty, and increase partnerships in research and teaching. The present-day leading universities in the world are the ones that long ago figured out the benefits of academic mobility and exchanges. As part of our internationalization efforts at UWO, I was keen on building ties with the renowned Indian Institute of Technology (IIT) campuses in India. I invited Professor Deshmukh of the IIT Madras to come to UWO to spend a term in residence and also teach one course. That course turned out to be Classical Mechanics. How enlightening to find out that Professor Deshmukh had already developed extensive lecture materials in this area, including a full videographed lecture course 'Special Topics in Classical Mechanics' that is available on YouTube. Not surprising then, the course he taught at UWO was a tour-de-force of classical mechanics that also, notably, included topics that are considered to be 'modern', including chaos theory and relativity. Many of our students appreciated, highly, the inclusion of special relativity and, for example, to be able to finally understand a resolution to the mind-bending Twin Paradox. We were lucky to have Professor Deshmukh bring his diverse expertise to Canada, and UWO in particular, and it established personal and research links between individuals, and more generally between UWO and the IITs, which continue today.

The FoCM book is a further extension of the broad approach that Professor Deshmukh has brought to his teaching. This is not just another book on Classical Mechanics, due to both its approach and extensive content. The chapters are written in a conversational style, with everyday examples, historical anecdotes, short biographical sketches, and pedagogical features included. The mathematical rigor of the book is also very high, with equations introduced and justified as needed. Students and professionals who want a resource that synthesizes the unity of classical and modern physics will want to have FoCM on their shelves. The classical principles of conservation laws are introduced through the beautiful ideas of symmetry that have found resonance in modern physics. FoCM also introduces the Lagrangian and Hamiltonian approaches and shows their connection with relativity and quantum theory, respectively. The book covers Fourier methods and the equations of fluid mechanics, plus Maxwell's equations, topics that are not always present in classical mechanics books. Chaos theory is a modern incarnation of classical physics and finds significant coverage. The inclusion of special relativity and even the basic tenets of general relativity makes the book a very special resource. There is a simplified derivation of the precession of the perihelion of Mercury's orbit as deduced from general relativity.

After reading this book I am convinced that classical physics is a very 'modern' subject indeed. Over the years, the offering of classical mechanics courses has been reduced at many physics departments, sometimes to just a single term. Physics departments around the world would be well advised to ensure a two-term series on the subject that makes the crucial connections between classical and modern physics as expounded in FoCM.

Shantanu Basu, Professor, Department of Physics and Astronomy,
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# PREFACE

It gives me great pleasure in presenting *Foundations of Classical Mechanics* (*FoCM*) to the undergraduate students of the sciences and engineering.

A compelling urge to comprehend our surroundings triggered inquisitiveness in man even in ancient times. Rational thought leading to credible science is, however, only about two thousand years old. Science must be considered young, bearing in mind the age of the homo sapiens on the planet Earth. Over two millennia, extricating science from the dogmas and superstitious beliefs of the earlier periods has been an inspiring struggle; a battle lamentably not over as yet. Progress in science has nonetheless been rapid in the last few hundred years, especially since the fifteenth and the sixteenth century.

To be considered *classical*, just as for the arts or literature, a scientific formalism needs to have weathered the tests of acceptability by the learned over centuries, not just decades. The work of Nicolas Copernicus (1473–1543), Tycho Brahe (1546–1601), and Johannes Kepler (1571–1630) in the fifteenth, sixteenth, and seventeenth centuries was quickly followed, in the seventeenth through the nineteenth centuries, by a galaxy of stalwarts, many of whom excelled in both experimental and theoretical sciences, and some among them were proficient also in engineering and technology. They developed a large number of specialized fields, including of course physics and mathematics, and also chemistry and life sciences. Among the brilliant contributions of numerous researchers who belong to this period, the influence of the works of Galileo Galilei (1564–1642), Isaac Newton (1642–1727), Leonhard Euler (1707–1783), Joseph Lagrange (1736–1813), Charles Coulomb (1736–1813), Thomas Young (1773–1829), Joseph Fourier (1768–1830), Carl Friedrich Gauss (1777–1855), Andre-Marie Ampere (1775–1836), Michael Faraday (1791–1867), William Hamilton (1805–1865), and James Clerk Maxwell (1831–1879) resulted in a lasting impact which has withstood the test of time for a few hundred years already. Their contributions are recognized as *classical physics*, or *classical mechanics*. The theory of relativity, developed later by Albert Einstein (1879–1955), is a natural fallout of Maxwell's formalism of the laws of electrodynamics, and is largely considered to be an intrinsic part of *classical* physics. The theory of chaos is also an integral part of it, though it is a little bit younger.

