Theory and Computation of Hydrodynamic Stability

The study of hydrodynamic stability is fundamental to many subjects, ranging from geophysics and meteorology through to engineering design. This treatise covers both classical and modern aspects of the subject, systematically developing it from the simplest physical problems, then progressing to the most complex, considering linear and nonlinear situations, and analyzing temporal and spatial stability. The authors examine each problem both analytically and numerically. Many relevant fluid flows are treated, including those where the fluid may be compressible, or those from geophysics, or those that require salient geometries for description. Details of initial-value problems are explored equally with those of stability.

The text includes copious illustrations and an extensive bibliography, making it suitable for courses on hydrodynamic stability or as an authoritative reference for researchers. In this second edition the opportunity has been taken to update the text and, most importantly, provide solutions to the numerous extended exercises.

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Theory and Computation of Hydrodynamic Stability

SECOND EDITION

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National Science Foundation
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Preface to the Second Edition

This second edition started soon after the first printing because it was recognized that this specialty field involves significant mathematics and the exercises at the end of each chapter may prove challenging to many. Therefore, it was determined that providing solutions to the exercises may be more useful to the learning process, and a number of solutions to the exercise problems are provided in Appendix C. We have also taken the opportunity to move the discussion of all of the numerical algorithms to one place, namely Appendix B. Also, many thanks to M. R. Malik and H. C. Kuhlmann for the published reviews of the first edition. Although the authors caught most of the issues presented in the reviews in the preprint process, the final printed version failed to address some typos and some poor quality images. It is our hope that this second edition will be more useful to the readers, especially with the addition of solutions to the exercises.