

Uncertainty Analysis for Engineers and Scientists

Build the skills for determining appropriate error limits for quantities that matter with this essential toolkit. Understand how to handle a complete project and how uncertainty enters into various steps. This book provides a systematic, worksheet-based process to determine error limits on measured quantities, and all likely sources of uncertainty are explored, measured, or estimated. It also features instructions on how to carry out error analysis using Excel and MATLAB[®], making previously tedious calculations easy. Whether you are new to the sciences or an experienced engineer, this useful resource provides a practical approach to performing error analysis. This book is suitable as a text for a junior- or senior-level laboratory course in aerospace, chemical, and mechanical engineering, and for professionals.

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Uncertainty Analysis for Engineers
and Scientists
A Practical Guide

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Preface

This is a new, short book on uncertainty analysis, targeted both to be a guide for practicing engineers and scientists and as a contribution to the undergraduate textbook market.

Hard numbers from measurements are the backbone of both scientific knowledge and engineering practice. Measurements must be interpreted in context, however: no measured value is infinitely precise, and measurements are certainly not always accurate. This book introduces basic aspects of uncertainty analysis as applicable to the physical sciences and engineering. We focus on one objective: enabling readers to determine appropriate uncertainty limits for measured and calculated quantities. Better conclusions can be drawn from results when they are reported with the appropriate uncertainty limits.

Uncertainty analysis is part of STEM education (science, technology, engineering, and mathematics) to varying degrees: early science education introduces the concept of significant figures; some disciplines teach error propagation in laboratory courses; and many STEM practitioners study statistics. In our advanced technological society, however, all practicing scientists and engineers need refined skills in assessing the reliability of measurements. More broadly, the public needs literacy in measurement of uncertainty, as the reliability of measurements impacts regulations, guilt or innocence in court, budget priorities, and many other issues in the public sphere. To compare “apples to apples” in political or scientific discussions that hinge on measurement accuracy or sufficiency, we need tools like the error-analysis methods emphasized in this book.

We have argued that error analysis is essential, but unfortunately error analysis can seem tedious and involved and, perhaps, of limited practical value. This book addresses that impression head-on. Reading formal texts on error analysis, one might infer that uncertainty is determined with complex statistical formulas that require prolonged study. This impression sets up a barrier to

the most important aspects of error analysis: understanding the quality and meaning of day-to-day measurements and using measurements to draw valid, quantitative conclusions on an ongoing basis. With this book, the reader is given the tools, the justification for using those tools, and clear guidance on the common ways to apply classic error analysis.

This book is not a compilation of formulas with no context. The text engages the reader in the issue of the quality of numbers that come from instruments and calculators. The presence of 16 digits in a calculator obscures the fact that the actual precision of the answer is probably quite small and may contain only one or two (and perhaps zero) significant digits. We also guide the reader to ask questions about calculations, such as how to identify the dominant source of uncertainty in the result. We show the reader how to use some modern tools (Microsoft Excel and MATLAB) to make error calculations, allowing us to run trial scenarios to see how uncertainty affects final answers.

Practicing engineers and scientists can be passionate about practical expertise, such as knowing which kind of meter or device is reliable in a given circumstance, or understanding that, in the appropriate context, almost any number can be “essentially zero” or “below our ability to measure.” That kind of expertise is often hard-won through experience, but it can be learned systematically with the study of error analysis. We see in this book how to interrogate our devices to determine the extent to which we can rely on them.

The book is divided into six chapters:

1. Introduction and Definitions
2. Quick Start: Replicate Error Basics
3. Reading Error
4. Calibration Error
5. Error Propagation
6. Model Fitting

The first chapter sets the stage for the discussion of error analysis and introduces the three categories of error presented in the subsequent chapters. **Replicate error** is classic random error, and Chapter 2 presents a brief introduction of the statistical topics that feed into error analysis, including the properties of continuous stochastic variables and their probability density distributions, and the definition of standard error, which allows us to combine independent contributing errors. In addition to replicate error we discuss **reading error** (Chapter 3) and **calibration error** (Chapter 4), including how to estimate errors when necessary (Type B errors). The book contributes worksheets associated with the three error types (Appendix A), which guide

the user to ask appropriate questions. When error estimates are used, the worksheets record and archive the decisions made, a practice that allows improvements to be incorporated as we learn more about our systems.

Chapter 5 presents error propagation and an error propagation worksheet. The equation for error propagation (combination of variances) is relatively straightforward, but to the uninitiated, the presence of second partial derivatives and confusing nomenclature makes the calculation seem harder than it is. In some texts, use of the full equation is replaced with several equations that apply to specific circumstances (for example, special equations for when two variables are added or subtracted or multiplied). In this book we prefer to use the worksheet and the full equation for two reasons: (1) it builds familiarity with the variance-combination equation in the mind of the user and (2) it is easily evaluated with software. Programming an error propagation into computer software allows us to readily identify the dominant error and also allows us to run scenarios with different values of contributing error. The computer approach thus enables the teaching of critical thinking about errors.

Chapter 6 focuses on ordinary least-squares regression and the errors associated with model predictions and with a model's parameters. Many undergraduates and practitioners use plotting functions and curve-fitting options, but with a bit of additional knowledge they can expand their use to full error analysis. We discuss 95% confidence error limits and 95% confidence prediction intervals and show how these can improve our understanding of correlations in our datasets.

In summary, the book puts error-analysis methods into the hands of students and practitioners in a way that encourages their use. The four worksheets (one for each type of error plus one for error propagation) become tools that guide the determination of error limits, and they form an archive of estimates made and errors determined. The book is short and is designed to become a go-to guide for assessing the quality of measurements and associated calculations.

Many people have helped in various aspects of this book. The junior chemical engineering laboratory facility at Michigan Tech that was the inspiration for this book was built and maintained for many years by David Caspary and Timothy Gasperich and they were instrumental in its success. Jim ('58) and Sally Brozzo provided the essential funding and moral support for the lab. Dr. Nam Kim was the original instructor. The following individuals were my teaching assistants over the years: Austin Bryan, Cho Hui Lim, Matt Chye, Danial Lopez Gaxiola, Ben Conard, Jithendar Guja, Shubham Borole, Hwiyoung Lee, Kaela Leonard, Neeva Benipal, Rachael Martin, Erik Leslie, and Yang Zhang. David Chadderton taught the lab during my 2012 sabbatical and did a great job. All these individuals impacted the material presented here.

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