The Dialogical Roots of Deduction

This comprehensive account of the concept and practices of deduction is the first to bring together perspectives from philosophy, history, psychology and cognitive science, and mathematical practice. Catarina Dutilh Novaes draws on all of these perspectives to argue for an overarching conceptualization of deduction as a dialogical practice: deduction has dialogical roots, and these dialogical roots are still largely present both in theories and in practices of deduction. Dutilh Novaes' account also highlights the deeply human and in fact *social* nature of deduction, as embedded in actual human practices; as such, it presents a highly innovative account of deduction. The book will be of interest to a wide range of readers, from advanced students to senior scholars, and from philosophers to mathematicians and cognitive scientists.

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The Dialogical Roots of Deduction

Historical, Cognitive, and Philosophical Perspectives on Reasoning

Catarina Dutilh Novaes

Vrije Universiteit Amsterdam and University of St Andrews



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Preface

This book has been in the making for over ten years or, to put it differently, I have been thinking about deduction from a dialogical perspective for over ten years. In February 2010, I submitted the grant proposal "The Roots of Deduction" to the Netherlands Organisation for Scientific Research (NWO) (VIDI 720225), which I had been working on for at least six months before that. At the time, I did not yet have a permanent academic position, and securing this grant was of vital importance for my career, as it would put me in a strong position to negotiate a permanent position somewhere (such is the system in the Netherlands, for better or worse). On October 7, 2010 (I remember the exact date, as it also happens to be the birthday of my older daughter), I received the happy news that my application had been successful. I could finally relax, knowing that my future in academia was more or less guaranteed from then on. I was then offered a permanent position at the Philosophy Faculty of the University of Groningen, where I started running the project in July 2011.

Ten years may sound like a long time for a book to be written, and indeed it should have been completed years ago (roughly coinciding with the end of the project funding in 2016). But life interfered with my plans, as it usually does (though these were particularly challenging years on the personal front for me). Still, late or not, here it is, finally; I owe it to the world (not to mention the funding agencies that generously supported me with taxpayer money) to share what I take to be the important findings presented here.

The main hypothesis defended throughout the book is the idea that deduction has dialogical roots, and that these dialogical roots are still largely present both in theories and in practices where deduction features prominently. Of course, this claim can be understood in many ways. The original inspiration for this approach came from the history of deduction, in particular the emergence of deduction in Ancient Greek philosophy and mathematics (discussed in Chapters 5 and 6). Defying the perils of the genetic fallacy, I hypothesized that these historical roots would still be present in current instantiations of conceptualizations and practices of deduction, albeit enmeshed with non-dialogical components picked up along the way. Thus, it seemed to me that the dialogical perspective would allow for a unified account of deductive

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theories and practices, which would explain at one stroke some puzzling and apparently disconnected features of the phenomena in question.

In the past years, I have published a number of papers where the dialogical hypothesis is applied to specific issues; however, the whole story requires a book-length treatment to bring all these different pieces together. Indeed, individually each of these findings only offers limited evidential support to the dialogical hypothesis; it is only when put together side by side that the strength of the dialogical account truly emerges. Put differently, the argumentative structure of this book can be described as an inference to the best explanation: the dialogical hypothesis is the best way to make sense of the variety of manifestations of deductive reasoning and argumentation across time and across domains – but, again, never forgetting the non-dialogical components picked up along the way.

To produce this grand dialogical narrative, I adopted the integrative methodology that I have deployed in most of my career (and which is described in detail in the conclusion of my monograph *Formal Languages in Logic*, Dutilh Novaes, 2012a). I had to approach deduction from a number of different angles, each of them requiring specific methods, such as 'traditional' philosophical conceptual analysis, historical analysis (mostly of philosophical texts, but also taking into account sociocultural and political factors), and empirically informed philosophical analysis, with extensive engagement with adjacent (empirical) disciplines including cognitive science, psychology, and education studies. (A fourth methodology, not extensively deployed in this book but present in some of the articles with related material, is philosophical analysis aided by formal tools.) Another term that I recently adopted to describe the kind of philosophy that I practice is 'synthetic philosophy' (Schliesser, 2019), which reflects the thought that philosophers are ideally placed to bring together findings from a number of disciplines that normally do not engage with each other much.