The quantum phenomena, discovered in the early parts of the twentieth century, exposed the limitations of the classical physics. The quantum theory cannot be developed as any kind of modification of the classical theory. It needs new tenets. To that extent, one may have to concede that classical physics has failed. This would be, however, a very unfair appraisal. It does no justice to the wide-scale adequacy of classical physics to account for a large number of physical phenomena; let alone to its beauty, elegance and ingenuity. Niels Bohr established the 'correspondence principle', according to which classical physics and quantum theory give essentially the same results, albeit only in a certain limiting situation. Besides, the dynamical variables of classical physics continue to be used in the quantum theory, even if as *mathematical operators* whose analysis and interpretation requires a new mathematical structure, developed by Planck, Louis de Broglie, Schrödinger, Heisenberg, Bohr, Einstein, Pauli, Dirac, Fermi, Bose, Feynman, and others. Notwithstanding the fact that classical physics is superseded by the quantum theory, it *continues* to be regarded as *classical*, outlasting the times when it was developed. *Many of the very same physical quantities of classical physics continue to be of primary interest in the quantum theory*. Classical physics remains commandingly relevant even for the interpretation of the quantum theory. The importance of classical mechanics is therefore stupendous.

The laws of classical mechanics are introduced early on to students, even in the high-school, where they are taught the three laws of Newton. Many kids can recite all the three laws in a single breath. The simple composition of these laws, however, hides how sophisticated and overpowering they really are. The subtleties are rarely ever touched upon in high-school physics. The first course in physics for students of science and engineering after high-school education is when Newton's laws must therefore be *re-learned* to appreciate their depth and scope. The central idea in Newton's formulation of classical mechanics is inspired by the principles of causality and determinism, which merit careful analysis. Besides, students need an exposure to the alternative, equivalent, formulation of classical mechanics based on the Hamilton–Lagrange principle of variation. The theory of fractals and chaos which patently involves classical determinism is also an intrinsic part of classical mechanics; it strongly depends on the all-important initial conditions of a dynamical system. The main topics covered in this book aim at providing essential insights into the general field of classical mechanics, and include Newtonian and Lagrangian formulations, with their applications to the dynamics of particles, and their aggregates including rigid bodies and fluids. An introduction to the theory of chaos is included, as well as an essential summary of electrodynamics, plus an introduction to the special and the general theory of relativity.

*FoCM* is the outcome of various courses I have offered at the Indian Institute of Technology Madras (IITM) for over three decades. During this period, I also had the opportunity to offer similar courses at the IIT Mandi. Over the National Knowledge Network, this course was offered also at the IIT Hyderabad. During the past four years, I have offered courses at the same level at the IIT Tirupati, and also at the Indian Institute of Science Education and Research Tirupati (IISER-T). Besides, during my sabbatical, I taught a course at the University of Western Ontario (UWO), London, where Professor Shantanu Basu had invited me to teach Classical Mechanics. I am indebted to IITM, IITMi, IITH, UWO, IITT, and IISERT for the teaching opportunities I had at these institutions. My understanding of the subject has been greatly enhanced by the discussions with many students, and colleagues, at the institutions where I taught, especially with Dr C. Vijayan and Dr G. Aravind at IITM, and Dr S.R.Valluri, Dr Shantanu Basu, and Dr Ken Roberts at the UWO. The gaps in my understanding of this unfathomable subject are of course entirely a result of my own limitations. The colleagues and students whom I benefited from are too numerous to be listed here; and even if I could, it would be impossible to thank them appropriately. Nevertheless, I will like to specially acknowledge Dr Koteswara Rao Bommisetti, Dr Srijanani Anurag Prasad, and Dr Girish Kumar Rajan for their support. Besides, I am privileged to acknowledge the National Programme for Technology Enhanced Learning (NPTEL) of the Ministry of Human Resource Development, Government of India, which videographed three of my lecture courses, including one on 'Special/Select Topics in Classical Mechanics: STiCM' which strongly overlaps with the contents of this book. The various courses that I offered at the four different IITs, the IISER-T, the UWO, and the STiCM (NPTEL) video-lecture courses have all contributed to the development of *FoCM*.

