

Principles of Optics

Principles of Optics is one of the most highly cited and most influential physics books ever published, and one of the classic science books of the twentieth century. To celebrate the 60th anniversary of this remarkable book's first publication, the seventh expanded edition has been reprinted with a special foreword by Sir Peter Knight. The seventh edition was the first thorough revision and expansion of this definitive text. Amongst the material introduced in the seventh edition is a section on CAT scans, a chapter on scattering from inhomogeneous media, including an account of the principles of diffraction tomography, an account of scattering from periodic potentials, and a section on the so-called Rayleigh-Sommerfield diffraction theory. This expansive and timeless book continues to be invaluable to advanced undergraduates, graduate students and researchers working in all areas of optics.

To the Memory of
Sir Ernest Oppenheimer

Principles of Optics

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Foreword by Sir Peter Knight

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Foreword

Sir Peter Knight

1 Introduction

Optics in the twenty-first century is a vibrant part of modern physics, with stunning developments in fundamental science (imaging, correlations, and coherence, and so much more), as well as underpinning our technological world, including providing high bit rate optical communications, and precision laser engineering. But, 75 years ago, optics as a major field of research had been regarded by many as a backwater. One of the leaders of my own department at Imperial College London had described it as “all pins and mirrors” and pushed to have it dropped from the undergraduate syllabus. How wrong he was and how fashions have changed: the field was by then poised for explosive development, starting with the realisation, very much pioneered by Emil Wolf, that the study of correlations in light fields unlocked new insights. Understanding partial coherence, the extension to higher-order correlations with the work of Hanbury Brown and Twiss, and then of course the realisation of the laser transformed our views of the optical world. And the magnificent monograph by Max Born and Emil Wolf was at the fore in this revolution. With the publication of B&W, at last we had a magisterial account of the fundamental principles and their application. What an achievement! It has become a major sourcebook used throughout the world.

2 Physical Optics Prior to the Appearance of Born and Wolf

In the early twentieth century, authoritative books on optics, developing the basic phenomena in a systematic fashion, were not plentiful, especially ones building up the theoretical basis from proper electromagnetic foundations. It was, it seems, hard to locate sound and rigorous analytic treatments of diffraction theory, let alone high-level discussions of image formation.

Born’s own monograph *Optik* was published by Springer Verlag in 1933, just as he was forced to leave Germany by the Nazis. *Optik* itself eventually formed the seed for what became Born and Wolf, and was wrongly thought at the time by Born to have had very limited sales. Springer’s scientific advisor in the 1930’s was Paul Rosbaud, an influential figure in pre-war German science, in contact with all the important figures in German physics and much valued by Born. He was later to be revealed as a highly valued British Intelligence agent throughout the Nazi years as described by Kramish (1986) and will appear again in this account of how Born and Wolf came about.

3 Max Born

Max Born, one of the greatest figures in twentieth-century science, is best known for his pioneering work in the creation of quantum mechanics in the 1920s in Goettingen, for which he was awarded the Nobel Prize much later and after an inexplicable delay where the citation read for “fundamental research in Quantum Mechanics, especially in the statistical interpretation of the wave function.” Born had led an extraordinarily talented group of theoretical physicists, including Werner Heisenberg and Pascual Jordan, in the 1920s, who had pioneered the development of quantum mechanics, developing the matrix mechanics approach, commutation relations, and much more that underpins our understanding of the microscopic world. Indeed, Hedwig and Max Born’s tombstone in the Goettingen Stadtfriedhof carries the famous p, q quantum commutation relation, one that Born himself considered to be his main single contribution to science, according to his son Gustav (Born 2002).

But Born was truly a polymath, active in an extraordinary range of physics, including continuum mechanics, solid state physics, and of course optics. Max Delbrück, Siegfried Flügge, Friedrich Hund, Pascual Jordan, Maria Goeppert-Mayer, Lothar Wolfgang Nordheim, Robert Oppenheimer, and Victor Weisskopf all received their Ph.D. degrees under Born at Goettingen, and his assistants included Enrico Fermi, Werner Heisenberg, Gerhard Herzberg, Friedrich Hund, Pascual Jordan, Wolfgang Pauli, Léon Rosenfeld, Edward Teller, Walter Heitler, and Eugene Wigner. The catastrophic rise of the Nazis at the start of the 1930s destroyed this wonderful centre: expulsions and a mass exodus dispersed this incredible talent around the world, and Born’s *Optik* appeared as a kind of last act from this Goettingen world.

Max Born, for some years after 1933, led a peripatetic life in Cambridge and elsewhere, before finally setting in Edinburgh as Tait Professor of Natural Philosophy, where his group members included Kellerman, Fuchs, Schlapp, Nisbet, and others. His Edinburgh “Natural Philosophers” – really the theoretical physics group – were housed in High School Yard on Drummond Street, a rather dingy back street behind Thin’s University Bookshop, with a small lecture room and a large room to house the entire group. Born would progress round each of his group – and especially his students – every morning, asking what progress had been made since the day before. I vividly remember Wolf explaining to me the tensions this progression induced in the young researchers!

Born had been a prolific textbook author, on relativity, atomic physics, optics, and crystal lattice dynamics, demonstrating his enormous breadth of interests and his encyclopaedic knowledge. He retired from his chair in 1952 and he and his wife returned to Germany in 1954, and he continued with active writing for many years. He finally, and very belatedly, received his Nobel Prize in 1954 for his fundamental work in quantum mechanics. Max Born died at age 87 in hospital in Goettingen on January 5, 1970.

Born’s very precise mathematical approach to fundamental physical phenomena must have stemmed in part from his early academic career in Goettingen as the assistant to David Hilbert, the doyen of mathematics at the turn of the twentieth century. Born and Wolf beautifully displays this approach: elegant, deep, and precise. Kemmer and Schlapp (1971), in their Royal Society Biographical Memoir of Born, captured this precisely: “Born’s approach here, as in most of his other work, was to face his problem in all its complexity, to devise a mathematical formulation of appropriate generality and then to descend to the simpler, more tractable (and usually physically most interesting) cases as clearly defined specialisations and approximations to the general formalisms.”

4 Emil Wolf

Wolf, the father of optical coherence theory, dominated optics for more than half a century. He was born in Prague in 1922 to Jewish parents and at age 16, following the 1939 German invasion of Czechoslovakia, became a refugee, initially in Paris, and then, after a perilous escape from Paris, arrived in England after the fall of France in 1940. He completed high school in England and studied at the University of Bristol for his B.Sc. in Mathematics and Physics (1945) and stayed on for his Ph.D. with E. H. Linfoot, with a dissertation entitled “A Contribution to the Theory of Aspheric Optical Systems.”

About the time of Wolf’s Bristol thesis completion, his advisor E. H. Linfoot moved to the Cambridge University Observatory, taking Wolf with him as his assistant for the next two years. During this time, Wolf participated in the regular meetings then held at Imperial College of the small UK optics community, and cemented his strong links with Dennis Gabor, G. P. Thomson, and others.

Between January 1951 and 1954, Wolf worked at the University of Edinburgh with Max Born, writing B&W. According to Wolf (2005), Born wrote to Appleton, the then Principal of Edinburgh, saying that he felt the decision about appointing his assistant should not be made by Born alone as he “would like to appoint a Wolf after a Fox” (a previous holder of his assistantship was the atom spy Klaus Fuchs – “fox” in German)! After Born’s retirement, Wolf led a peripatetic career for a while. After a period on the Faculty of the University of Manchester, notably forming his close and highly successful collaboration on partial coherence with Brian Thompson (later to be Dean in Rochester), Wolf moved to the United States in 1959 to take a position at the University of Rochester where he supervised many Ph.D. students who went on to highly successful careers. He eventually became a naturalised US citizen and became the Wilson Professor of Optical Physics at the University of Rochester. My own stay in the group of Joseph Eberly at the University of Rochester (with an office along the corridor from Emil) in the early 1970s was enlivened by our daily group lunches at the University Faculty Club, where new developments in optics were vigorously dissected, and Emil showed his extraordinary grasp of the whole swathe of optical science. In 1978 he became President of the Optical Society of America, his spiritual home, and attended without fail the OSA Annual Meetings, always making a point of meeting up with student members to learn about the latest developments in optics.

