

Introduction

We believe that physics would make it possible for us to comprehend the nature of the physical universe from our observations and investigations. We expect the methods of physics to equip us with a capability to take cognizance of a physical reality, and describe it in terms of crisp, succinct *laws*. Physics endeavors to recognize consistent patterns in natural events, and formulate them in a precise, unambiguous, and verifiable manner. When you look around, you would notice that there is mostly *irregularity* in much of what we experience in our daily life. When dust is raised by blowing winds, the dust particles seem to move in random directions, chaotic; the tree leaves flutter unpredictably. If a dense forest were to catch fire, it would be hard to tell which the next tree would be that the fire would gut. On the other hand, if you hold an object and let go of it, it always falls down, accelerating toward the ground at 9.8 m/s^2 (with only minor alterations over the Earth's surface). This would be so no matter what the mass or the size of the falling object is—let alone what its color or smell—in fact, just no matter which object was dropped. The *free fall* occurs at an acceleration which is even quantitatively *predictable*, of course only if the object fell under gravity alone, undisturbed by any other interaction, including friction with air-molecules or their buoyant effects. Likewise, if you have two electric charges, Q_1 and Q_2 at a distance r , they would repel or attract each other, depending only on the nature of the charges being respectively like or unlike. No matter what the individual values Q_1 and Q_2 of the two charges are or who performed the experiment, where it was performed, or for that matter, at what distance the charges were initially held from each other, the magnitude of the force between the charges would always be proportional to $\frac{Q_1 Q_2}{r^2}$.

If you look at the sky on a clear night, you would see lovely stars, even galaxies, and supernovae, sometimes amid recognizable patterns and shapes that we call constellations. Further, if you measure the Doppler shifts of the spectral lines present in the light from the stars, you could compile some systematic information and discover that the universe is actually expanding. By further carrying out some clever calculations, you can possibly discover at least some of the *physical laws* which describe the expansion of the universe. In fact, you can even determine the age of the universe from the big bang by studying such observable phenomena. On the other hand, in some *local* part of the cosmos, we may as well observe galaxies not flying away from each other; they may even be on a collision path

toward each other. Detailed observations and calculations have now established that the Andromeda galaxy would actually collide with ours, the ‘Aakaash Ganga’, commonly called the ‘Milky Way’, in about four billion years. Notwithstanding such irregular events prompted by rather special situations, *most* of the galaxies, nonetheless, fly *away* from each other.

As Eugene P. Wigner described in his Nobel Prize Lecture (1963), what we call as a *law of nature*, is a statement on the *regularities* in nature. Physicists arrive at the laws of nature after systematically separating the infrequent departures from the same, like the collision of the galaxies. Systematic data analysis from extensive studies shows that when not affected by buoyant effects and wind currents, even feathers fall just as do stones, toward the earth at an acceleration of $\sim 9.8 \text{ m/s}^2$, and that the universe is actually expanding; in fact, the galaxies seem to be not merely moving *away* from each other, but are actually *accelerating away* from each other. It is then the *regularities in natural phenomena* that physicists revere as the laws of nature.

It is, however, necessary, every once so often, to review and reassess, what we believe the laws of nature are. This is of course mandatory when a physical event is detected in a reliable experiment that does *not* conform to one of the hitherto-known-laws. We must then improvise the formulation of that law. We must even be prepared to completely abandon its basis if a mere modification to it might turn out to be insufficient. Our understanding of the universe thus progresses incrementally, requiring improvisations every now and then. The process of discovering the laws of nature, modifying, or even completely replacing them by new enunciations as and when necessary, is the very signature of the scientific pursuit.

The backdrop canvas of the scientific inquiry is huge. The range of values which the fundamental physical quantities—*mass* (M), *length* (L) and *time* (T)—take in the universe is mind-boggling. It is well beyond the day-to-day human experience: from extremely tiny to absolutely humongous. On a cognizable mass scale, the photons and the gluons are virtually massless. In the tiny world of the elementary particles alone, we already have a range of mass values covering more than 10^{10} orders of magnitudes. The mass of an electron is of the order of $\sim 10^{-30} \text{ kg}$, a one-rupee coin has a mass of $\sim 10^{-3} \text{ kg}$, the Sun has a mass of $\sim 10^{30} \text{ kg}$, and the mass of the universe (including ‘dark’ matter) is estimated to be $\sim 10^{52} \text{ kg}$. On the length scale, we have again an enormously vast range. The tiniest of elementary particles have a size of the order of an *am* (an ‘attometer’ being 10^{-18} m), the Sun has a diameter of $\sim 10^9 \text{ m}$, and the radius of our, cognizable, universe is $\sim 4 \times 10^{26} \text{ m}$, and viable speculations on multiverses are strongly building even as I write. The range of the order of magnitude numbers for mass and size is staggering, from the exceedingly tiny to the enormously gigantic. Physicists thus deal with objects from the very tiny to the very big.