The topics included in *FoCM* are deliberately chosen to speed up young minds into the foundation principles and prepare them for important applications in engineering and technology. Possibly in the next twenty, or fifty, or a hundred years, mankind may make contact with extra-terrestrial life, understand dark matter, figure out what dark energy is, and may comprehend comprehensively the big bang and the very-early universe. The 'multiverse', if there were any, and possibly new physics beyond the 'standard' model, will also be explored, and possibly discovered. Students of science must therefore rapidly prepare themselves for major breakthroughs that seem to be just around the corner in the coming decades. *FoCM* aspires to provide a fruitful initiation in this endeavour. Engineering students also need a strong background in the foundations of the laws of physics. The GPS system would not function without accounting for the special theory of relativity, and also the general theory of relativity. GPS navigation, designing and tracking trajectories of ships, airplanes, rockets, missiles, and satellites, would not be possible without an understanding, and nifty adaptation, of the principles of classical mechanics. Emerging technologies of driverless cars, drone technology, robotic surgery, quantum teleportation, etc., would require the strategic manipulation of classical dynamics, interfacing it with devices which run on quantum theory and the theory of relativity. *FoCM* is therefore designed for both students of science and engineering, who together will innovate tomorrow's technologies.

A selection of an assorted set of equations from various chapters in *FoCM* appears on the cover of this book. This choice is a tribute to Dirac, who said "a physical law must possess a mathematical beauty", and to Einstein, who said "... an equation is for eternity". Of the top ten most-popular (and most-beautiful) equations in physics that are well known to internet users, two belong to the field of thermodynamics, and three are from the quantum theory. The remaining five, *and also some more which are of comparable importance and beauty*, are introduced and discussed in *FoCM*.

*FoCM* can possibly be covered in two undergraduate semesters. Contents from other books may of course be added to supplement *FoCM*. Among other books at similar level, those by J. R. Taylor (*Classical Mechanics*), David Morin (*Introduction to Classical Mechanics*), and by S. T. Thornton and J. B. Marion (*Classical Dynamics of Particles and Systems*) are my favourites. Along with these books, I hope that *FoCM* will offer a far-reaching range of teaching and learning option.

It has been a pleasure working with the Cambridge University Press staff, specially Mr Gauravjeet Singh Reen, Ms Taranpreet Kaur, Mr Aniruddha De, and Mr Gunjan Hajela on the production of this book. At every stage, their handling reflected their admirable expertise and generous helpfulness. The compilation of problems that are included in *FoCM* has been possible due to superb assistance from Dr G. Aarthi, Dr Ankur Mandal, Mr Sourav Banerjee, Mr Soumyajit Saha, Mr Uday Kumar, Dr S. Sunil Kumar, Mr Krishnam Raju, Mr Aakash Yadav, Mr Aditya Kumar Choudhary, Mr Pranav P. Manangath, Ms Gnaneswari Chitikela, Mr Naresh Chockalingam S, Ms Abiya R., Mr Harsh Bharatbhai Narola, Mr Parth Rajauria, Mr Kaushal Jaikumar Pillay, Mr Mark Robert Baker, and Ms Gayatri Srinivasan. A number of books and internet sources were consulted to compile the problem sets.

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