5 Postwar Situation and Translation Plans for *Optik*

Paul Rosbaud, whom we met in an earlier section, was thanked in the preface of the first edition of B&W for having been closely associated with the project in its early days. Rosbaud had been involved in the earlier Born monograph *Optik* as a former editor for Springer, and was by then interested in translating German texts into English. Rosbaud after the war had moved to England, where he helped set up a publishing company, Butterworth-Springer, with a distinguished Scientific Advisory Board that included Alfred Egerton, Charles Galton Darwin (Born’s predecessor as Tait Professor of Natural Philosophy in Edinburgh), Edward Salisbury, and Alexander Fleming. When the Butterworth Company decided to pull out of the English/German liaison, Robert Maxwell (like Wolf, a Czech wartime refugee) acquired 75 percent of the shares of the company, while 25 percent rested with Rosbaud. The company name was changed

to Pergamon Press; the partners, with their considerable language skills, cooperated in establishing new academic journals until 1956, when, after an inevitable disagreement, Rosbaud left.

Maxwell from then on dominated Pergamon, with unhappy implications described below. Maxwell himself was ejected from the board of Pergamon in October 1969. An inquiry by the UK Government Department of Trade and Industry reported in mid-1971: “We regret having to conclude that, notwithstanding Mr Maxwell’s acknowledged abilities and energy, he is not in our opinion a person who can be relied on to exercise proper stewardship of a publicly quoted company.” Nevertheless, Maxwell reacquired Pergamon in 1974, although it was sold to Elsevier in 1991 after Maxwell’s strange drowning from his yacht in the Atlantic led to the collapse of his very extensive publishing group.

6 The Move from an Update of *Optik* to a New Book

As Born’s plans for a translation and updating of *Optik* were developing, he became aware of a curious involvement of the US Government in the rights for the book. The US had spent considerable sums in acquiring access to German scientific publications before the war. Then, during the war, they had reproduced many foreign journals and books under the aegis of the “Office of Alien Property Custodian,” which allowed US publishers with licences to print without royalty payments to authors or original publishers. Born, of course, had been a British citizen since before the war, yet was caught up in all this and had made no progress in restoring his rights to *Optik*, despite many appeals to the authorities. Indeed, according to Nancy Thorndike Greenspan, Born’s biographer (Greenspan 2005), Thomas H. Creighton of the Office of Alien Property insisted the rights were vested in the US under the Trading with the Enemy Act, that he would need to apply to the US Government for a licence if he wanted to use portions of *Optik* in the new book - and, what’s more, had to pay 2 percent royalties on the new book as they owned the copyright! The US Government finally relented, presumably realising that Born was far from ever being an enemy alien and had for many years been a citizen of an allied country! They returned to Born his copyright and, belatedly, the royalties on what he discovered were an unexpected 1,000 sales. As we will see, this should have alerted Born to be wary in future about reliable sales figures and royalties.

7 Update and Co-authorship

The (quite sparse in those pre-laser days) scientists working in optics in the 1940s and 1950s would gather regularly at Imperial College London for meetings of what had been called the “Optical Society of London,” and then became the Optical Group of the Physical Society, now the Institute of Physics. Regular attendees included Born, Dennis Gabor, Harold Hopkins, E. H. Linfoot, and, of course, Emil Wolf. Later attendees included Leonard Mandel, who became Wolf’s closest collaborator over many years. The early plans envisaged Born contributing material from *Optik*, with new sections contributed by proposed co-authors Dennis Gabor and Harold Hopkins. The initial plan was to complete the book by late 1951, before Born’s retirement from the Tait chair, although of course the writing took eight years in the end. Hopkins withdrew from the

project early in 1950, and in October 1950 Gabor, encouraged by Born, wrote to Linfoot and Wolf asking if they could take Hopkins' place (Wolf 2005). Eventually, Born, Gabor, and Wolf agreed to author the new book. Wolf moved from Cambridge at the end of January 1951 to focus on the book. But then Gabor, like Hopkins earlier, decided he really did not have the time to devote to the writing as a full author but agreed he would contribute a section on electron optics. So, at that point, we see the emergence of the Born and Wolf collaboration.

The book was intended from the outset to have sections on various specialist topics contributed by others (Wolf himself was initially drawn into the project to write one on the diffraction theory of aberrations!). Distinguished contributors included Clemmow on rigorous diffraction theory (and the appendix on steepest descent and stationary phase), Wilcock on interferometers, Wayman on image-forming optics, Bhatia on ultrasonic diffraction, Gabor on the link between geometrical optics and classical mechanics – especially for electron optics – and so on. An appendix on the calculus of variations is based on unpublished lectures by David Hilbert, Born's early mentor in Goettingen, providing a link going back a century by then to one of the greatest mathematicians in the world.

Most of the writing was done in Edinburgh and Manchester, and finally completed when Wolf was a guest at the Institute of Mathematical Sciences at New York University. Born was always able to write quickly, and according to Wolf was often none too pleased with the slow progress made overall on the Principles of Optics project. The delays in part stemmed from the new developments in optical coherence developed principally by Wolf. By 1957, Wolf received a letter from Born asking why the book was still unfinished. Wolf replied that it was essentially completed, except for the chapter on partial coherence. According to Wolf (2005), Born wrote back to ask “who apart from you is interested in partial coherence. Leave that chapter out and send the rest of the manuscript to the printers.” Fortunately, he resisted, and within a couple of years the laser revolution was upon us and optical coherence became centre stage in the subject.

One of the features of the book from the outset was the careful discussion of optical correlations, both of amplitudes and of intensities. The early Manchester experiments carried out by Brian Thompson on the effects of partial coherence on two-beam interference were included to illustrate the importance of first-order coherence. The dramatic discovery of intensity correlations by Hanbury Brown and Twiss also appeared at this time and featured in the book.

8 The First Born and Wolf

The first edition appeared in January 1959, by which time Max Born had retired from his chair in Edinburgh to live in Bad Pyrmont in Germany. Emil Wolf was then working in Manchester University. This first edition of Born and Wolf was very well received for its unique comprehensiveness and depth: to quote Kemmer and Schlapp (1971), “it presents a systematic treatment based on electromagnetic theory of all optical phenomena that can be described in terms of a continuous distribution of matter.”

Born and Wolf appeared at an extremely opportune time: just before the realisation of the laser, where its spatial and temporal coherence and ability to transform image science and information technology. Suddenly, everyone needed the insights that Born and Wolf provided. Gabor himself stated that Born and Wolf was the first systematic account

of holography in an authoritative text. Serendipity played its role too: for example, as lasers were used to explore nonlinear optics, it was necessary to understand the spatial distribution of intensity and phase of focused laser beams, and there in B&W already was a beautiful discussion of the very isophotes the pioneers needed to understand phase matching.

9 The Reception of Born and Wolf

Born and Wolf was very warmly received from the outset. University teachers quarried it for insights in their courses, researchers used it as a source of rigorous reliable information in optical science, and the resultant excellent sales reflected the real value the world community placed on this treasure.

10 Updates

Updates and new editions appeared on a regular basis as new developments were carefully incorporated by Wolf. The authors had considerable difficulties for some years with Pergamon Press over royalties, with discrepancies over sales figures and the emergence of perhaps previously unknown editions; this led to complex legal arbitration, described in the biography by Greenspan and in detail by Max Born's son, Professor Gustav Born, in an article written shortly after Maxwell's death, entitled "Pilfering from the Professors" in the UK magazine *The Oldie*, edited then by Richard Ingrams. The British satirical magazine *Private Eye*, also edited by Ingrams, had previously lampooned Maxwell as the "bouncing Czech," a nickname originally coined by Prime Minister Harold Wilson when Maxwell had been Labour MP for Buckingham. The happy transition to Cambridge University Press for this edition of B&W (and the previous two editions) put an end to what can only be described as a sorry story of the collisions of two worlds, one of academia and what had sadly been revealed as one of a predatory publisher. What a contrast this revealed between two Czech refugees from Nazi tyranny with such different characters – Emil Wolf being one, and Robert Maxwell the other, entangled over Born and Wolf! The first five revised editions were published by Pergamon Press (1959–1975). Cambridge University Press took over the publishing of the monograph in 1980 with a seventh expanded edition published in 1999. I still treasure my own Pergamon and Cambridge editions complete with a handwritten greeting from Emil.

Plans were already expressed in the preface of the first edition of B&W for a volume II on Molecular and Atomic Optics, and volume III on Quantum Optics (one of the earliest uses of this term, to my knowledge). Rather touchingly, the authors expressed the hope that the CGS system of units would have returned to favour by the time these volumes might appear. Readers of the famous 1995 monograph *Optical Coherence and Quantum Optics* by Leonard Mandel and Emil Wolf, representing in itself – in a sense – this long-awaited "volume 3," will have noted a partial fulfillment of this hope!

11 Lasting Value, Scholarship, and Reliable Knowledge

Here one continues to find in this masterpiece of lucid authoritative writing the most complete account of modern classical optical physics. Born and Wolf remains one of

the most influential science books of the past 75 years. Here you will find the most precise accounts of the Kirchhoff theory of diffraction, the theory of image formation and aberrations, of partial coherence, and the like. You will find here the principles of diffraction tomography, of scattering by inhomogeneous media – I could go on, of course! Its impact can be measured by the many editions and reprints it has gone through: a book that has a treasured place on the shelves of anyone working seriously in optics.