It seems therefore almost arrogant that physics aims at understanding the dynamics of an object in such a multifarious, intricate, and enormous universe in terms of only a few parameters, which would not merely describe the ‘state of a physical system’ but also account for its dynamical evolution in terms of an appropriate equation of motion. What is breath-taking is that advances in physics have already provided a huge amount of insight and understanding about the universe, *and* its evolution, in terms of just a few parameters and a

very few ‘laws’. What is especially remarkable is that these advances have been made merely in the last few hundred years, despite the much longer life-time (about 200,000 years) of the human species on our planet.

The first known conception of what is referred to as the atomistic approach to analyze the material world is perhaps found in the works of Maharshi Kanad (~600 BCE), an Indian sage who developed the *Vaishesikha* philosophy. This scheme seeks to explore the cognizable universe in terms of nine elements: air, earth, fire, mind, self, sky, space, time, and water. The Vaishesikha scheme has of course now been superseded by the periodic table of elements, and further by the elementary particles of the standard model of physics, which would perhaps be upgraded someday to include what is now considered to be ‘dark matter’ (and may be even the ‘dark energy’?) What has come to be regarded as the scientific method is also based on a somewhat similar principle in the sense that it begins with a set of *ansatz*, which is an initial assumption or an axiom that we make. It is a basic tenet from which we develop our physical model. The *ansatz* is, in some sense, an intelligent guess at an underlying fundamental physical principle that is intuitively recognized and verifiable, even if not derivable from anything else. It needs to be robust enough to stand scrutiny by physical observations, and must not lead to logical inconsistencies. A scientific theory is then built in a logically consistent formalism based on the *ansatz*.

Advances in science have been phenomenal. It is now possible to fathom distances from as tiny as 10^{-35} m to very vast that even light would take billions of years to traverse. Even as physicists are toiling to determine what may have happened in the first 10^{-43} s since the beginning of the universe, they estimate that the universe itself is over 10^{+17} s (~13.8 billion years) old. Major events during the period over which the universe has evolved are depicted in Fig. I.1 [1]. The physical events and the dynamics in the ponderable evolution of the universe are absolutely stupefying. The universe is made up of just a few elementary particles. The atoms were once thought to be the smallest ingredients of matter. In the early part of the twentieth century, following the experiments such as those conducted by J. J. Thomson and E. Rutherford, it became known that the atom itself has an internal structure consisting of the central nuclei and the electrons. Even tinier ingredients exist, namely the quarks and the leptons (and their anti particles), which together with the so-called gauge bosons and the Higgs boson, constitute the fundamental particles as we know now. The quarks participate in the making up of the protons and the neutrons, which along with the electrons are ingredients of atoms, which further partake in the composition of the molecules and clusters which constitute condensed matter that make up both ‘beings’ and ‘things’ – living and inanimate. From grains of sand to mountains, from planets and stars to the millions of galaxies in the universe, and from life-cells to flowers and trees, from microbes to animals and humans, every *thing* and every *being* is made up of elementary particles. By and large, physics does not directly endeavor to address just what constitutes *life* and *consciousness*, *at least not quite as yet*. Nevertheless, attempts to address even these mind-boggling mysteries have also been undertaken already by physicists. Broadly speaking, physicists seek to discover ‘*what*’ the laws of nature are, and ‘*how*’ they influence the physical events; rather than ‘*why*’ the laws are what they are. Toward this study, physics aims at discovering the *minimal* set of laws of

nature in terms of which all else that happens in the universe can be described. It should now be clear why the most general enunciations of the fundamental principles are worthy of being called the ‘fundamental universal laws of nature’.

