## Rigid Body Kinematics

Master the conceptual, theoretical, and practical aspects of kinematics with this exhaustive text, which provides a rigorous analysis and description of general motion in mechanical systems, with numerous examples, from spinning tops to wheeled ground vehicles. Over 400 figures illustrate the main ideas and provide a geometrical interpretation and a deeper understanding of concepts, and quiz questions and problems throughout the text provide additional hands-on practice. Ideal for students taking courses on rigid body kinematics, and an invaluable reference for researchers.

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# Contents

# Preface

## Origin and Scope

This textbook, together with *Rigid Body Dynamics* (Cambridge University Press, forthcoming), is the result of a whole professional life devoted to Newtonian Mechanics, both from an educational and a research point of view. Through more than 40 years, our way of teaching this subject to second-year undergraduate students (UG) in Industrial Engineering at the Universitat Politècnica de Catalunya (UPC) has evolved progressively. To find the right words and examples to make every concept clear, to discover the misconceptions that lead to the most frequent errors, and to kindle the students' curiosity for this discipline have been our major concerns throughout all these years.

We have found our personal point of view to present this classical subject, and have generated hundreds of exercises, questionnaires, and interesting case studies. Sharing this material with others involved in the higher education field seems nearly an obligation!

Our regular course on Newtonian Mechanics at UPC spans over 15 weeks and contains three main parts: Kinematics, Dynamics, and Energy. Students enrolling in this course already have a background in fundamental physics (mainly focused on 2D cases) and mathematics, and they will not be studying any other course on general mechanics (though they may follow some particular applications as "Machine and mechanism theory" and "Vibrations"). This context conditions the syllabus of our course: it has to be complete and provide the main tools to face any problem that may be encountered in Mechanical Engineering.

A percentage of our regular students are really enthusiastic about this discipline. For them we propose an additional and optional 15-week course where some concepts presented in the regular course are revisited and enlarged, and some well-known methods to perform dynamical analysis of mechanical systems are introduced. The course includes a final section devoted to an interesting problem: percussive dynamics.

Our goal when writing the book was not just to cover the syllabus of both courses but also to provide answers to questions raised by the best students (often out of scope of the course). The result has been a long text that includes more material than is likely to be covered in formal lectures to second-year undergraduates (including both the regular and the optional courses), and thus may serve as a reference also for graduate students, professional engineers working in the industry, and even researchers in robotics and biomechanics.

Though the main target are the undergraduate students, enlarging the potential audience was the main reason for splitting it into two volumes (*Rigid Body Kinematics* and *Rigid Body Dynamics*). We are aware that some people working in the fields of robotics and biomechanics and game developers are mainly interested in the kinematical description of mechanical systems. Thus, devoting a first volume dealing exclusively with kinematics seemed appropriate.

The contents of the UG standard course on Mechanics is not the same in the United States and in Europe. In the United States, this text may be better suited for intermediate (and even master) courses rather than for UG courses. Conversely, classical US textbooks may correspond to first-year UG students in Europe but may fall below the level we seek in second-year students.

## Main Features

This textbook presents the general problem of rigid body kinematics in a very rigorous and precise way, and with a particularly suitable approach for engineering students.

Rigor is not at odds with the very visual and geometric approach we pursue throughout the book. We have included hundreds of illustrations which emphasize the geometrical properties of rigid body kinematics and help understand the concepts visually.

The mathematical notation throughout the book is highly explicit not only to avoid any ambiguity but also to allow an automatic and straightforward interpretation.

Every chapter starts with a short introduction that provides an overview of its content. Then, the different concepts and methods are introduced. The proofs are clearly isolated from the main text so that the reader may skip them. Every single concept is illustrated through figures (expressly designed for this work) and many fully worked-out examples (nothing is left to the reader as a further exercise!). Some of them are strictly pedagogical exercises, but many others correspond to simplified models of usual mechanical systems.

Advanced topics are presented in appendices, and they may be overlooked without impairing the understanding of the following chapters.

The book includes an extensive collection of multiple-choice quiz questions (with answers) and exercises (with final results). The quiz questions do not require long calculations, but a clear understanding of the fundamental concepts. Each of them is illustrated through a figure showing the mechanical system under study plus the precisions an engineer would add to complete the description (as dimensions and motion). Thus, it is not strictly necessary to read the text as the figures are self-explanatory.