Acknowledgements

In writing this preface to the anniversary edition, I have drawn on many years of discussions with Emil Wolf and his colleagues, and with Max Born's son G. V. R. Born (Gus), as well as from the many publications of and about Born and Wolf – but especially from a lifetime of consulting this magnificent book!

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*Sir Peter Knight
Imperial College*

Preface to corrected reprint of the seventh edition

As mentioned in the Preface to the seventh edition of this work, a change to a new publisher has made it possible to reset the whole text. Not surprisingly such a large amount of typesetting introduced some typographical errors. Those found by now have been corrected, as has been other inaccuracies which the reviewers and readers of the book brought to my attention. A small number of additional references have also been included. For the sake of completeness the Prefaces to the third, fourth and fifth editions have also been added.

I am particularly indebted to Dr E. Hecht who in a thorough review of the seventh edition noted several errors and inaccuracies that have now been corrected. I am also obliged to Dr S. H. Wiersma and Mr Damon Diehl who read much of the text very carefully and supplied me with long lists of misprints and other errors. I must also thank my friends and colleagues Professor Taco Visser, Dr Daniel F. V. James, Dr Peter Milonni, Professor Richard M. Sillitto, Mrs Winifred Sillitto and Dr Andrei Shchegrov for drawing my attention to a number of errors. Finally I wish to express my indebtedness to my colleague and former student Dr Greg Gbur for much help with the preparation of the corrected version.

Rochester, New York
August 2001

EW

Preface to the first edition

THE idea of writing this book was a result of frequent enquiries about the possibility of publishing in the English language a book on optics written by one of us* more than twenty-five years ago. A preliminary survey of the literature showed that numerous researches on almost every aspect of optics have been carried out in the intervening years, so that the book no longer gives a comprehensive and balanced picture of the field. In consequence it was felt that a translation was hardly appropriate; instead a substantially new book was prepared, which we are now placing before the reader. In planning this book it soon became apparent that even if only the most important developments which took place since the publication of *Optik* were incorporated, the book would become impracticably large. It was, therefore, deemed necessary to restrict its scope to a narrower field. *Optik* itself did not treat the whole of optics. The optics of moving media, optics of X-rays and γ -rays, the theory of spectra and the full connection between optics and atomic physics were not discussed; nor did the old book consider the effects of light on our visual sense organ – the eye. These subjects can be treated more appropriately in connection with other fields such as relativity, quantum mechanics, atomic and nuclear physics, and physiology. In this book not only are these subjects excluded, but also the classical molecular optics which was the subject matter of almost half of the German book. Thus our discussion is restricted to those optical phenomena which may be treated in terms of Maxwell's phenomenological theory. This includes all situations in which the atomistic structure of matter plays no decisive part. The connection with atomic physics, quantum mechanics, and physiology is indicated only by short references wherever necessary. The fact that, even after this limitation, the book is much larger than *Optik*, gives some indication about the extent of the researches that have been carried out in classical optics in recent times.

We have aimed at giving, within the framework just outlined, a reasonably complete picture of our present knowledge. We have attempted to present the theory in such a way that practically all the results can be traced back to the basic equations of Maxwell's electromagnetic theory, from which our whole consideration starts.

In Chapter I the main properties of the electromagnetic field are discussed and the effect of matter on the propagation of the electromagnetic disturbance is described formally, in terms of the usual material constants. A more physical approach to the

* Max Born, *Optik* (Berlin, Springer, 1933).

question of influence of matter is developed in Chapter II: it is shown that in the presence of an external incident field, each volume element of a material medium may be assumed to give rise to a secondary (scattered) wavelet and that the combination of these wavelets leads to the observable, macroscopic field. This approach is of considerable physical significance and its power is illustrated in a later chapter (Chapter XII) in connection with the diffraction of light by ultrasonic waves, first treated in this way by A. B. Bhatia and W. J. Noble; Chapter XII was contributed by Prof. Bhatia himself.

A considerable part of Chapter III is devoted to showing how geometrical optics follows from Maxwell's wave theory as a limiting case of short wavelengths. In addition to discussing the main properties of rays and wave-fronts, the vectorial aspects of the problem (propagation of the directions of the field vectors) are also considered. A detailed discussion of the foundations of geometrical optics seemed to us desirable in view of the important developments made in recent years in the related field of microwave optics (optics of short radio waves). These developments were often stimulated by the close analogy between the two fields and have provided new experimental techniques for testing the predictions of the theory. We found it convenient to separate the mathematical apparatus of geometrical optics – the calculus of variations – from the main text; an appendix on this subject (Appendix I) is based in the main part on unpublished lectures given by D. Hilbert at Göttingen University in the early years of this century. The following appendix (Appendix II), contributed by Prof. D. Gabor, shows the close formal analogy that exists between geometrical optics, classical mechanics, and electron optics, when these subjects are presented in the language of the calculus of variations.

We make no apology for basing our treatment of geometrical theory of imaging (Chapter IV) on Hamilton's classical methods of characteristic functions. Though these methods have found little favour in connection with the design of optical instruments, they represent nevertheless an essential tool for presenting in a unified manner the many diverse aspects of the subject. It is, of course, possible to derive some of the results more simply from *ad hoc* assumptions; but, however valuable such an approach may be for the solution of individual problems, it cannot have more than illustrative value in a book concerned with a systematic development of a theory from a few simple postulates.

The defect of optical images (the influence of aberrations) may be studied either by geometrical optics (appropriate when the aberrations are large), or by diffraction theory (when they are sufficiently small). Since one usually proceeds from quite different starting points in the two methods of treatment, a comparison of results has in the past not always been easy. We have attempted to develop a more unified treatment, based on the concept of the deformation of wave-fronts. In the geometrical analysis of aberrations (Chapter V) we have found it possible and advantageous to follow, after a slight modification of his eikonal, the old method of K. Schwarzschild. The chapter on diffraction theory of aberrations (Chapter IX) gives an account of the Nijboer–Zernike theory and also includes an introductory section on the imaging of extended objects, in coherent and in incoherent illumination, based on the techniques of Fourier transforms.

Chapter VI, contributed by Dr P. A. Wayman, gives a brief description of the main image-forming optical systems. Its purpose is to provide a framework for those parts of the book which deal with the theory of image formation.

Chapter VII is concerned with the elements of the theory of interference and with interferometers. Some of the theoretical sections have their nucleus in the corresponding sections of *Optik*, but the chapter has been completely re-written by Dr W. L. Wilcock, who has also considerably broadened its scope.

Chapter VIII is mainly concerned with the Fresnel–Kirchhoff diffraction theory and with some of its applications. In addition to the usual topics, the chapter includes a detailed discussion of the central problem of optical image formation – the analysis of the three-dimensional light distribution near the geometrical focus. An account is also given of a less familiar alternative approach to diffraction, based on the notion of the boundary diffraction wave of T. Young.

The chapters so far referred to are mainly concerned with perfectly monochromatic (and therefore completely coherent) light, produced by point sources. Chapter X deals with the more realistic case of light produced by sources of finite extension and covering a finite frequency range. This is the subject of partial coherence, where considerable progress has been made in recent years. In fact, a systematic theory of interference and diffraction with partially coherent light has now been developed. This chapter also includes an account of the closely related subject of partial polarization, from the standpoint of coherence theory.

Chapter XI deals with rigorous diffraction theory, a field that has witnessed a tremendous development over the period of the last twenty years,* stimulated largely by advances in the ultra-shortwave radio techniques. This chapter was contributed by Dr P. C. Clemmow who also prepared Appendix III, which deals with the mathematical methods of steepest descent and stationary phase.

The last two chapters, ‘Optics of metals’ (Chapter XIII) and ‘Optics of crystals’ (Chapter XIV) are based largely on the corresponding chapters of *Optik*, but were revised and extended with the help of Prof. A. M. Taylor and Dr A. R. Stokes respectively. These two subjects are perhaps discussed in less detail than might seem appropriate. However, the optics of metals can only be treated adequately with the help of quantum mechanics of electrons, which is outside the scope of this book. In crystal optics the centre of interest has gradually shifted from visible radiation to X-rays, and the progress made in recent years has been of a technical rather than theoretical nature.

