CAMBRIDGE

Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

Index

ascending ladder MAP, 294
  excursion measure, 294
  matrix Laplace exponent, 294

beta function, 431, 432
β-subordinator, 78
  Lamperti-stable, 81
Borel–Cantelli Lemma, 438
Brownian motion, 28
  hitting unit sphere, 363
  Spitzer's winding limt, 412
  subordination, 71
  walk-on-spheres algorithm, 381

Cauchy density, 16
Cauchy process, 63
  transition density, 63
Cauchy's estimates, 430
censored stable process, 136
  lower envelope, 277
  upper envelope, 278
characteristic exponent, 2
  Cauchy distribution, 3
  d-dimensional, 21
  isotropic stable, 24
  isotropic stable process, 71
  Lévy–Khintchine formula, 2
  normalised, 24
  normalised stable, 12
  stable distribution, 3
  stable process, 59
compensation formula, 444
creeping, 49

deep factorisation, 295, 330, 331, 403
  d-dimensional, 404
  in dimension $d \geq 0$, 403
  inverse ladder MAP exponent, 334
  inverse matrix factorisation, 332
  ladder MAP exponent, 348
  ladder MAP potential matrix, 335
Donsker-type convergence, 39
double gamma function, 96, 432
double sine function, 184, 434
drifting, 35
duality, 41
  duality lemma, 41
  Hunt–Nagasawa duality, 120, 442
  Markov additive process, 290
  q-resolvent, 42
  semigroup, 42

entrance law, 117
  censored stable process, 144
  duality, 119
  pssMp, 118
  radial part of isotropic stable process, 151
  recurrent extension, 122
  stable process conditioned to stay positive, 133
  time reversal, 120
Esscher transform
  Lévy process, 38
  Markov additive process, 291
excursion, 44
excursion theory
  excursion measure, 45, 177
  Markov additive process, 292
exit problem
  hitting points, 160
  interval, 154, 156, 159

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

exponential functional
  log moments, 253

Feller process, 441
  Lévy process, 34
  regular, 115, 121, 286, 300, 441
Feller semigroup, 441
filtration
  les conditions habituelles, 440
  MAP, 287, 300
  natural, 440
  natural enlargement, 122, 152, 176, 287, 300, 302, 308, 331, 440
  ssMp, 302
  stable, 308
  stable process, 122, 313, 331
first entrance from a ball
  triple law, 395
first entrance in a ball, 366
  triple law, 395
first entrance into a bounded interval, 162, 163, 318
first exit from a ball, 366
first exit from an interval, 154, 156, 159
first hitting of a sphere, 355
first hitting of a two-point set, 168
first passage problem, 50
  asymptotic overshoot subordinator, 52
  Lévy process, 50
  reflected stable process, 177
  stable process, 69
  stable subordinator, 68
  subordinator, 51
Fourier transform, 430

gamma function, 431
  duplication formula, 432
  infinite product representation, 431
  recursion formula, 431
  reflection formula, 432
generalised last exit time, 441
generalised polar coordinates, 301

hitting points, 42
  first hitting of a two-point set, 168
  in an interval for stable process, 160
  Mellin transform of hitting time for stable process, 174
  stable process, 65
hitting points in an interval, 160
Hunt–Nagasawa duality, 120, 419, 442
hypergeometric function, 435
  linear combination of, 435
  special formulae for, 436
hypergeometric Lévy process, 78, 84, 252
  admissible parameters, 84
  first passage problem, 89

infinitesimal generator, 35, 55
  Lévy process, 35

Jacobian
  sphere inversion, 362
  sphere inversion with reflection, 362

Lévy measure, 2
  isotropic stable, 25
  stable Cartesian coordinates, 25
  stable distribution, 5
  stable polar coordinates, 25
  stable process, 58
Lévy process, 27
  asymmetry, 31
  β-subordinator, 78
  Brownian motion, 28
  Cauchy process, 63
  characteristic exponent, 27, 54
  compound Poisson with drift, 29, 32
  creeping, 49
  definition, 27
  Donsker-type convergence, 39
  drifting, 35
  duality, 41, 55
  Esscher transform, 38
  excursion theory, 44
  exponential change of measure, 37
  exponential moments, 37
  Feller property, 34, 54
  first passage problem, 50
  hitting points, 42
  hypergeometric, 78, 84, 252
  in dimension $d \geq 2$, 54
  infinitesimal generator, 35, 55
  killing, 30
  Lévy–Itô decomposition, 29
  Lévy–Khintchine formula, 28
  Lamperti-stable, 87
  Markov property, 34, 35
  moments, 36, 55
  non-arithmetic, 228, 260
  oscillating, 35
  path variation, 31
  polarity of points, 55
  recurrence, 40, 55
  regularity, 44
  resolvent with killing at first passage, 52

