

Contents

<i>Introduction</i>	<i>page</i> 1
Part I Category Theory	5
1 Basic Notions in Category Theory	7
1.1 Definition of a Category and Examples	7
1.2 EI Categories and Groupoids	12
1.3 Epi- and Monomorphisms	14
1.4 Subcategories and Functors	18
1.5 Terminal and Initial Objects	25
2 Natural Transformations and the Yoneda Lemma	27
2.1 Natural Transformations	27
2.2 The Yoneda Lemma	32
2.3 Equivalences of Categories	37
2.4 Adjoint Pairs of Functors	38
2.5 Equivalences of Categories via Adjoint Functors	46
2.6 Skeleta of Categories	47
3 Colimits and Limits	49
3.1 Diagrams and Their Colimits	49
3.2 Existence of Colimits and Limits	63
3.3 Colimits and Limits in Functor Categories	65
3.4 Adjoint Functors and Colimits and Limits	66
3.5 Exchange Rules for Colimits and Limits	68
4 Kan Extensions	71
4.1 Left Kan Extensions	71
4.2 Right Kan Extensions	77
4.3 Functors Preserving Kan Extensions	79

4.4	Ends	81
4.5	Coends as Colimits and Ends as Limits	85
4.6	Calculus Notation	86
4.7	“All Concepts are Kan Extensions”	87
5	Comma Categories and the Grothendieck Construction	91
5.1	Comma Categories: Definition and Special Cases	91
5.2	Changing Diagrams for Colimits	97
5.3	Sifted Colimits	100
5.4	Density Results	102
5.5	The Grothendieck Construction	106
6	Monads and Comonads	109
6.1	Monads	109
6.2	Algebras over Monads	112
6.3	Kleisli Category	120
6.4	Lifting Left Adjoints	125
6.5	Colimits and Limits of Algebras over a Monad	127
6.6	Monadicity	137
6.7	Comonads	140
7	Abelian Categories	141
7.1	Preadditive Categories	141
7.2	Additive Categories	145
7.3	Abelian Categories	146
8	Symmetric Monoidal Categories	149
8.1	Monoidal Categories	149
8.2	Symmetric Monoidal Categories	157
8.3	Monoidal Functors	161
8.4	Closed Symmetric Monoidal Categories	167
8.5	Compactly Generated Spaces	169
8.6	Braided Monoidal Categories	175
9	Enriched Categories	180
9.1	Basic Notions	180
9.2	Underlying Category of an Enriched Category	184
9.3	Enriched Yoneda Lemma	188
9.4	Cotensored and Tensored Categories	192
9.5	Categories Enriched in Categories	194
9.6	Bicategories	196
9.7	Functor Categories	200
9.8	Day Convolution Product	201

Part II From Categories to Homotopy Theory	209
10 Simplicial Objects	211
10.1 The Simplicial Category	211
10.2 Simplicial and Cosimplicial Objects	213
10.3 Interlude: Joyal's Category of Intervals	216
10.4 Bar and Cobar Constructions	220
10.5 Simplicial Homotopies	225
10.6 Geometric Realization of a Simplicial Set	227
10.7 Skeleta of Simplicial Sets	232
10.8 Geometric Realization of Bisimplicial Sets	233
10.9 The Fat Realization of a (Semi)Simplicial Set or Space	235
10.10 The Totalization of a Cosimplicial Space	236
10.11 Dold–Kan Correspondence	238
10.12 Kan Condition	239
10.13 Quasi-Categories and Joins of Simplicial Sets	243
10.14 Segal Sets	246
10.15 Symmetric Spectra	248
11 The Nerve and the Classifying Space of a Small Category	251
11.1 The Nerve of a Small Category	251
11.2 The Classifying Space and Some of Its Properties	254
11.3 π_0 and π_1 of Small Categories	257
11.4 The Bousfield Kan Homotopy Colimit	261
11.5 Coverings of Classifying Spaces	266
11.6 Fibers and Homotopy Fibers	268
11.7 Theorems A and B	273
11.8 Monoidal and Symmetric Monoidal Categories, Revisited	278
12 A Brief Introduction to Operads	285
12.1 Definition and Examples	285
12.2 Algebras Over Operads	290
12.3 Examples	294
12.4 E_∞ -monoidal Functors	302
13 Classifying Spaces of Symmetric Monoidal Categories	304
13.1 Commutative H-Space Structure on BC for \mathcal{C} Symmetric Monoidal	304
13.2 Group Completion of Discrete Monoids	307
13.3 Grayson–Quillen Construction	310
13.4 Group Completion of H-Spaces	313
14 Approaches to Iterated Loop Spaces via Diagram Categories	316
14.1 Diagram Categories Determine Algebraic Structure	316
14.2 Reduced Simplicial Spaces and Loop Spaces	320

14.3	Gamma-Spaces	320
14.4	Segal K-Theory of a Permutative Category	325
14.5	Injections and Infinite Loop Spaces	328
14.6	Braided Injections and Double Loop Spaces	329
14.7	Iterated Monoidal Categories as Models for Iterated Loop Spaces	331
14.8	The Category Θ_n	334
15	Functor Homology	346
15.1	Tensor Products	347
15.2	Tor and Ext	348
15.3	How Does One Obtain a Functor Homology Description?	350
15.4	Cyclic Homology as Functor Homology	355
15.5	The Case of Gamma Homology	357
15.6	Adjoint Base-Change	359
16	Homology and Cohomology of Small Categories	361
16.1	Thomason Cohomology and Homology of Categories	361
16.2	Quillen's Definition	363
16.3	Spectral Sequence for Homotopy Colimits in Chain Complexes	364
16.4	Baues–Wirsching Cohomology and Homology	365
16.5	Comparison of Functor Homology and Homology of Small Categories	367
	<i>References</i>	372
	<i>Index</i>	384