Though we have aimed at producing a book which in its methods of presentation and general approach would be similar to *Optik*, it will be evident that the present book is neither a translation of *Optik*, nor entirely a compilation of known data. As regards our own share in its production, the elder coauthor (M. B.) has contributed that material from *Optik* which has been used as a basis for some of the chapters in the present treatise, and has taken an active part in the general planning of the book and in numerous discussions concerning disputable points, presentation, etc. Most of the compiling, writing and checking of the text was done by the younger coauthor (E. W.).

Naturally we have tried to use systematic notation throughout the book. But in a book that covers such a wide field, the number of letters in available alphabets is far too limited. We have, therefore, not always been able to use the most elegant notation but we hope that we have succeeded, at least, in avoiding the use in any one section of the same symbol for different quantities.

* The important review article by C. J. Bouwkamp, *Rep. Progr. Phys.* (London, Physical Society), **17** (1954), 35, records more than 500 papers published in the period 1940–1954.

In general we use vector notation as customary in Great Britain. After much reflection we rejected the use of the nabla operator alone and employed also the customary ‘div’, ‘grad’ and ‘curl’. Also, we did not adopt the modern electrotechnical units, as their main advantage lies in connection with purely electromagnetic measurements, and these play a negligible part in our discussions; moreover, we hope, that if ever a second volume (*Molecular and Atomic Optics*) and perhaps a third volume (*Quantum Optics*) is written, the C.G.S. system, as used in theoretical physics, will have returned to favour. Although, in this system of units, the magnetic permeability μ of most substances differs inappreciably from unity at optical frequencies, we have retained it in some of the equations. This has the advantage of greater symmetry and makes it possible to derive ‘dual’ results by making use of the symmetry properties of Maxwell’s equations. For time periodic fields we have used, in complex representation, the factor $\exp(-i\omega t)$ throughout.

We have not attempted the task of referring to all the relevant publications. The references that are given, and which, we hope, include the most important papers, are to help the reader to gain some orientation in the literature; an omission of any particular reference should not be interpreted as due to our lack of regard for its merit.

In conclusion it is a pleasure to thank many friends and colleagues for advice and help. In the first place we wish to record our gratitude to Professor D. Gabor for useful advice and assistance in the early stages of this project, as well as for providing a draft concerning his ingenious method of reconstructed wave-fronts (§8.10). We are also greatly indebted to Dr F. Abelès, who prepared a draft, which is the backbone of §1.6, on the propagation of electromagnetic waves through stratified media, a field to which he himself has made a substantial contribution. We have also benefited by advice on this subject from Dr B. H. Billings.

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The main part of the writing was done at the Universities of Edinburgh and Manchester. The last stages were completed whilst one of the authors (E. W.) was a guest at the Institute of Mathematical Sciences, New York University. We are grateful to Professor M. Kline, Head of its Division of Electromagnetic Research, for his helpful interest and for placing at our disposal some of the technical facilities of the Institute.

We gratefully acknowledge the loan of original photographs by Professor M. Françon and Dr M. Cagnet (Figs 7.4, 7.26, 7.28, 7.60, 15.24, 15.26), Professor H. Lipson and his coworkers at the Manchester College of Science and Technology (Figs. 8.10, 8.12, 8.15), Dr O. W. Richards (Figs. 8.34, 8.35), and Professor F. Zernike and Dr K. Nienhus (Figs. 9.4, 9.5, 9.8, 9.10, 9.11). Fig. 7.66 is reproduced by courtesy of the Director of the Mount Wilson and Palomar Observatories. The blocks of Fig. 7.42 were kindly loaned by Messrs Hilger and Watts, Ltd, and those of Figs. 7.64 and 7.65 by Dr K. W. Meissner.

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Finally, it is a pleasure to thank our publishers and in particular Mr E. J. Buckley, Mr D. M. Lowe and also Dr P. Rosbaud, who as a former Director of Pergamon Press was closely associated with this project in its early stages, for the great care they have taken in the production of the book. It is a pleasure to pay tribute also to the printers, Pitman Press of Bath, for the excellence of their printing.

Bad Pyrmont and Manchester
January 1959

Max Born
Emil Wolf

Preface to the second edition

ADVANTAGE has been taken in the preparation of a new edition of this work to make a number of corrections of errors and misprints, to make a few minor additions and to include some new references.

Since the appearance of the first edition almost exactly three years ago, the first optical masers (lasers) have been developed. By means of these devices very intense and highly coherent light beams may be produced. Whilst it is evident that optical masers will prove of considerable value not only for optics but also for other sciences and for technology, no account of them is given in this new edition. For the basic principles of maser action have roots outside the domain of classical electromagnetic theory on which considerations of this book are based. We have, however, included a few references to recent researches in which light generated by optical masers was utilized or which have been stimulated by the potentialities of these new optical devices.

We wish to acknowledge our gratitude to a number of readers who drew our attention to errors and misprints. We are also obliged to Dr B. Karczewski and Mr C. L. Mehta for assistance with the revisions.

Bad Pyrmont and Rochester
November 1962

M.B.
E.W.

Preface to the third edition

A number of errors and misprints which were present in the earlier editions of this work have been corrected and references to some recent publications have been added.

A new figure (8.54) which relates to an interesting recent development in the wavefront reconstruction technique was also included. We are indebted to E. N. Leith and J. Upatnieks for a loan of the original photographs.

Bad Pyrmont and Rochester
July 1965

M. B.
E. W.

Preface to the fourth edition

Owing to the appreciable size of this work, it was found impractical to incorporate in this new edition additional material relating to the most recent developments in optics. We have, however, made further corrections and improvements in the text and have added references to some recent publications.

We are indebted to Dr E. W. Marchand and Mr. T. Kusakawa for supplying us with lists of misprints and errors found in the previous editions. We are also obliged to Dr G. Bédard for assistance with the revisions.

Bad Pyrmont and Rochester
August 1968

M. B.
E. W.

Preface to the fifth edition

Some further errors and misprints that were found in the earlier editions of this work have been corrected, the text in several sections has been improved and a number of references to recent publications have been added. More extensive changes have been made in §§13.1–13.3, dealing with the optical properties of metals. It is well known that a purely classical theory is inadequate to describe the interaction of an electromagnetic field with a metal in the optical range of the spectrum. Nevertheless, it is possible to indicate some of the main features of this process by means of a classical model, provided that the frequency dependence of the conductivity is properly taken into account and the role that the free, as well as the bound, electrons play in the response of the metal to an external electromagnetic field is understood, at least in qualitative terms. The changes in §§13.1–13.3 concern mainly these aspects of the theory and the revised sections are believed to be free of misleading statements and inaccuracies that were present in this connection in the earlier editions of this work and which can also be commonly found in many other optical texts.

I am grateful to some of our readers for informing me about misprints and errors. I wish to specifically acknowledge my indebtedness to Prof. A. D. Buckingham, Dr D. Canals Frau and, once again, Dr E. W. Marchand, who supplied me with detailed lists of corrections and to Dr É. Lalor and Dr G. C. Sherman for having drawn my attention to the need for making more substantial changes in Chapter XIII. I am also much obliged to Mr J. T. Foley for assistance with the revisions.

Rochester
January 1974

E. W.

Preface to the sixth edition

THIS edition differs from its immediate predecessor chiefly in that it contains corrections of a small number of errors and misprints.

Rochester
September 1985

E. W.

Preface to the seventh edition

Forty years ago this month Max Born and I dispatched to the publishers the Preface to the first edition of *Principles of Optics*. Since that time the book has been published in six editions and has been reprinted seventeen times (not counting unauthorized editions and several translations), usually with only a small number of corrections. A recent change to a new publisher, who expressed willingness to reset the whole text, has given me the opportunity to make more substantial changes.

The first edition was published a year before the invention of the laser, an event which triggered an explosion of activities in optics and soon led to the creation of entirely new fields, such as non-linear optics, fiber optics and opto-electronics. Numerous applications followed, in medicine, in optical data storage, in information transfer and in many other areas. On a more fundamental level, quantum optics emerged as a vibrant and rapidly expanding field, which has provided new ways of testing some basic assumptions of quantum physics relating, for example, to localization and indistinguishability. The progress made in these fields has been rapid and broad and some of the newer areas have themselves become the subjects of books.