The exercises are pencil-and-paper problems to be solved symbolically: symbolic results allow to detect possible errors (undetectable in numerical solutions) and give the whole picture of the system behavior (showing which parameters may be responsible for a bifurcation or transition from one regime to another). The student may implement the results in a computer, but that is not the main purpose of the exercises. Computer simulations may be time-consuming, and the required skills (programming, numerical calculation, etc.) do not fall within the objectives of our textbook.

The results for the exercises are given at the end of the chapters, but we do not provide the detailed resolution. Thus, the students are obliged to work on their approach and find by themselves the mistakes leading to wrong results: you have to do kinematics to learn kinematics!

A few puzzles complete the application material given at the end of the chapters. They confront the student with formally simple systems and situations found in everyday life. Justifying their design and/or understanding their kinematic behavior may be far from intuitive and calls for a rigorous application of the concepts introduced in the chapter. A complete explanation is given for each puzzle.

## Content Description

The book presents a rigorous analysis of the kinematics of rigid body systems. Emphasis is made in 3D motion, though planar (2D) motion is reviewed as a particular case.

As the potential students have already acquired the basic notions in a fundamental physics course, units and error propagation are not included. However, some basic mathematical concepts are explained to make the book self-contained.

Chapter 1 sets the basic concept of reference frame and the mathematical representations of space and time. We review the usual mathematical vector operations, and provide both a geometrical and an analytical method to carry them out. The main concern in that chapter is the orientation of rigid bodies (and reference frames and vector bases) in 3D. We introduce the Euler angles, which will be extensively used throughout the book, and the angular rotation vector. Two appendices present alternative methods to describe the rotation and orientation of rigid bodies.

Chapter 2 reviews the kinematics of particles with a special focus on the composition of movements, which establishes the relationship between the kinematics of the same particle in two different reference frames with a general 3D relative motion. Absolute, relative, and transportation movements are introduced when considering the velocity composition. The acceleration composition adds a third term with no particularly useful physical interpretation: the Coriolis acceleration. The inertial guidance is presented in an appendix as an interesting application problem.

Chapter 3 is the core of the book. It is devoted to the general 3D motion of rigid bodies. The definition of rigid body (as a set of points mutually fixed) implies the existence of a relationship between the velocity and the acceleration of any pair of points in the rigid body. The geometry of the velocity distribution is thoroughly analyzed, and the concepts of Instantaneous Screw Axis (ISA), fixed axode, and moving axode are introduced as interesting tools to understand the general motion and the kinematical equivalence of mechanical systems (regardless their particular shape).

Planar motion is treated as a particular case, and the ISA and the axodes are substituted by the Instantaneous Centre of Rotation (ICR) and the centrodes.

The appendices present a very complete and unique coverage of the kinematics and maneuvering possibilities of wheeled vehicles (both with conventional and omnidirectional wheels), modeled as systems of rigid bodies, with non-sliding wheels rolling on a flat ground.

As the usual mechanical systems are multibody systems formed by rigid bodies with mutual kinematic restrictions, Chapter 4 is an introduction to the kinematics of such systems. The concepts of generalized coordinates and velocities, independent coordinates, degrees of freedom, holonomy, and constraint equations are presented. The main focus, however, is on the rigorous and systematic analysis of the kinematic constraints between pairs of rigid bodies. We provide a powerful tool to discover redundancy and all the associated drawbacks based on the analysis of kinematic torsors and the geometrical designs not included in any standard textbook.

In short, this textbook sets a powerful framework that will allow the student to implement a complete kinematic analysis of mechanical systems, which is the starting point to undertake their dynamic description.

## Acknowledgments

This book is the result of many years of interactions with other professors in our teaching team. The regular discussions at the end of the academic year to assess what had been a success and what could be improved have been precious to us. The interaction with the students who have followed our course has also been of invaluable help. They have not only forced us to be precise and find the right examples but also contributed with interesting questions that have compelled us to address problems not specifically treated in the literature so far. Our deepest gratitude to all of them.