

Contents

<i>Preface</i>	<i>page xi</i>
<i>Acknowledgements</i>	xv
PART ONE THEORY	1
1 Fractal Geometry and Dimension Theory	3
1.1 The Emergence of Fractal Geometry	3
1.2 Dimension Theory	5
2 The Assouad Dimension	10
2.1 The Assouad Dimension and a Simple Example	10
2.2 A Word or Two on the Definition	13
2.3 Some History	15
2.4 Basic Properties: <i>The Greatest of All Dimensions</i>	17
3 Some Variations on the Assouad Dimension	22
3.1 The Lower Dimension	22
3.2 The Quasi-Assouad Dimension	24
3.3 The Assouad Spectrum	25
3.4 Basic Properties: <i>Revisited</i>	37
4 Dimensions of Measures	56
4.1 Assouad and Lower Dimensions of Measures	56
4.2 Assouad Spectrum and Box Dimensions of Measures	60
5 Weak Tangents and Microsets	64
5.1 Weak Tangents and the Assouad Dimension	64
5.2 Weak Tangents for the Lower Dimension?	72
5.3 Weak Tangents for Spectra?	73
5.4 Weak Tangents for Measures?	75

PART TWO	EXAMPLES	79
6	Iterated Function Systems	81
6.1	IFS Attractors and Symbolic Representation	81
6.2	Invariant Measures	84
6.3	Dimensions of IFS Attractors	86
6.4	Ahlfors Regularity and Quasi-Self-Similarity	88
7	Self-Similar Sets	93
7.1	Self-Similar Sets and the Hutchinson–Moran Formula	93
7.2	The Assouad Dimension of Self-Similar Sets	95
7.3	The Assouad Spectrum of Self-Similar Sets	102
7.4	Dimensions of Self-Similar Measures	105
8	Self-Affine Sets	110
8.1	Self-Affine Sets and Two Strands of Research	110
8.2	Falconer’s Formula and the Affinity Dimension	111
8.3	Self-Affine Carpets	114
8.4	Self-Affine Sets with a Comb Structure	124
8.5	A Family of Worked Examples	127
8.6	Dimensions of Self-Affine Measures	129
9	Further Examples: Attractors and Limit Sets	137
9.1	Self-Conformal Sets	137
9.2	Invariant Sets for Parabolic Interval Maps	140
9.3	Limit Sets of Kleinian Groups	145
9.4	Mandelbrot Percolation	154
10	Geometric Constructions	160
10.1	Products	160
10.2	Orthogonal Projections	166
10.3	Slices and Intersections	178
11	Two Famous Problems in Geometric Measure Theory	181
11.1	Distance Sets	181
11.2	Keakeya Sets	187
12	Conformal Dimension	190
12.1	Lowering the Assouad Dimension by Quasi-Symmetry	190
PART THREE	APPLICATIONS	197
13	Applications in Embedding Theory	199
13.1	Assouad’s Embedding Theorem	200

Contents

ix

13.2	The Spiral Winding Problem	203
13.3	Almost Bi-Lipschitz Embeddings	212
14	Applications in Number Theory	215
14.1	Arithmetic Progressions	215
14.2	Diophantine Approximation	219
14.3	Definability of the Integers	224
15	Applications in Probability Theory	226
15.1	Uniform Dimension Results for Fractional Brownian Motion	226
15.2	Dimensions of Random Graphs	229
16	Applications in Functional Analysis	230
16.1	Hardy Inequalities	230
16.2	$L^p \rightarrow L^q$ Bounds for Spherical Maximal Operators	232
16.3	Connection with L^p -Norms	234
17	Future Directions	237
17.1	Finite Stability of Modified Lower Dimension	237
17.2	Dimensions of Measures	237
17.3	Weak Tangents	238
17.4	Further Questions of Measurability	239
17.5	IFS Attractors	240
17.6	Random Sets	242
17.7	General Behaviour of the Assouad Spectrum	243
17.8	Projections	245
17.9	Distance Sets	246
17.10	The Hölder Mapping Problem and Dimension	247
17.11	Dimensions of Graphs	248
	<i>References</i>	250
	<i>List of Notation</i>	264
	<i>Index</i>	267