Accordingly, the book is divided into three main parts. Part I (Chapters 1 to 4) presents the philosophical roots of deduction, relying predominantly on conceptual analysis and engagement with previous authors who have addressed similar issues. Part II (Chapters 5 to 7) is dedicated to the historical roots of deduction, and there the main method used is traditional textual analysis of philosophical texts of the past. (In the future, I would like to add techniques from digital humanities to my toolkit as a historian of philosophy.) Part III (Chapters 8 to 11) focuses on the cognitive roots of deduction, deploying the method of empirically informed philosophical analysis by engaging extensively with findings from cognitive science and mathematical education studies, as well as quasi-sociological observations on the practices of mathematicians.

Chapter 1 starts by defining the explanandum of the whole book, i.e. the phenomenon (or phenomena) that it is about. There, I introduce deduction as

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having three main characteristics: necessary truth-preservation (which is perhaps the most central one, distinguishing deduction from other forms of inference and argument such as induction and abduction), perspicuity, and beliefbracketing. I also discuss a number of puzzling features of deduction, i.e. philosophical issues pertaining to deduction that remain open questions, as they have not yet been adequately 'solved.' In other words, there is a problem (rather a number of problems) with deduction, which justifies a book-length treatment of the topic such as this one. In Chapter 2, I argue that what is needed is a 'roots' approach, and briefly present the four main senses in which deduction has dialogical roots treated in this book: philosophically, historically, cognitively, and pertaining to mathematical practices. In Chapter 3, I discuss the prominent dialogical accounts of deduction and deduction proposed by Paul Lorenzen, Jaakko Hintikka, and Imre Lakatos. This discussion then allows me to offer a precise formulation of the dialogical model that I defend, the Prover-Skeptic model. In Chapter 4, I present a dialogical rationale based on the Prover-Skeptic model for the three main features of deduction identified in Chapter 1, and address a number of ongoing debates in the philosophy of logic - the normativity of logic, logical pluralism, logical paradoxes, and logical consequence. I argue that the Prover-Skeptic model sheds new light on these debates.

Part II starts with Chapter 5, where I turn to the historical roots of deduction in Ancient Greek philosophy and mathematics. I rely extensively on the work of G.E.R. Lloyd and Reviel Netz to argue that dialogical, debating practices in a democratic city-state like Athens were causally instrumental for the emergence of the axiomatic-deductive method in mathematics. The same sociocultural political background was decisive for the emergence of practices of dialectic, the kinds of dialogical interactions famously portrayed in Plato's dialogues. In turn, dialectic provided the background for the emergence of the first fully-fledged theory of deduction in history (that we know of at this point, at least), namely Aristotle's syllogistic. In Chapter 6, I turn to Aristotle and argue that syllogistic did indeed emerge from the dialectical matrix as well as from considerations pertaining to scientific demonstration and demonstration in mathematics. This means that, even early on, non-dialogical components motivated and were integrated into practices and theory of deduction. Chapter 6 also briefly discusses two other formidable ancient intellectual traditions, namely the Indian tradition and the Chinese tradition. It is argued that, while these were indeed highly sophisticated, fully-fledged theories of deduction are not to be found in them (in the early stages, at least; later developments in India may well count as theories of deduction). Finally, Chapter 7 retraces the genealogical development of deduction from the Ancient Greeks (unfortunately, not covering the late antiquity period - a regrettable lacuna) in the Latin and Arabic medieval traditions, in the early modern period, and finally

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with the emergence of mathematical logic in the nineteenth century. This chapter thus explains why we (i.e. twenty-first century philosophers) have by and large forgotten the dialogical roots of deduction, as mentalistic conceptions of logic and deduction became increasingly prominent.