It is clear that a fully updated new edition of *Principles of Optics* would require that it be expanded into several volumes. Consequently, in order to preserve a single-volume, only a few new topics have been added; they were selected to some extent so as not to necessitate major revisions of the original text. Specifically, the following new material has been added:

- (1) Section 4.11, which presents the principles of computerized axial tomography, generally referred to as CAT. This subject originated in the early 1960s and has revolutionized diagnostic medicine. The section also includes an account of the Radon transform, introduced as early as 1917, which underlies the theory of computerized axial tomography, although this was not known to its inventors. The fact that three Nobel Prizes have been awarded for this invention and its applications attests to its importance. More recently the theory underlying the CAT scan is finding much broader usage, for example in connection with the reconstruction of quantum states.
- (2) Section 8.11 gives an account of the so-called Rayleigh–Kirchhoff diffraction theory. This theory has become rather popular after it was introduced in the book *Optics* by A. Sommerfeld, published in 1954. It is preferred by some optical scientists to the much older classic Kirchhoff diffraction theory. However, which of these theories better describes various diffraction effects is still an open question.
- (3) Section 10.5 discusses some effects discovered relatively recently arising on

superposition of broad-band light beams of any state of coherence. The analysis of such effects has demonstrated that even though under such circumstances interference fringes may not be seen in the region of superposition, the light distribution in that region may nevertheless contain important physical information which is revealed when the light is spectrally analyzed. One may then find that the spectrum of the light is different at different points of observation and from such spectral changes one may determine coherence properties of the light. This effect is an example of a coherence phenomenon in the space-frequency domain, which must be distinguished from the more familiar coherence effects in the space-time domain. The quantitative measure of space-frequency coherence phenomena is the so-called spectral degree of coherence, which is introduced in this new edition in the somewhat revised Section 10.5. The experiment which is analyzed in that section also provides an elementary introduction to the phenomenon of correlation-induced spectral changes, discovered just over 10 years ago and studied extensively since that time.

(4) A new Chapter 13 presents the theory of scattering of light by inhomogeneous media. In the context of optics this subject was largely developed in relatively recent times although essentially the same theory was well established many years ago in connection with quantum mechanical potential scattering. The chapter presents the basic integral equation of light scattering on linear isotropic bodies, discusses the solution of the equation in series form and includes a detailed account of the first Born and the first Rytov approximations. The chapter includes a brief description of the classic theory of scattering by a medium with periodic structure, which is the basis of the theory of X-ray diffraction by crystals. The chapter also covers the von Laue equations, Bragg's law, the Ewald sphere of reflection and the Ewald limiting sphere. In recent years these subjects have become important in the broad area of inverse light scattering and they have found new applications, for example in connection with holographic gratings. The chapter also contains a detailed account of the optical cross-section theorem (usually known as the optical theorem), which, in spite of its name, is seldom discussed in the optical literature.

Another topic treated in Chapter 13 is diffraction tomography. In computerized axial tomography discussed in the new Section 4.11, the finite wavelength of the radiation is ignored. This is usually justified when the technique is used with X-rays, but the approximation is often inadequate in applications where light waves or sound waves are used. Diffraction tomography takes the finite wavelength of the radiation into account and, therefore, provides better resolution.

In addition to the new material already mentioned, this new edition also contains several new appendices (VIII, XI and XII), many new references, some of which replace older ones, and a few relatively small changes have been made in the text, usually with the aim of improving clarity and updating information.

It is a pleasure to thank several colleagues, friends and students for their help. I am obliged to Professor Harrison H. Barrett for helpful suggestions which led to improvements of Sections 4.11 and 13.2 on computerized axial tomography and on diffraction tomography. I am indebted to my long-time friend and colleague Professor Leonard Mandel for useful comments on some parts of the manuscript. Dr Doo Cho and Dr V. N. Mahajan were kind enough to draw my attention to some inaccuracies which they found in earlier editions and which have now been taken care of. My collaborators Dr

Yajun Li, Dr Taco D. Visser and my former students Dr Avshalom Gamliel, Dr David G. Fischer and Dr Kisik Kim have read some of the new sections and have made helpful comments leading to improvements. My former student Dr Weijian Wang kindly prepared most of the new figures. I am particularly grateful to two of my present graduate students, P. Scott Carney and Greg J. Gbur, who provided invaluable help by checking calculations and references, weeding out inaccuracies, suggesting improvements in the presentation and helping with proof-reading.

I wish to express my appreciation to Mrs Patricia Sulouff, Head Librarian of the Physics–Optics–Astronomy library at the University of Rochester for helping to trace some of the more obscure references and for locating papers and books that were not easily accessible. I am also obliged to Mrs Ellen Calkins for typing and re-typing several drafts of the manuscript of the new sections and for preparing the author index.

Most of the revisions were prepared whilst I was on a visiting appointment at the Center for Research and Education in Optics and Research (CREOL) at the University of Central Florida during the 1998 Spring Semester. I wish to thank Professors M. J. Soileau, M. G. Moharam and G. I. Stegeman of the CREOL faculty for providing a congenial environment and facilities for this work.

The family life of most authors suffers during the time when one of the marriage partners is engaged in book writing. Mine was no exception, but I am glad to say that my wife, Marlies, survived this period cheerfully and without complaints. She has helped with checking the manuscript and the proofs, for which I am grateful.

I acknowledge with thanks the fine cooperation that I received from the staff of Cambridge University Press. I am particularly appreciative of the considerable help provided by Dr Simon Capelin, the publishing director for Physical Sciences. I am also obliged to Mrs Maureen Storey for thorough copy-editing of the whole book and to Ms Miranda Fyfe and particularly to Mrs Jayne Aldhouse for their cooperation in meeting the rather stringent production deadlines.

I am sometimes asked about my collaboration with Max Born, which resulted in the publication of *Principles of Optics*. Interested readers can find it discussed in my article ‘Recollections of Max Born,’ published in *Optics News* **9**, 10–16 (November/December, 1983).*

Rochester, New York
January 1999

Emil Wolf

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Historical introduction

THE physical principles underlying the optical phenomena with which we are concerned in this treatise were substantially formulated before 1900. Since that year, optics, like the rest of physics, has undergone a thorough revolution by the discovery of the quantum of energy. While this discovery has profoundly affected our views about the nature of light, it has not made the earlier theories and techniques superfluous; rather, it has brought out their limitations and defined their range of validity. The extension of the older principles and methods and their applications to very many diverse situations has continued, and is continuing with undiminished intensity.

In attempting to present in an orderly way the knowledge acquired over a period of several centuries in such a vast field it is impossible to follow the historical development, with its numerous false starts and detours. It is therefore deemed necessary to record separately, in this preliminary section, the main landmarks in the evolution of ideas concerning the nature of light.*

The philosophers of antiquity speculated about the nature of light, being familiar with burning glasses, with the rectilinear propagation of light, and with refraction and reflection. The first systematic writings on optics of which we have any definite knowledge are due to the Greek philosophers and mathematicians [Empedocles (c. 490–430 BC), Euclid (c. 300 BC)].

Amongst the founders of the new philosophy, René Descartes (1596–1650) may be singled out for mention as having formulated views on the nature of light on the basis of his metaphysical ideas.† Descartes considered light to be essentially a pressure transmitted through a perfectly elastic medium (the aether) which fills all space, and he attributed the diversity of colours to rotary motions with different velocities of the particles in this medium. But it was only after Galileo Galilei (1564–1642) had, by his

* For a more extensive account of the history of optics, reference may be made to: J. Priestley, *History and Present State of Discoveries relating to Vision, Light and Colours* (2 Vols., London, 1772); Thomas Young, *A Course of Lectures on Natural Philosophy and the Mechanical Arts* Vol. 1 (London, 1845), pp. 374–385; E. Wilde, *Geschichte der Optik vom Ursprung dieser Wissenschaft bis auf die gegenwärtige Zeit*, 2 Vols. (Berlin, 1838, 1843); Ernst Mach, *The Principles of Physical Optics*, A historical and philosophical treatment (First German edition 1913. English translation 1926, reprinted by Dover Publications, New York, 1953); E. Hoppe, *Geschichte der Optik* (Leipzig, Weber, 1926); V. Ronchi, *Storia della Luce* (Bologna: Zanichelli, 2nd. Ed., 1952). A comprehensive historical account up to recent times is E. T. Whittaker's *A History of the Theories of Aether and Electricity*, Vol. I (*The Classical Theories*), revised and enlarged edition 1952; Vol. II (*The Modern Theories 1900–1926*), 1953, published by T. Nelson and Sons, London and Edinburgh. The first volume was used as the chief source for this introductory section.

† R. Descartes, *Dioptrique, Météores* (published (anonymously) in Leyden in 1637 with prefatory essay 'Discours de la méthode'). *Principia Philosophiae* (Amsterdam, 1644).

development of mechanics, demonstrated the power of the experimental method that optics was put on a firm foundation. The law of reflection was known to the Greeks; the law of refraction was discovered experimentally in 1621 by Willebrord Snell* (Snellius, *c.* 1580–1626). In 1657 Pierre de Fermat (1601–1665) enunciated the celebrated *Principle of Least Time*† in the form ‘Nature always acts by the shortest course’. According to this principle, light always follows that path which brings it to its destination in the shortest time, and from this, in turn, and from the assumption of varying ‘resistance’ in different media, the law of refraction follows. This principle is of great philosophical significance, and because it seems to imply a teleological manner of explanation, foreign to natural science, it has raised a great deal of controversy.

