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A COMPREHENSIVE INTRODUCTION TO SUB-RIEMANNIAN GEOMETRY

Sub-Riemannian geometry is the geometry of a world with nonholonomic constraints. In such a world, one can move, send and receive information only in certain admissible directions but eventually one can reach every position from any other.

In the last two decades sub-Riemannian geometry has emerged as an independent research domain impacting on several areas of pure and applied mathematics, with applications to many areas such as quantum control, Hamiltonian dynamics, robotics and PDEs.

This comprehensive introduction proceeds from classical topics to cutting-edge theory and applications, assuming only a standard knowledge of calculus, linear algebra and differential equations. The book may serve as a basis for an introductory course in Riemannian geometry or an advanced course in sub-Riemannian geometry, covering elements of Hamiltonian dynamics, integrable systems and Lie theory. It will also be a valuable reference source for researchers in various disciplines.

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A Comprehensive Introduction to Sub-Riemannian Geometry

From the Hamiltonian Viewpoint

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To Irina, Michela and Giusy

Contents

Preface

This book presents material taught by the authors in graduate courses at SISSA, Trieste, at Institut Henri Poincaré, Paris, at Paris Diderot, and at several summer schools, in the period 2008–2018. It contains material for an introductory course in sub-Riemannian geometry at master or PhD level, as well as material for a more advanced course.

We have attempted to be as elementary as possible, but, although the main concepts are recalled, a certain ability in managing objects in differential geometry is desirable (vector fields, differential forms, symplectic manifolds etc.). We have tried to avoid the use of functional analysis as much as possible (some is required starting from Chapter 6).

The reader does not require any knowledge in Riemannian geometry. Actually from the book one can extract an introductory course in Riemannian geometry as a special case of the sub-Riemannian one, starting from the geometry of surfaces in Chapter 1.

There are few other books on sub-Riemannian geometry available. Besides the pioneering book edited by A. Bellaïche and J.-J. Risler [BR96], the book of R. Montgomery [Mon02] is nowadays a classical reference, and inspired several of our chapters. More recent books, written in a language similar to the one we use, are those of F. Jean [Jea14] and L. Rifford [Rif14]; see also the collection of lecture notes [BBS16a, BBS16b]. Other related books, although with a different approach, are the monographs [BLU07] and [CDPT07].

Example of an introductory course of sub-Riemannian geometry:
Chapters 2, 3 (without the appendices), 4, 7 (without Section 7.1), 9, 13, 21.

Example of an advanced course of sub-Riemannian geometry:
Chapters 2, 3 (with the appendices), 4, 6, 7 (together with Section 7.1), 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, the appendix by Zelenko.

Example of a course of Riemannian geometry: Chapters 1, 2, 3 (without the appendices), 4, 5, 7, 8, 11, 14 (without Section 14.4–14.6), 15, 16, Section 21.1.

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