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

semigroup, 34
spectrally one-sided, 33
strong Markov property, 34
subordinator, 33
transience, 40, 55
triple law, 50
Wiener–Hopf factorisation, 44, 47
Lévy–Khintchine formula, 2
in dimension $d \geq 2$, 54
isotropic stable, 24
regularisation function, 3
stable distribution, 3
Lamperti transform, 115
Lamperti–Kiu transform, 287
in dimension $d \geq 2$, 301
Lamperti-stable MAP, 307
dual process, 309
Lamperti-stable process, 87
admissible parameters, 88
subordinator, 81
Laplace exponent
spectrally negative stable process, 63
stable subordinator, 64
Laplace–Fourier transform, 430
last exit time, 442
generalised, 441
law of the iterated logarithm
radial maximum of a stable process, 221
spectrally negative stable process, 217
stable process conditioned to limit to 0 from above, 274
stable subordinator, 208
Liouville's theorem, 430
lower envelope
censored stable process, 277
pssMp, 235, 239
radial part of isotropic stable process, 280, 283
stable process conditioned to hit 0 continuously, 328
stable processes, 219
stable processes conditioned to limit to 0 from above, 273

MAP, *see* Markov additive process
Markov additive Poisson point process, 444
compensation formula, 445
Markov additive process, 287
ascending ladder MAP, 293
compensation formula, 311
deep factorisation, 295
dual ladder MAP, 337
dual to Lamperti-stable MAP, 309
Esscher transform, 291
excursion theory, 292
filtration, 287, 300
isotropic self-similar Markov process, 302
ladder MAP dual potential, 337
ladder MAP potential, 335, 387
Lamperti-stable MAP, 307
matrix exponent, 289, 291
matrix Laplace exponent, 294
modulator, 287
ordinator, 287
reflected, 292
strong law of large numbers, 292
time change, 291
Wiener–Hopf factorisation, 295, 403, 404
with discrete modulation, 287
Markov process
Hunt–Nagasawa duality, 442
Markov property
Lévy process, 34, 35
Mellin transform, 430
first hitting time of point for stable process, 174
Morera's Theorem, 430

Nagasawa duality, *see* Hunt–Nagasawa duality
natural filtration, 440
naturally enlarged filtration, 440
Newtonian Poisson formula, 363, 368

oscillating, 35

perpetuity, 437
tail distribution, 438
Pochhammer symbol, 435
point of closest reach, 166, 389
point of furthest reach, 166
Poisson point process, 443
Markov additive Poisson point process, 444
Poisson random measure, 443
positive self-similar Markov process, 115, 227, 260
censored stable process, 136
conservative, 117
entrance at 0, 117
envelopes, 227
last passage time, 228
non-conservative, 117
path functional distributions, 260
radial part of stable process, 144
radius of an isotropic self-similar Markov process, 303

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE
Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

recurrent extension, 121
stable process conditioned to limit to 0 from above, 133
positivity parameter, 61
pssMp, *see* positive self-similar Markov process
path decomposition, 227

radial excursion theory, 383
excursion measure, 385, 398
radial part of isotropic stable process
lower envelope, 280, 283
upper envelope, 281, 284
radial part of stable process, 144
radial reflection, 406
limiting distribution, 406
radially reflected stable process, 331
limiting distribution, 331
real self-similar Markov process, 287, 288
recurrence, 40, 55
criterion for Lévy processes, 40
stable process, 64
recurrent extension, 122
regular Feller process, 300
regularisation function, 3
regularity of half-line, 44
renewal measure, 50
residue, 429
resolvent, 41
adjusted resolvent, 74
entrance into a bounded interval, 165, 319
exit from a half-line, 159
exit from an interval, 157
first entry into ball, 370
first exit from ball, 370
first hitting of a sphere, 364
first hitting of point, 174
first passage of reflected process, 180
killed process, 42
Lévy process killed at first passage, 52
ladder height, 50
q-excessive, 43
q-resolvent, 41
stable process, 72
Riesz–Bogdan–Żak transform
in dimension $d \geq 2$, 324
one dimensional, 316
rssMp, *see* real self-similar Markov process

