Classical Field Theory

Classical field theory predicts how physical fields interact with matter and is a logical precursor to quantum field theory. This introduction focuses purely on modern classical field theory, helping graduate students and researchers build an understanding of classical field theory methods before embarking on future studies in quantum field theory. It describes various classical methods for fields with negligible quantum effects – for instance, electromagnetism and gravitational fields. It focuses on solutions that take advantage of classical field theory methods as opposed to applications or geometric properties. Other fields covered include fermionic fields, scalar fields, and Chern–Simons fields. Methods such as symmetries, global and local methods, the Noether theorem, the energy-momentum tensor, and important solutions of the classical equations – soliton solutions, in particular – are also discussed.

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To the memory of my mother,
who inspired me to become a physicist

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Preface

The methods of classical field theory have been around for a long time, but it mostly meant electromagnetism and general relativity. The classic book that developed this point of view is Landau and Lifshitz's *The Theory of Classical Fields*, which, however, was first published almost 70 years ago. In the meantime, quantum field theory has developed into a very large subject. Electromagnetism, or more precisely classical electrodynamics, is a subject geared towards many physical applications, while general relativity has developed into a subject rich in Riemannian and differential geometry, as well as topology. Thus, in graduate school there usually is a specialized course that deals with electrodynamics – for instance, following the standard textbook of J. D. Jackson – as well as a specialized course in general relativity – perhaps following a textbook like Wald. On the other hand, in most physics departments in the United States and Europe, one usually jumps directly into a course in quantum field fheory that offers a bit of classical methods interspersed here and there but mostly focuses in on the quantum aspects. It is my experience that this means students have a hard time adjusting to the large conceptual jump from classical mechanics to quantum field theory, and most modern methods of classical field theory don't make it into the standard curriculum.

At the Institute for Theoretical Physics of UNESP, we took a point of view that I find more sensible: we teach a semester of classical field theory, followed by two semesters of quantum field theory. This way, the transition is smoother, and one has time to learn some more modern methods of classical field theory. The course deals with electromagnetism and general relativity *only as examples of classical field theories*, so there are no applications of electromagnetism to a real medium and no geometry and topology for general relativity. Therefore, it is meant as a *complement* – and not a substitute – to electromagnetism and general relativity courses, one that focuses on the classical field theory aspects. The course I gave forms the basis for this book, to which I added more chapters, denoted here with an asterisk, to make it of use also to more advanced graduate students and researchers looking for a modern classical field theory reference. Thus, if one intends to use the book for teaching a one-semester course, one can safely drop the chapters with asterisk, which can be skipped until a second reading. There are other books on classical field theory out there, of which the most relevant I find Burgess's [5], which, however, spends little time on specific field theories and none on classical solutions; Rubakov's [58], which spends most of the time on gauge fields and on some quantum aspects; and Manton and Sutcliffe's [14], which deals only with solitons and is geared mostly towards researchers.

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I want to thank all those who have guided me on my journey as a physicist, starting with my mother Ligia, the first physicist I knew who first introduced me to the wonderful world of physics. My high school physics teacher, Iosif Sever Georgescu, showed me that I can make physics into a career. My student exchange advisor at the Niels Bohr Institute, Poul Olesen, introduced me to string theory, my chosen field of study, and everybody at NBI showed me how interesting physics research can be. My PhD advisor Peter van Niewenhuizen showed me how to be a complete theoretical physicist; he introduced me to the value of rigor and the beauty in calculations; and he taught me that one should strive to understand field theory as a whole, no matter your specialization.

This book is an expanded version of a course I gave at the IFT in São Paulo, so I would like to thank all of the students in the class for their questions and input, which helped to shape this book.

I want to thank all of my collaborators on topics of research that I worked on and appear in this book. I want to thank my wife Antonia for her patience and encouragement while I wrote this book, mostly at home in the evenings. I also want to thank my students and postdocs for dealing with my reduced time for them during the period I wrote this book. A big thanks to my editor at Cambridge University Press, Simon Capelin, who has been always supportive of me and helped me get this book published. To all the staff at CUP, thanks for making sure this book, as well as my previous ones, is as good as it can be.

Introduction

As I said in the Preface, this book deals with classical aspects of field theory, including electromagnetism and general relativity, but it is not meant as a substitute for a course in either – rather a complement to them. It is meant as a bridge between the study of graduate classical mechanics and quantum field theory, albeit one that comes with many interesting problems of its own. As such, I assume a good knowledge of classical mechanics, in particular Lagrangean and Hamiltonian formulations. I mostly stay out of quantum topics, except when dealing with the spin statistics theorem and representations of the Lorentz group – since those are topics needed to describe the kinds of fields we will be using – and in the case of collective coordinate quantization – since this topic goes hand in hand with the topic of solitons. In these cases, we will deal with quantum *mechanics*, not quantum *field theory* issues. As such, I also assume a working knowledge of quantum mechanics at the advanced undergraduate (if not beginning graduate) level.

The representations of the Lorentz group and the spin statistics theorem are necessary to describe the types of fields we can have: scalars, gauge fields, symmetric and antisymmetric tensors, spinors, and so on. I will start in Part I by describing general properties of the fields and, after describing the classification, focusing on scalars and gauge fields; I will also describe the basic ideas of hydrodynamics, as viewed from the perspective of field theory. In Part II, I will continue with solitons and topological issues, which were previewed in Part I by the Hopfion solution of electromagnetism, as well as with non-Abelian gauge theory. In Part III, I will treat spinors, p-forms and anyons, and general relativity as a field theory only. That means that I will not describe standard geometrical and topological issues, only issues that can be easily described in terms of Lagrangeans and their equations of motion. With respect to spinors, I will only consider "classical" issues like solutions to the equations of motion – though, as I will explain, spinors are in some sense always quantum.

Central to classical field theory is the notion of *field*, which is a function of space and time that describes some physical interaction. The easiest example to understand is the electromagnetic field, which is responsible for electromagnetic (electric and magnetic) interactions. Classical field theory is the way to describe, using classical mechanics, these fields and their interactions via Lagrangeans and their equations of motion. That is why, besides some general formal aspects, a lot of the book will be about equations of motion for these Lagrangeans and their solutions. In the case of "soliton" solution, a solution can be on a similar footing as a "particle," or fundamental excitation, of the theory (in the case of electromagnetism, the photon). We will therefore study such solitons in all the theories that we will consider.

Another importand notion is the notion of symmetry, encapsulated in the form of the Lagrangean of the system. That is why we will devote a lot of time to it, also, and we will introduce at the beginning of the book some notions of group theory and explain how we can write Lagrageans invariant under a symmetry. Throughout Part I we will deal with Abelian, or commuting, symmetries mostly and will analyze the more complicated non-Abelian symmetries in Part II. In Part III, besides the more standard fermions and general relativity, we will also deal with some more modern field theory issues: anyons and Chern–Simons fields have appeared a lot recently in condensed matter, while *p*-forms are mostly used in higher-dimensional theories (Kaluza Klein, supergravity, and string theory), though they also appear in some descriptions of QCD and other theories in 3+1 dimensions.