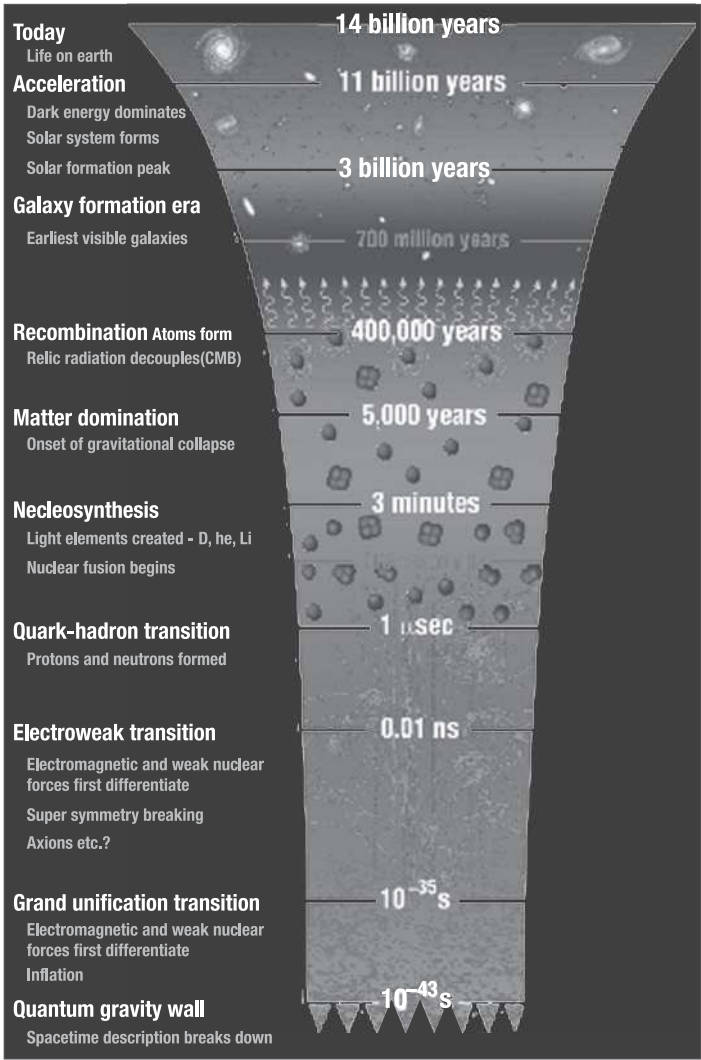


Fig. I.1 Major events in the evolution of the universe [1]. Even as much is known now, much also remains to be known. Some of this ignorance is referred to as ‘dark matter’ and ‘dark energy’, whose effects have been sensed, but it poses itself as a huge challenge to physics. This is only a small part of the reason why getting into a career in physics is a very exciting proposition today. Mankind is in an exciting time-slot in which exciting discoveries are just round the corner.

The laws of nature that account for the effects of relativity and quantum physics are relatively recent (considering the age of man), just over a hundred years old. Until about the seventeenth century, even ‘gravity’ was a total mystery. In earlier times, the Greeks *explained away* why

an object falls when let to do so by stating that the earth is the natural abode of things, and objects fall just as horses return to their stables. Earth was earlier believed to be flat, and other planets in our solar system were considered to be wandering stars. Ignorance led to fear and superstition. Shadows cast in eclipses influenced life-styles and contaminated human thought, forming prejudices that are regrettably hard to break, even as science has scooped out much of the truth from the shadows of ignorance and fear.

A few early deductions known to some ancient Indian and Greek astronomers have of course turned out to be correct. For example, Aryabhata, in the fifth century CE, had inferred and proclaimed that Earth has a spherical shape, not flat. Brahmagupta, in the seventh century, had estimated to an extremely high degree of accuracy the circumference of Earth. His estimate of ~36,210 km is remarkably close to the value obtained using modern technology, which is 40,075 km (~24,900 miles). The Kerala astronomers [2], in the fifteenth century CE, *even before Copernicus*, were quite aware of the Sun’s central position in our solar system, even if this knowledge is often referred to as the *Copernican* revolution. Nevertheless, many other thinkers continued to maintain incorrect, often absurd, ideas about the universe, including about our solar system. It was in the backdrop of such prejudices that Nicolus Copernicus (1473–1543) advanced his heliocentric frame of reference to describe our solar system. It was indeed courageous of him to do so, for a system that did not have Earth at the center of the universe was in sharp contradiction with the belief promoted at that time by the church. Copernicus’ book could be published only well after he died, so that he would not be persecuted by the church. When it was finally published, the church prohibited it [3]. Copernican view was upheld by Galileo Galilee (1564–1642; Fig. I.2), who was then predictably prosecuted by the Catholic Church for going against its doctrine. The trial of Galileo is one of the most famous ones [4]. It is against the dogmas of his times that Galileo arrived at robust scientific conclusions based on systematic measurements. Apart from upholding the Copernican viewpoint, Galileo carried out ingenious experiments which laid the very foundations of classical mechanics. Galileo is thus fittingly regarded as the *father of experimental physics*.