The first phenomenon of interference, the colours exhibited by thin films now known as ‘Newton’s rings’, was discovered independently by Robert Boyle‡ (1627–1691) and Robert Hooke§ (1635–1703). Hooke also observed the presence of light in the geometrical shadow, the ‘diffraction’ of light but this phenomenon had been noted previously by Francesco Maria Grimaldi|| (1618–1663). Hooke was the first to advocate the view that light consists of rapid vibrations propagated instantaneously, or with a very great speed, over any distance, and believed that in an homogeneous medium every vibration will generate a sphere which will grow steadily.¶ By means of these ideas Hooke attempted an explanation of the phenomenon of refraction, and an interpretation of colours. But the basic quality of colour was revealed only when Isaac Newton (1642–1727) discovered** in 1666 that white light could be split up into component colours by means of a prism, and found that each pure colour is characterized by a specific refrangibility. The difficulties which the wave theory encountered in connection with the rectilinear propagation of light and of polarization (discovered by Huygens††) seemed to Newton so decisive that he devoted himself to the development of an emission (or corpuscular) theory, according to which light is propagated from a luminous body in the form of minute particles.

At the time of the publication of Newton’s theory of colour it was not known whether light was propagated instantaneously or not. The discovery of the finite speed of light was made in 1675 by Olaf Römer (1644–1710) from the observations of the eclipses of Jupiter’s satellites.‡‡

The wave theory of light which, as we saw, had Hooke amongst its first champions was greatly improved and extended by Christian Huygens†† (1629–1695). He enunciated the principle, subsequently named after him, according to which every point of the ‘aether’ upon which the luminous disturbance falls may be regarded as the centre of a new disturbance propagated in the form of spherical waves; these secondary waves

* Snell died in 1626 without making his discoveries public. The law was first published by Descartes in his *Dioptrique* without an acknowledgement to Snell, though it is generally believed that Descartes had seen Snell’s manuscript on this subject.

† In a letter to Cureau de la Chambre. It is published in *Oeuvres de Fermat*, Vol. 2 (Paris, 1891) p. 354.

‡ *The Philosophical Works of Robert Boyle* (abridged by P. Shaw), Vol. II (Second ed. London, 1738), p. 70.

§ R. Hooke, *Micrographia* (1665), 47.

|| F. M. Grimaldi, *Physico-Mathesis de lumine, coloribus, et iride* (Bologna, 1665).

¶ The early wave theories of Hooke and Huygens operate with single ‘pulses’ rather than with wave trains of definite wavelengths.

**I. Newton, *Phil. Trans.* No. 80 (Feb. 1672), 3075.

††Chr. Huygens, *Traité de la lumière* (completed in 1678, published in Leyden in 1690).

‡‡Olaf Römer, *Mém. de l’Acad. Sci. Paris*, 10 (1666–1699), 575; *J. de Sav.* (1676), 223.

combine in such a manner that their envelope determines the wave-front at any later time. With the aid of this principle he succeeded in deriving the laws of reflection and refraction. He was also able to interpret the double refraction of calc-spar [discovered in 1669 by Erasmus Bartholinus (1625–1698)] by assuming that in the crystal there is, in addition to a primary spherical wave, a secondary ellipsoidal wave. It was in the course of this investigation that Huygens made the fundamental discovery of polarization: each of the two rays arising from refraction by calc-spar may be extinguished by passing it through a second crystal of the same material if the latter crystal be rotated about the direction of the ray. It was, however, left to Newton to interpret these phenomena; he assumed that rays have ‘sides’; and indeed this ‘transversality’ seemed to him an insuperable objection to the acceptance of the wave theory, since at that time scientists were familiar only with longitudinal waves (from the propagation of sound).

The rejection of the wave theory on the authority of Newton led to its abeyance for nearly a century, but it still found an occasional supporter, such as the great mathematician Leonhard Euler (1707–1783).*

It was not until the beginning of the nineteenth century that the decisive discoveries were made which led to general acceptance of the wave theory. The first step towards this was the enunciation in 1801 by Thomas Young (1773–1829) of the principle of interference and the explanation of the colours of thin films.† However, as Young’s views were expressed largely in a qualitative manner, they did not gain general recognition.

About this time, polarization of light by reflection was discovered by Étienne Louis Malus‡ (1775–1812). Apparently, one evening in 1808, he observed the reflection of the sun from a window pane through a calc-spar crystal, and found that the two images obtained by double refraction varied in relative intensities as the crystal was rotated about the line of sight. However, Malus did not attempt an interpretation of this phenomenon, being of the opinion that current theories were incapable of providing an explanation.

In the meantime the emission theory had been developed further by Pierre Simon de Laplace (1749–1827) and Jean-Baptiste Biot (1774–1862). Its supporters proposed the subject of diffraction for the prize question set by the Paris Academy for 1818, in the expectation that a treatment of this subject would lead to the crowning triumph of the emission theory. But their hopes were disappointed, for, in spite of strong opposition, the prize was awarded to Augustin Jean Fresnel (1788–1827), whose treatment§ was based on the wave theory, and was the first of a succession of investigations which, in the course of a few years, were to discredit the corpuscular theory completely. The substance of his memoir consisted of a synthesis of Huygens’ Envelope Construction with Young’s Principle of Interference. This, as Fresnel showed, was sufficient to explain not only the ‘rectilinear propagation’ of light but also the minute deviations from it – diffraction phenomena. Fresnel calculated the diffraction caused by straight edges, small apertures and screens; particularly impressive was the

* *L. Euleri Opuscula varii argumenti* (Berlin, 1746), p. 169.

† Th. Young, *Phil. Trans. Roy. Soc., London* xcii, 12 (1802) 387. *Miscellaneous works of the late Thomas Young*, Vol. I (London, J. Murray, 1885) pp. 140, 170.

‡ É. L. Malus, *Nouveau Bull. d. Sci., par la Soc. Philomatique*, Vol. 1 (1809), 266. *Mém. de la Soc. d’Arcueil*, Vol. 2 (1809).

§ A. Fresnel, *Ann. Chim. et Phys.*, (2), 1 (1816) 239; *Oeuvres*, Vol. 1, 89, 129.

experimental confirmation by Arago of a prediction, deduced by Poisson from Fresnel's theory, that in the centre of the shadow of a small circular disc there should appear a bright spot.

In the same year (1818), Fresnel also investigated the important problem of the influence of the earth's motion on the propagation of light, the question being whether there was any difference between the light from stellar and terrestrial sources. Dominique François Arago (1786–1853) found from experiment that (apart from aberration) there was no difference. On the basis of these findings Fresnel developed his theory of the partial convection of the luminiferous aether by matter, a theory confirmed in 1851 by direct experiment carried out by Armand Hypolite Louis Fizeau (1819–1896). Together with Arago, Fresnel investigated the interference of polarized rays of light and found (in 1816) that two rays polarized at right angles to each other never interfere. This fact could not be reconciled with the assumption of longitudinal waves, which had hitherto been taken for granted. Young, who had heard of this discovery from Arago, found in 1817 the key to the solution when he assumed that the vibrations were transverse.

Fresnel at once grasped the full significance of this hypothesis, which he sought to put on a more secure dynamical basis* and from which he drew numerous conclusions. For, since only longitudinal oscillations in a fluid are possible, the aether must behave like a solid body; but at that time a theory of elastic waves in solids had not yet been formulated. Instead of developing such a theory and deducing the optical consequences from it, Fresnel proceeded by inference, and sought to deduce the properties of the luminiferous aether from the observations. The peculiar laws of light propagation in crystals were Fresnel's starting point; the elucidation of these laws and their reduction to a few simple assumptions about the nature of elementary waves represents one of the greatest achievements of natural science. In 1832, William Rowan Hamilton† (1805–1865), who himself made important contributions to the development of optics, drew attention to an important deduction from Fresnel's construction, by predicting the so-called conical refraction, whose existence was confirmed experimentally shortly afterwards by Humphrey Lloyd‡ (1800–1881).

It was also Fresnel who (in 1821) gave the first indication of the cause of dispersion by taking into account the molecular structure of matter§, a suggestion elaborated later by Cauchy.

Dynamical models of the mechanism of aether vibrations led Fresnel to deduce the laws which now bear his name, governing the intensity and polarization of light rays produced by reflection and refraction.||

Fresnel's work had put the wave theory on such a secure foundation that it seemed almost superfluous when in 1850 Foucault¶ and Fizeau and Breguet** undertook a crucial experiment first suggested by Arago. The corpuscular theory explains refraction

* A. Fresnel, *Oeuvres Complètes d'Augustin Fresnel*, Vol. 2 (Paris, Imprimerie Imperiale, 1866–1870), pp. 261, 479.