Schwartz's Reflection Principle, 430
self-similar Markov process, 287
censored stable process, 136
entrance at 0, 118, 303, 305
in dimension $d \geq 2$, 300
isotropic, 302
radial part of stable process, 144
stable process conditioned to avoid the origin, 314
stable process conditioned to hit 0 continuously, 314
stable process conditioned to limit to 0 from above, 133
stable process conditioned to stay positive, 128
stable process killed on entering $(-\infty, 0)$, 122
self-similarity, 60
stable process, 60
semigroup, 440
Feller property, 441
Lévy process, 34, 440
Skorokhod space, 118, 265, 304, 439
Skorokhod topology, 440
weak convergence, 440
spectrally negative stable process
law of the iterated logarithm, 217
sphere inversion, 351
with reflection, 354
ssMp, *see* self-similar Markov process
stable density, 14, 16
Cauchy, 16
power series, 16
stable distribution, 1
Cauchy, 3
Cauchy density, 16
definition, 1
density function, 14, 16
higher dimensions, 20
isotropic, 23
Lévy measure, 5
moment formula, 14
moments, 9, 14
normalised, 11
one dimension, 1
parameters, 12
positivity parameter, 11, 15
symmetric, 22
Zolotarev's duality, 15
stable process, 4, 58
adjusted resolvent, 74
admissible parameters, 61
Cauchy process, 63
censored, 136

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

characteristic exponent, 59, 71
conditioned to avoid the origin, 314
conditioned to hit 0 continuously, 314
conditioned to limit to 0 from above, 133
conditioned to stay positive, 128
deep factorisation, 404
drifting and oscillating, 64
excursion theory, 177
filtration, 122, 308, 313, 331
first entrance from a ball, 395
first entrance in a ball, 366, 395
first entrance into a bounded interval, 162, 163, 318
first exit from a ball, 366
first exit from an interval, 154, 156, 159
first hitting of a sphere, 355
first hitting of a two-point set, 168
first passage, 68
first passage problem, 69
hitting points, 65
hitting points in an interval, 160
in dimension $d \geq 2$, 70
isotropic, 70
killed on entering $(-\infty, 0)$, 122
Lévy measure, 58
moments, 64
normalised, 60
one dimension, 58
path variation, 62
point of closes and furthest reach, 166
polarity, 72
positivity parameter, 61
radial process, 144
radial reflection, 406
reflected, 176
regularity of half-line, 66
resolvent density, 72
Riesz–Bogdan–Żak transform, 316, 324
self-similarity, 60
spectrally negative, 63
subordinator, 64, 206
transience and recurrence, 64, 72
transition density, 60
up-crossings of origin, 422
Wiener–Hopf factorisation, 66
winding at infinity, 414
winding at origin, 418
stable process conditioned to hit 0 continuously
lower envelope, 328
upper envelope, 328
stable process conditioned to limit to 0 from above
law of the iterated logarithm, 274
lower envelope, 273
upper envelope, 274
stable process conditioned to stay positive
law of the iterated logarithm, 270
lower envelope, 266
upper envelope, 268, 269
stable subordinator, 206
law of the iterated logarithm, 208
stopping time, 441
strong Markov process, 441
subadditive functions, 436
submultiplicative function, 37
subordinator, 33
asymptotic first passage, 52
β-subordinator, 78
Lamperti-stable subordinator, 81
Laplace exponent, 33
stable subordinator, 64

transience, 40, 55
criterion for Lévy processes, 40
stable process, 64
triple law, 50

up-crossings of stable process, 422
upper envelope
censored stable process, 278
pssMp, 241, 248, 250
radial part of isotropic stable process, 281, 284
stable process, 214
stable process conditioned to hit 0 continuously, 328
stable process conditioned to limit to 0 from above, 274
stable process conditioned to stay positive, 268, 269

walk-on-spheres, 374
Brownian motion, 381
first exit problem, 375
first exit problem with resolvent, 379
Monte Carlo, 375
number of steps to completion, 375
stable process, 375
Wiener–Hopf factorisation, 47
censored stable, 143
deep factorisation, 295, 403
Doney classes, 196
Doney's factorisation, 196, 203

© in this web service Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
978-1-108-48029-1 — Stable Lévy Processes via Lamperti-Type Representations
Andreas E. Kyprianou , Juan Carlos Pardo
Index
More Information

Doney–Kuznetsov factorisation, 184
Kuznetsov's factorisation, 183, 185
Lévy process, 47
radius of stable process, 150
space-time factorisation, 47, 48
spatial factorisation, 48
stable conditioned to limit to 0 from above, 135
stable conditioned to stay positive, 132
stable killed on exiting $(-\infty, 0)$, 125
stable process, 66
temporal factorisation, 47, 48
winding number, 412
winding of Brownian motion, 412
winding of stable process
at infinity, 414
at the origin, 418

Zolotarev's duality, 15

© in this web service Cambridge University Press www.cambridge.org