Nicolaus Copernicus (1473–1543)



Galileo Galilee (1564–1642)

Fig. I.2 The scientific methods are old, but often the most significant beginnings are referenced from the works in the fifteenth and the sixteenth centuries. The names of Copernicus and Galileo are among the tallest in this period.

Classical mechanics refers to the developments in science mostly during the fifteenth to the nineteenth century through the *classic* works of Nicolaus Copernicus (1473–1543), Tycho Brahe (1546–1601), Johannes Kepler (1571–1630), Galileo Galilei (1564–1642), Isaac Newton (1643–1727), Leonhard Euler (1707–1783), Joseph-Louis Lagrange (1736–1813),

William Rowan Hamilton (1805–1865), Charles Augustine de Coulomb (1736–1806), Andre-Marie Ampere (1775–1836), Michael Faraday (1791–1867), Heinrich Lenz (1804–1865), James Clerk Maxwell (1831–1879), etc.

Several results of the scientific studies based on classical mechanics were subsequently found, nevertheless, to be only approximately correct, no matter how good the approximation is. The redress required revolutionary ideas which led to the development of the quantum theory in the twentieth century. Classical mechanics, however, provides the essential nucleus, and continues to be adequate in a large number of situations. Even the special theory of relativity developed by Albert Einstein (1889–1955) connects intimately to the classical electromagnetic theory and thus integrates seamlessly into the philosophy of classical mechanics.

This book aims at laying down the foundations of classical mechanics. Methodologies developed to account for the analysis of motion and dynamics of simple physical systems like point-particles are dealt with in Chapters 1 through 6. Extensions of these formalisms to study the dynamics of composite rigid bodies are discussed in Chapters 7 and 8. Chapter 9 provides an introduction to the theory of chaos and fractals, which judiciously belongs to the increasingly important domain of classical mechanics which is very sensitive to the initial conditions that describe the physical system under investigation. Chapters 10 and 11 provide the framework for studying fluids. The formalism in these two chapters is applicable to study the state of *matter* that flows, and it also provides the framework to describe imperative properties of the electromagnetic *field*. A condensed summary of the laws of electromagnetism is provided in Chapter 12 and 13. Finally, in Chapters 13 and 14, the foundations of the *special* and the *general* theories of relativity are laid down. The role of symmetry in natural laws is exhibited in several chapters, permeating through the whole book. The astute reader will hopefully find this insightful. This approach links prefatory concepts in the first course in Physics to contemporary research in frontiers areas, even if the intermediate links are challenging. As the students advance through the contents of this book, it is earnestly hoped that they will enjoy the romance in physics and beauty in its simplicity. Also, very importantly, the students will, I hope, get adept at the necessary rigor in the formulation of the physical laws. Mathematics lends itself as a constructive collaborator in the development of the physical laws. No wonder it is then that Newton developed the law of gravity and differential calculus together. In this book too, then, mathematical methods are introduced as and where required, sometimes as an important technique, but more often as an inseparable collaborator in the very nature of the laws of physics.

The mind-boggling, counter-intuitive, quantum theory is, by the very connotation of *classical mechanics*, outside the scope of this book. *Classical mechanics* provides the *prerequisite* foundation on which the student must stand firmly before she/he may leap into the quantum world. An attempt is therefore made in this book to comprehensively elucidate the path that led to the *classical*, non-quantum, laws of nature. I hope that the student-readers will rediscover the educative excitement felt by physicists in the last few hundred years in learning how simple enunciations of the classical laws account for a really vast range of phenomena in the physical universe. This text-book aims at providing a robust

under-structure to students of physics and engineering, to prepare them to appreciate advance discoveries in science, and also developments in new technologies, whose frontiers are pushed by the hour even as these pages are written.

References

- [1] *The Universe in a Nutshell*. 2015. <http://cjuniverseinanutshell.blogspot.com/2015/04/a-brief-history-of-relativity-cont.html>. Accessed on 26 April 2019.
- [2] Ramasubramanian, K. M. D. S., M. D. Srinivas, and M. S. Sriram. 1994. 'Modification of the earlier Indian planetary theory by the Kerala astronomers (c. 1500 CE) and the implied heliocentric picture of planetary motion.' *Current Science* 66(10): 784–790.
- [3] *Vatican Bans Copernicus' Book*. <https://physicstoday.scitation.org/doi/10.1063/PT.5.030911/full/>. Accessed on 26 April 2019.
- [4] Carlo Maccagni. 2009. *Galileo Galilei: A New Vision of the Universe*. http://www.ncert.nic.in/publication/journals/pdf_files/school_science/sc_June_2009.pdf. Accessed on 26 April 2019.