† W. R. Hamilton, *Trans. Roy. Irish Acad.*, **17** (1833), 1. Also *Hamilton's Mathematical Papers*, eds. J. L. Synge and W. Conway, Vol. 1 (Cambridge, Cambridge University Press, 1931) p. 285.

‡ H. Lloyd, *Trans. Roy. Irish Acad.*, **17** (1833), 145.

§ A. Fresnel, *ibid.*, p. 438.

|| A. Fresnel, *Mém. de l'Acad.*, **11** (1832), 393; *Oeuvres*, **1**, 767.

¶ L. Foucault, *Compt. Rend. Acad. Sci. Paris*, **30** (1850), 551.

H. Fizeau and L. Breguet, *Compt. Rend. Acad. Sci. Paris*, **30 (1850), 562, 771.

in terms of the attraction of the light-corpuscles at the boundary towards the optically denser medium, and this implies a greater velocity in the denser medium; on the other hand the wave theory demands, according to Huygens' construction, that a smaller velocity obtains in the optically denser medium. The direct measurement of the velocity of light in air and water decided unambiguously in favour of the wave theory.

The decades that followed witnessed the development of the elastic aether theory. The first step was the formulation of a theory of the elasticity of solid bodies. Claude Louis Marie Henri Navier (1785–1836) developed such a theory*, discerning that matter consists of countless particles (mass points, atoms) exerting on each other forces along the lines joining them. The now customary derivation of the equations of elasticity by means of the continuum concept is due to Augustine Louis Cauchy† (1789–1857). Of other scientists who participated in the development of optical theory, mention must be made of Siméon Denis Poisson‡ (1781–1840), George Green§ (1793–1841), James MacCullagh|| (1809–1847) and Franz Neumann¶ (1798–1895). Today it is no longer relevant to enter into the details of these theories or into the difficulties which they encountered; for the difficulties were all caused by the requirement that optical processes should be explicable in mechanical terms, a condition which has long since been abandoned. The following indication will suffice. Consider two contiguous elastic media, and assume that in the first a transverse wave is propagated towards their common boundary. In the second medium the wave will be resolved, in accordance with the laws of mechanics, into longitudinal and transverse waves. But, according to Arago's and Fresnel's experiments, elastic longitudinal waves must be ruled out and must therefore be eliminated somehow. This, however, is not possible without violating the laws of mechanics expressed by the boundary conditions for strains and stresses. The various theories put forward by the authors mentioned above differ in regard to the assumed boundary conditions, which always conflicted in some way with the laws of mechanics.

An obvious objection to regarding the aether as an elastic solid is expressed in the following query: How is one to imagine planets travelling through such a medium at enormous speeds without any appreciable resistance? George Gabriel Stokes (1819–1903) thought that this objection could be met on the grounds that the planetary speeds are very small compared to the speeds of the aetherial particles in the vibrations constituting light; for it is known that bodies like pitch or sealing wax are capable of rapid vibrations but yield completely to stresses applied over a long period. Such controversies seem superfluous today since we no longer consider it necessary to have mechanical pictures of all natural phenomena.

A first step away from the concept of an elastic aether was taken by MacCullagh,** who postulated a medium with properties not possessed by ordinary bodies. The latter store up energy when the volume elements change shape, but not during rotation. In MacCullagh's aether the inverse conditions prevail. The laws of propagation of waves

* C. L. M. H. Navier, *Mém. de l'Acad.*, **7**, (submitted in 1821, published in 1827), 375.

† A. L. Cauchy, *Exercice de Mathématiques*, **3** (1828), 160.

‡ S. D. Poisson, *Mém. de l'Acad.*, **8** (1828), 623.

§ G. Green, *Trans. Camb. Phil. Soc.* (1838); *Math. Papers*, 245.

|| J. MacCullagh, *Phil. Mag.* (3), **10** (1837), 42, 382; *Proc. Roy. Irish Acad.*, **18** (1837).

¶ F. Neumann, *Abh. Berl. Akad., Math. Kl.* (1835), **1**.

** J. MacCullagh, *Trans. Roy. Irish Acad.*, **21**, Coll. Works, Dublin (1880), 145.

in such a medium show a close similarity to Maxwell's equations of electromagnetic waves which are the basis of modern optics.

In spite of the many difficulties, the theory of an elastic aether persisted for a long time and most of the great physicists of the nineteenth century contributed to it. In addition to those already named, mention must be made of William Thomson* (Lord Kelvin, 1824–1908), Carl Neumann† (1832–1925), John William Strutt‡ (Lord Rayleigh, 1842–1919), and Gustav Kirchhoff§ (1824–1887). During this period many optical problems were solved, but the foundations of optics remained in an unsatisfactory state.

In the meantime researches in electricity and magnetism had developed almost independently of optics, culminating in the discoveries of Michael Faraday|| (1791–1867). James Clerk Maxwell¶ (1831–1879) succeeded in summing up all previous experiences in this field in a system of equations, the most important consequence of which was to establish the possibility of electromagnetic waves, propagated with a velocity which could be calculated from the results of purely electrical measurements. Actually, some years earlier Rudolph Kohlrausch (1809–1858) and Wilhelm Weber** (1804–1891) had carried out such measurements and the velocity turned out to be that of light. This led Maxwell to conjecture that light waves are electromagnetic waves; a conjecture verified by direct experiment in 1888 by Heinrich Hertz†† (1857–1894). In spite of this, Maxwell's electromagnetic theory had a long struggle to gain general acceptance. It seems to be a characteristic of the human mind that familiar concepts are abandoned only with the greatest reluctance, especially when a concrete picture of the phenomena has to be sacrificed. Maxwell himself, and his followers, tried for a long time to describe the electromagnetic field with the aid of mechanical models. It was only gradually, as Maxwell's concepts became more familiar, that the search for an 'explanation' of his equations in terms of mechanical models was abandoned; today there is no conceptual difficulty in regarding Maxwell's field as something which cannot be reduced to anything simpler.

But even the electromagnetic theory of light has attained the limits of its serviceability. It is capable of explaining, in their main features, all phenomena connected with the propagation of light. However, it fails to elucidate the processes of emission and absorption, in which the finer features of the interaction between matter and the optical field are manifested.

The laws underlying these processes are the proper object of modern optics, indeed of modern physics. Their story begins with the discovery of certain regularities in spectra. The first step was Josef Fraunhofer's (1787–1826) discovery‡‡ (1814–1817)

* W. Thomson, *Phil. Mag.*, (5), **26** (1888), 414. Baltimore Lectures (London, 1904).

† C. Neumann, *Math. Ann.*, **1** (1869), 325, **2** (1870), 182.

‡ J. W. Strutt, (Lord Rayleigh), *Phil. Mag.*, (4) **41** (1871), 519; **42** (1871), 81.

§ G. Kirchhoff, *Berl. Abh. Physik., Abteilg.* **2** (1876), 57; *Ges. Abh.* 352; *Berl. Ber.* (1882), 641; *Pogg. Ann. Physik. u. Chem.* (2), **18** (1883), 663; *Ges. Abh., Nachtrag.* 22.

|| M. Faraday, *Experimental Researches in Electricity* (London, 1839).

¶ J. C. Maxwell, *A Treatise on Electricity and Magnetism*, 2 Vols. (Oxford, 1873).

R. Kohlrausch and W. Weber, *Pogg. Ann. Physik u. Chem.* (2), **99 (1856), 10.

††H. Hertz, *Sitzb. Berl. Akad. Wiss.*, Feb. 2, 1888; *Wiedem. Ann.* **34** (1888), 551; English translation in his *Electric Waves* (London, Macmillan, 1893), p. 107.

‡‡J. Fraunhofer, *Gilberts Ann.*, **56** (1817), 264. W. H. Wollaston (1766–1828) observed these lines in 1802 (*Phil. Trans. Roy. Soc.*, London (1802), 365) but had not appreciated his discovery and interpreted them incorrectly.

of the dark lines in the solar spectrum, since named after him; and their interpretation as absorption lines given in 1861 on the basis of experiments by Roger Wilhelm Bunsen (1811–1899) and Gustav Kirchhoff* (1824–1887). The light of the continuous spectrum of the body of the sun, passing the cooler gases of the sun's atmosphere, loses by absorption just those wavelengths which are emitted by the gases. This discovery was the beginning of spectrum analysis, which is based on the recognition that every gaseous chemical element possesses a characteristic line spectrum. The investigation of these spectra has been a major object of physical research up to and including the present; and since light is its subject, and optical methods are employed, it is often considered as a part of optics. The problem of how light is produced or destroyed in atoms is, however, not exclusively of an optical nature, as it involves equally the mechanics of the atom itself; and the laws of spectral lines reveal not so much the nature of light as the structure of the emitting particles. Thus, from being a part of optics, spectroscopy has gradually evolved into a separate discipline which provides the empirical foundations for atomic and molecular physics. This field is, however, beyond the scope of this book.