Part III focuses on deduction in human cognition. It starts in Chapter 8 with a review of experimental work on deductive reasoning, which has shown that human reasoners do not seem to reason spontaneously according to the deduction canons. However, there are also experimental results suggesting that, when tackling deductive tasks in groups, performance comes much closer to these canons. These findings already offer a partial vindication of the dialogical conception of deduction insofar as they show that, when given the opportunity to engage in dialogues with others, humans become better deductive reasoners. In Chapter 9, I look at the three main features of deduction defined in Chapter 1 from a cognitive, empirically informed perspective. I discuss experimental findings that lend support to the dialogical conceptualization of these three features presented in Chapter 4. I also discuss the notion of internalization formulated by Vygotsky, which allows for an explanation of how deductive practices can also take place in purely mono-agent situations: as an intrapersonal enactment of interpersonal dialogues. The upshot of Chapter 9 is that framing deductive practices dialogically provides cognitive scaffolding that facilitates the ontogenetic development of deductive reasoning in an individual. Chapter 10 in turns focuses on the 'phylogeny' of deduction, i.e. how deductive reasoning may have emerged given the genetically endowed cognitive apparatus of humans. I argue that the emergence of deduction should not be viewed as genetically encoded, but rather as a product of cultural processes. Finally, Chapter 11 may seem a bit of an outlier in Part III, but it investigates deductive practices in (what I take to be) their main current instantiations, namely practices of mathematical proofs. Here again the dialogical hypothesis delivers a compelling account of a number of features of these practices; as it turns out, the fictive characters Prover and Skeptic are in fact instantiated by real-life mathematicians, e.g. referees for journals acting as Skeptics.

This book is the result of countless 'dialogues' I've engaged in over the years, so there are many people to thank for their contributions of different kinds. First of all, I want to thank the funding agencies that made this research possible, in particular NWO, but also the European Research Council by awarding me a Consolidator Grant for the project "The Social Epistemology of Argumentation" (SEA) (ERC-17-CoG 771074) in 2017, which has allowed me to dedicate more time to research since the project started in July 2018, and thus to complete this book. I am also grateful for the trust and appreciation that Cambridge University Press, in particular Hilary Gaskin, continues to place in my work.

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Other than institutions, the humans who most contributed to the findings reported in this book are undoubtedly the members of the "Roots of Deduction" team in Groningen: the official ones, Leon Geerdink, Matthew Duncombe, and Rohan French, and the unofficial ones, Erik Krabbe (I cannot emphasize enough what a privilege it has been to be able to count on Erik's expertise and infinite knowledge on all matters pertaining to Aristotle's logic, argumentation theory, dialogical logic, etc.), my Ph.D. students Job de Grefte, Herman Veluwenkamp, Bianca Bosman, and César Frederico dos Santos, and various colleagues at the Faculty of Philosophy in Groningen, Jan Albert van Laar and Barteld Kooi in particular. Other colleagues in Groningen have contributed more indirectly but equally importantly, in particular Martin Lenz, Leah Henderson, and Jan-Willem Romeijn. Thanks also to my new colleagues at Vrije Universiteit Amsterdam, who since July 2018 have provided an inspiring philosophical environment and a warm welcome. Thanks especially to my new SEA team, Hein Duijf, Merel Talbi, and Elias Anttila, who fortunately also enjoy my communal style of doing philosophy.

A number of people have read drafts of chapters and offered insightful comments, in particular Matthew Duncombe, Leon Geerdink, Marije Martijn, Fenner Tanswell, Line Andersen, Benoît Castelnérac, Tushar Irani, Koji Tanaka, Andrew Buskell, and Keith Stenning. (Of course, the remaining shortcomings are all my own.) Drafts of chapters were also fruitfully discussed in seminars at a number of places: Bristol, ANU, and VU Amsterdam. I thank those who participated in these sessions for their comments. I've also benefited tremendously from sustained engagement with readers of my blog posts at NewAPPS and M-Phi between 2010 and 2016; it is a shame that the 'golden age' of philosophical blogging seems to have come to an end, but I am grateful to have been part of it.

Of course, there is also life outside philosophy, and many people have inspired me and given meaning to my life in these ten years. My daughters, Sophie and Marie, remain the main energy-drainers in my life, while also inspiring me and giving me joy every single day. I consider it my biggest accomplishment that they are becoming strong, loud feminist ladies who think that the sky is the limit for their ambitions and plans. A number of other people – friends, family, partners – have been in my life and supported me in different ways over the last ten years: Stephen Read, Ole Hjortland, Richard Pettigrew, Florian Steinberger, Marije Martijn, and Eric Schliesser; my mother, Maria, and my brother, Frederico; Reinout, Izaak, and Jan Roel.

Just as it takes a village to raise a child, it takes a village – more like a midsized city, really – to write a book.

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