

Contents

Preface	<i>page</i> xi
Acknowledgments	xv
Introduction	1
What is Combinatorics?	1
Typical Problems	1
How Do We “Count”?	2
1 Induction and Recurrence Relations	6
1.1 Induction	6
Problems	9
1.2 Strong Induction	15
Problems	16
1.3 Recurrence Relations	19
Problems	22
1.4 Linear Recurrence Relations; Unwinding a Recurrence Relation*	27
Problems	35
1.5 Open Problems and Conjectures	40
2 The Pigeonhole Principle and Ramsey Theory	42
2.1 The Pigeonhole Principle	42
Problems	48
2.2 Multisets and Graphs	52
Problems	58
2.3 Ramsey Theory	61
Problems	71
2.4 Schur, Van der Waerden, and Graph Ramsey Numbers*	75
Problems	78
2.5 Open Problems and Conjectures	80
3 Counting, Probability, Balls and Boxes	83
3.1 The Addition, Multiplication, and Subtraction Principles	83
Problems	86
3.2 Probability	87
Problems	89

viii	Contents	
	3.3 A Framework for Counting Questions: The Counting Table Problems	93 97
	3.4 Bijective Maps and Counting Problems	98 100
	COLLABORATIVE MINI-PROJECT 1: Counting Monochromatic Triangles	103
	COLLABORATIVE MINI-PROJECT 2: Binomial Coefficients	107
	COLLABORATIVE MINI-PROJECT 3: Stirling Numbers	111
4	Permutations and Combinations	115
	4.1 Permutations of a Set and Falling Factorials Problems	115 120
	4.2 Combinations of Sets and Binomial Coefficients Problems	121 123
	4.3 Permutations of Multisets and Multinomial Coefficients Problems	127 132
	4.4 Combinations of Multisets and Counting Integer Solutions Problems	134 140
	4.5 More Problems Problems	143 143
5	Binomial and Multinomial Coefficients	147
	5.1 Binomial Coefficients Problems	147 158
	5.2 The Binomial Theorem Problems	164 167
	5.3 Multinomials and the Multinomial Theorem Problems	170 172
	5.4 Binomial Inversion, Sums of Powers, Lattice Paths, Ming–Catalan Numbers, and More* Problems	173 173
	5.5 Open Problems and Conjectures	191
6	Stirling Numbers	193
	6.1 Stirling Numbers of the Second Kind Problems	193 206
	6.2 Stirling Numbers of the First Kind Problems	210 217
	6.3 How Are the Stirling Numbers Related? Problems	222 226

	Contents	ix
7 Integer Partitions		230
7.1 Partitions of an Integer		230
Problems		237
7.2 Asymptotics, Pentagonal Number Theorem, and More*		240
Problems		241
7.3 Open Problems and Conjectures		247
COLLABORATIVE MINI-PROJECT 4: Generating Functions		249
COLLABORATIVE MINI-PROJECT 5: Graphic Sequences and Planar Graphs		253
COLLABORATIVE MINI-PROJECT 6: Connectivity of Graphs		257
8 The Inclusion–Exclusion Principle		261
8.1 The Inclusion–Exclusion Principle		261
Problems		269
8.2 Combinations of a Multiset		275
Problems		277
8.3 Permutations with Forbidden Positions		278
Problems		284
9 Generating Functions		292
9.1 Ordinary Generating Functions		292
Problems		304
9.2 Combinations of Multisets and Solving Recurrence Relations		307
Problems		315
9.3 Exponential Generating Functions		321
Problems		327
9.4 Generating Functions for Partitions, Stirling Numbers, Bernoulli Numbers, and More*		330
Problems		330
Interregnum: Counting Table Completed		349
COLLABORATIVE MINI-PROJECT 7: Ming–Catalan Numbers		351
COLLABORATIVE MINI-PROJECT 8: Sperner’s Theorem		355
10 Graph Theory		361
10.1 Graphic Sequences		362
Problems		367
10.2 Paths, Cycles, and Trees		369
Problems		374

x	Contents	
	10.3 Bipartite Graphs	381
	Problems	383
	10.4 Eulerian Trails and Circuits	385
	Problems	389
	10.5 Hamiltonian Paths and Cycles	392
	Problems	396
	10.6 Planar Graphs and the Tiling of the Plane	401
	Problems	413
	10.7 Graph Coloring and More*	424
	Problems	426
	10.8 Open Problems and Conjectures	439
	COLLABORATIVE MINI-PROJECT 9: Cayley's Tree Formula	443
	COLLABORATIVE MINI-PROJECT 10: Incidence Matrices and Bipartite Graphs	449
	11 Posets, Matchings, and Boolean Lattices	453
	11.1 Posets, Total Orders, and Hasse Diagrams	453
	Problems	460
	11.2 Chains, Antichains, and Dilworth's Theorem	463
	Problems	469
	11.3 Matchings and the Marriage Theorem	473
	Problems	479
	11.4 Boolean Lattices: Symmetric Chains, Theorems of Sperner, and Erdős–Ko–Rado*	488
	Problems	497
	11.5 Boolean Lattices and Graphs: Ramsey Theory Extended*	499
	Problems	503
	11.6 Möbius Inversion: Inclusion–Exclusion Extended*	505
	Problems	514
	11.7 Open Problems and Conjectures	518
	Appendix A Short Answers for Warm-Up Problems	522
	Appendix B Hints for Selected Problems	524
	Appendix C Short Answers for Selected Problems	531
	Appendix D Complete Solutions for Selected Problems	533
	Bibliography	602
	Index	609
	<i>The plate section can be found between pages 248 and 249</i>	