Concerning methods, it has become apparent that classical mechanics is inadequate for a proper description of events occurring within the atom and must be replaced by the quantum theory, originated in 1900 by Max Planck† (1858–1947). Its application to the atomic structure, led in 1913, to the explanation by Niels Bohr‡ (1885–1962) of the simple laws of line spectra of gases. From these beginnings and from the ever increasing experimental material, modern quantum mechanics developed (Heisenberg, Born, Jordan, de Broglie, Schrödinger, Dirac).§ By its means considerable insight has been obtained into the structure of atoms and molecules.

However, our concept of the nature of light has also been greatly influenced by quantum theory. Even in its first form due to Planck there appears a proposition which is directly opposed to classical ideas, namely that an oscillating electric system does not impart its energy to the electromagnetic field in a continuous manner but in finite amounts, or 'quanta' $\epsilon = h\nu$, proportional to the frequency ν of the light, where $h = 6.55 \times 10^{-27}$ erg/s is Planck's constant. We may say that the occurrence of the constant h is the feature which distinguishes modern physics from the old.

It was only gradually that the paradoxical, almost irrational, character of Planck's equation $\epsilon = h\nu$ was fully realized by physicists. This was brought about mainly by the work of Einstein and Bohr. On the basis of Planck's theory, Einstein (1879–1955) revived in 1905 the corpuscular theory of light in a new form|| by assuming that Planck's energy quanta exist as real light-particles, called 'light quanta' or 'photons'. He thereby succeeded in explaining some phenomena which had been discovered more recently in connection with the transformation of light into corpuscular energy,

* R. Bunsen and G. Kirchhoff, *Untersuchungen über das Sonnenspektrum und die Spektren der Chemischen Elemente.*, *Abh. kgl. Akad. Wiss.* (Berlin, 1861) p. 1863.

† M. Planck, *Verh. d. deutsch phys. Ges.*, **2** (1900) 202, 237. *Ann. d. Physik* (4), **4** (1901), 553.

‡ N. Bohr, *Phil. Mag.* (6), **26** (1913), 1, 476, 857.

§ W. Heisenberg, *Z. Phys.*, **33** (1925), 879; M. Born and P. Jordan, *ibid.*, **34** (1925), 858; M. Born, W. Heisenberg, and P. Jordan, *ibid.*, **35** (1926), 557; L. de Broglie, *Thèse* (Paris, 1924); *Ann. de Physique* (10), **3** (1925), 22; E. Schrödinger, *Ann. d. Physik* (4), **79** (1926), 361, 489 and 734; **80** (1926), 437; **81** (1926), 109. English translation: *Collected Papers on Wave Mechanics* by E. Schrödinger (London and Glasgow, Blackie, 1928); P. A. M. Dirac, *Proc. Roy. Soc. A*, **109** (1925), 642; *ibid.*, **110** (1926), 561.

|| A. Einstein, *Ann. d. Physik* (4), **17** (1905), 132; **20** (1906), 199.

phenomena which were inexplicable by the wave theory. Chief among these are the so-called photoelectric effect and the fundamentals of photochemistry. In phenomena of this type light does not impart to a detached particle an energy proportional to its intensity, as demanded by the wave theory, but behaves rather like a hail of small shots. The energy imparted to the secondary particles is independent of the intensity, and depends only on the frequency of the light (according to the law $\varepsilon = h\nu$). The number of observations confirming this property of light increased year by year and the situation arose that the simultaneous validity of both wave and corpuscular theories had to be recognized, the former being exemplified by the phenomena of interference, the latter by the photoelectric effect. It is only in more recent years that the development of quantum mechanics has led to a partial elucidation of this paradoxical state of affairs, but this has entailed giving up a fundamental principle of the older physics, namely the principle of deterministic causality.

The detailed theory of the interaction between field and matter required the extension of the methods of quantum mechanics (field quantization). For the electromagnetic radiation field this was first carried out by Dirac* and these investigations form the basis of quantum optics. However, it was mainly the development of radically new light sources, the lasers, in the 1960s that led to the emergence of quantum optics as a new discipline. The invention of the laser provided new sources of very intense, coherent and highly directional light beams. Such sources are analogous to devices known as masers, developed a few years earlier, for generating and amplifying microwave radiation by conversion of atomic and molecular energy by the process of stimulated emission. Pioneering contributions which led to the invention of these devices were made notably by Basov, Prokhorov, Townes, Schawlow and Maiman.† Apart from providing an important tool for research in quantum optics, the invention of the laser led to numerous applications and originated several new fields such as quantum-electronics, nonlinear optics, fiber optics and others.

Another branch of optics, not touched upon in this work, is the optics of moving bodies. Like the quantum theory it has grown into a vast independent field of study. The first observed phenomenon in this field, recorded in 1728 by James Bradley‡ (1692–1762), was the aberration of ‘fixed stars’, i.e. the observation of slightly different angular positions of the stars according to the motion of the earth relative to the direction of the light ray. Bradley correctly interpreted this phenomenon as being due to the finite velocity of light and thus was able to determine the velocity. We have already mentioned other phenomena belonging to optics of moving media: Fresnel was the first to enquire into the convection of light by moving bodies and to show that it behaved as if the luminiferous aether participated in the movement only with a fraction of the speed of the moving bodies; Fizeau then demonstrated this partial convection experimentally with the aid of flowing water. The effect of the motion of the light source or of the observer was investigated by Christian Doppler§ (1803–1853) who formulated the well-known principle named after him. So long as the elastic theory of light held the field and the precision of measurements was sufficiently limited,

* P. A. M. Dirac, *Proc. Roy. Soc.*, **A114** (1927), 243, 710.

† N. G. Basov and A. M. Prokhorov, *Zh. Eksp. Teor. Fiz.*, **27** (1954), 431; A. L. Schawlow and C. H. Townes, *Phys. Rev.*, **112** (1958), 1940; T. H. Maiman, *Nature*, **187** (1960), 493.

‡ J. Bradley, *Phil. Trans.*, **35** (1728), 637.

§ Chr. Doppler, *Abh. Königl. böhm. Geselsch.*, **2** (1842), 466; *Pogg. Ann.*, **68** (1847), 1.

Fresnel's ideas on partial convection sufficed for a satisfactory explanation of all the phenomena. But the electromagnetic theory of light encountered difficulties of a fundamental nature. Hertz was the first to attempt to generalize Maxwell's laws to moving bodies. His formulae were, however, in conflict with some electromagnetic and optical experiments. Of great importance was the theory of Hendrik Antoon Lorentz (1853–1928) who assumed an 'aether in a state of absolute rest' to be the carrier of the electromagnetic field and deduced the properties of material bodies from the interaction of elementary electric particles – the electrons. He was able to show that Fresnel's coefficient of convection could be obtained correctly from his theory and that in general all phenomena known at the time (1895) lent themselves to explanation by this hypothesis.* But the enormous increase of precision in the determination of optical paths by means of the interferometer of Albert Abraham Michelson (1852–1931) led to a new anomaly: it proved impossible to demonstrate the existence of an 'aether drift' required by the theory of the 'stationary aether'.† The anomaly was resolved by Albert Einstein‡ in 1905 in his special theory of relativity. The theory is founded on a critique of the concepts of time and space and leads to the abandonment of Euclidian geometry and the intuitive conception of simultaneity. Its further development into the so-called general theory of relativity§ led to a completely new conception of gravitational phenomena by a 'geometrization' of the space-time manifold. The application of this theory involves the use of special mathematical and physical methods which, although relevant to optics in many cases, may easily be considered separately from it. The number of optical phenomena in which the motion of bodies (e.g. light sources) plays a significant part is rather small.

* H. A. Lorentz, *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern* (Leiden, E. J. Brill 1895; reprinted by Teubner, Leipzig, 1906).

† A. A. Michelson, *Amer. Jour. Sci.* (3), **22** (1881), 20; A. A. Michelson and E. W. Morley, *Amer. Jour. Sci.* (3), **34** (1887), 333; *Phil. Mag.*, **24** (1887), 449.

‡ A. Einstein, *Ann. d. Physik* (4), **17** (1905), 891.

§ A. Einstein, *Berl. Sitz.* (1915), 778, 799, 831, 844. *Ann. d. Physik* (4) **49** (1916), 769.