

COPULAS AND THEIR APPLICATIONS IN WATER RESOURCES ENGINEERING

Complex environmental and hydrological processes are characterized by more than one correlated random variable. These events are multivariate and their treatment requires multivariate frequency analysis. Traditional analysis methods are, however, too restrictive and do not apply in many cases. Recent years have therefore witnessed numerous applications of copulas to multivariate hydrologic frequency analyses. This book describes the basic concepts of copulas and outlines current trends and developments in copula methodology and applications. It includes an accessible discussion of the methods alongside simple step-by-step sample calculations. Detailed case studies with real-world data are included, and are organized based on applications, such as flood frequency analysis and water quality analysis. Illustrating how to apply the copula method to multivariate frequency analysis, engineering design, and risk and uncertainty analysis, this book is ideal for researchers, professionals, and graduate students in hydrology and water resources engineering.

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To

LZ: Mother Shuyuan, husband Bret, son Caelan

VPS: Wife Anita, son Vinay, daughter Arti, daughter-in-law Sonali, son-in-law
Vamsi, and grandsons Ronin, Kayden, and Davin

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Preface

Complex environmental and hydrological processes, such as floods, droughts, rainstorms, hurricanes, tornadoes, windstorms, weather extremes, and tides, are characterized by more than one correlated random variable. These events are multivariate and their treatment requires multivariate frequency analysis. Traditional multivariate frequency analysis methods are too restrictive and do not even apply in many cases. Recent years have therefore witnessed numerous applications of copulas to multivariate hydrologic frequency analyses.

Since the advent of Sklar theorem in 1959, several books have been written on copulas, but these books have been written by mathematicians and statisticians for students in mathematics and statistics. The book titled *Extremes in Nature: An Approach Using Copulas*, by Salvadori et al. (2007), is the only book discussing the copula theory and its application to natural events, but since its publication new types of copulas as well as new applications have been introduced. Therefore, there is a need for a book that describes basic concepts of copulas, illustrates them in an easy-to-understand manner, presents different types of copulas, and discusses their applications.

This book on copulas and their applications in water resources engineering covers current trends in copula applications in hydrological sciences and water engineering. Many copula-based approaches have been developed in econometrics that can be extended to hydrology and water resources engineering.

The book is organized into two parts. Part I introduces theoretical aspects of copulas, including copula properties and statistics, and different copula families. This part comprises nine chapters. Beginning with a short discussion of different methods of parameter estimation, Chapter 1 presents a short introduction to the history, development, and general applications of the copula theory. It also presents the theme of the entire book. Chapter 2 briefly discusses preliminaries for univariate and bivariate analyses. Chapter 3 deals with copulas and their properties. Starting with the definition of copula and its properties, it goes on to discussing bivariate copula, trivariate copula, methods of copula construction, copula families, dependence measures and properties, parameter estimation, copula simulation, goodness-of-fit tests, and return periods. Chapter 4 introduces the famous and well-accepted symmetric Archimedean copulas, including their properties and extension from two-dimensional to higher-dimensional analyses. Chapter 5 deals with asymmetric Archimedean copulas. Starting with nested Archimedean copula, this chapter discusses the

properties, parameter estimation, copula random variable simulation, and goodness-of-fit statistics for both nested Archimedean copula and vine copulas. The Plackett copula family is presented in Chapter 6. This chapter also discusses the disadvantage of extending the two-dimensional Plackett copula to higher-dimensional analysis. Chapter 7 presents meta-elliptic copulas. The meta-elliptic copula (especially the famous meta-Gaussian and metastudent t copulas) are easy to construct and well accepted in spatial analysis with high dimensions. Defining univariate constraints based on the Shannon entropy theory, Chapter 8 discusses the constraints necessary to construct the most-entropic copula and presents the uniqueness of the most-entropic canonical copula with examples. Chapter 9 presents the theoretical aspects of applying the copula theory to study multivariate and univariate time series.

Part II, comprising eight chapters, covers applications of copulas with case studies. Chapter 10 focuses on rainfall analysis. The case studies in this chapter include the depth-duration-frequency analysis from partial durations series and spatial rainfall depth analysis. Chapter 11 deals with flood analysis for both at-site and spatial flood frequency analyses. Chapter 12 focuses on the copula application to water quality analysis, including multivariate and univariate water quality time series. Chapter 13 presents the application of copulas to drought analysis using at-site drought characteristics. Risk and compound extreme (i.e., temperature and precipitation) are presented in Chapter 14. Using rain gauges from Louisiana, Chapter 15 discusses network design using the copula approach. Chapter 16 introduces the application of copulas to sediment yield analysis through the construction of sediment discharge rating curve and at-site trivariate suspended sediment yield analysis. The last chapter of the book presents the application of copulas to interbasin water transfer analysis.

This book covers important theoretical and practical aspects of the copula theory and its applications. It is hoped that the book will be useful to graduate students and faculty members who are interested in stochastic hydrology and environmental research and risk analyses. In the long term, copula-based methodologies may help improve engineering design and risk analysis practice.

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