

Econometric Analysis of Stochastic Dominance

This book offers an up-to-date, comprehensive coverage of stochastic dominance and its related concepts in a unified framework. A method for ordering probability distributions, stochastic dominance has grown in importance recently as a way to measure comparisons in welfare economics, inequality studies, health economics, insurance wages, and trade patterns. Whang pays particular attention to inferential methods and applications, citing and summarizing various empirical studies in order to relate the econometric methods with real applications and using computer codes to enable the practical implementation of these methods. Intuitive explanations throughout the book ensure that readers understand the basic technical tools of stochastic dominance.

Yoon-Jae Whang is Professor of Economics at Seoul National University. He is an elected fellow of the Econometric Society and the *Journal of Econometrics* and is Co-Director of the Center for Econometrics at Seoul National University.

Themes in Modern Econometrics

Series Editor

PETER C. B. PHILLIPS, *Sterling Professor of Economics, Yale University*

Themes in Modern Econometrics provides an organized sequence of advanced textbooks in econometrics aimed directly at the student population and is the first series in the discipline to have this as its express aim. Written at a level accessible to those who have completed an introductory course in econometrics, each book addresses topics and themes that students and researchers encounter daily. All areas of econometrics are covered within the series. Particular emphasis is given to theory fundamentals and practical implementation in breaking research areas that are relevant to empirical applications. Each book stands alone as an authoritative survey in its own right. The distinct emphasis throughout is on pedagogic excellence and accessibility.

Books in the Series

Structural Vector Autoregressive Analysis (2017) LUTZ KILIAN and
 HELMUT LÜTKEPOHL

Almost All About Unit Roots (2015) IN CHOI

Granularity Theory with Applications to Finance and Insurance (2014)
 PATRICK GAGLIARDINI and CHRISTIAN GOURIÉROUX,

Econometric Modeling with Time Series (2012) VANCE MARTIN, STAN
 HURN, and DAVID HARRIS

Economic Modeling and Inference (2007) JEAN-PIERRE FLORENSE,
 VELAYOUDOM MARIMOUTOU, and ANNE PEGUIN-FEISSOLLE

Translated by JOSEF PERKTOLD and MARINE CARRASCO

*Introduction to the Mathematical and Statistical Foundations of
 Econometrics* (2004) HERMAN J. BIERENS

Applied Time Series Econometrics (2004) HELMUT LÜTKEPOHL and
 MARKUS KRÄTZIG

Semiparametric Regression for the Applied Econometrician (2003)
 ADONIS YATCHEW

The Econometric Analysis of Seasonal Time Series (2001) ERIC GHYSELS
 and DENISE R. OSBORN

Econometrics of Qualitative Dependent Variables (2000) CHRISTIAN
 GOURIEROUX *Translated by* PAUL B. KLASSEN

Nonparametric Econometrics (1999) ADRIAN PAGAN and AMAN ULLAH

Generalized Method of Moments Estimation (1999) *Edited by* LÁSZLÓ
 MÁTYÁS

Unit Roots, Cointegration, and Structural Change (1999) G.S. MADDALA
 and IN-MOO KIM

Time Series and Dynamic Models (1997) CHRISTIAN GOURIEROUX and
 ALAIN MONFORT *Translated and edited by* GIAMPIERO GALLO

Statistics and Econometric Models: Volumes 1 and 2 (1995) CHRISTIAN
 GOURIEROUX and ALAIN MONFORT *Translated by* QUANG VUONG

ECONOMETRIC ANALYSIS OF STOCHASTIC DOMINANCE

Concepts, Methods, Tools, and Applications

YOON-JAE WHANG
Seoul National University



Cambridge University Press & Assessment
 978-1-108-47279-1 — Econometric Analysis of Stochastic Dominance
 Yoon-Jae Whang
 Frontmatter
[More Information](#)



Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
 One Liberty Plaza, 20th Floor, New York, NY 10006, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
 103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
 Information on this title: www.cambridge.org/9781108472791

DOI: 10.1017/9781108602204

Yoon-Jae Whang © 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press & Assessment.

First published 2019

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Names: Whang, Yoon-Jae, author.

Title: Econometric analysis of stochastic dominance: concepts, methods, tools, and applications / Yoon-Jae Whang, Seoul National University.

Description: New York : Cambridge University Press, [2018] |

Includes bibliographical references and index.

Identifiers: LCCN 2018012045 | ISBN 9781108472791 (alk. paper)

Subjects: LCSH: Economics, Mathematical. | Stochastic processes. | Mathematical statistics.

Classification: LCC HB135 .W464 2018 | DDC 330.01/51923—dc23

LC record available at <https://lcn.loc.gov/2018012045>

ISBN 978-1-108-47279-1 Hardback

Cambridge University Press & Assessment has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press & Assessment
978-1-108-47279-1 — Econometric Analysis of Stochastic Dominance
Yoon-Jae Whang
Frontmatter
[More Information](#)

*To my parents
and
Mi Kyung, Sun Young, and Soo Young*

Contents

<i>List of figures</i>	<i>page</i> xi
<i>List of tables</i>	xii
<i>Preface</i>	xiii
<i>Abbreviations and Notation</i>	xv
1 Introduction	1
1.1 Concepts of Stochastic Dominance	1
1.1.1 Definitions	1
1.1.2 Basic Properties of Stochastic Dominance	7
1.1.3 A Numerical Example	9
1.1.4 Extensions and Some Related Concepts	11
1.2 Applications of Stochastic Dominance	15
1.2.1 Welfare Analysis	15
1.2.2 Finance	17
1.2.3 Industrial Organization	18
1.2.4 Labor Economics	20
1.2.5 International Economics	20
1.2.6 Health Economics	21
1.2.7 Agricultural Economics	21
1.3 Outline of Subsequent Chapters	22
2 Tests of Stochastic Dominance: Basic Results	24
2.1 Introduction	24
2.2 Null of Dominance against Non-Dominance	25
2.2.1 Tests Based on Multiple Comparisons	26
2.2.2 Supremum-Type Tests	32
2.2.3 Integral-Type Tests	45
2.2.4 Quantile-Based Tests	48
2.2.5 Neyman’s Smooth Tests	53
2.3 Null of Non-Dominance against Dominance	58
	vii

viii	Contents	
	2.3.1 Infimum t-Test	59
	2.3.2 Empirical Likelihood Test	61
2.4	Null of Equality against Dominance	64
2.5	Empirical Examples	65
	2.5.1 Comparison of Income Distributions	65
	2.5.2 Testing for Monday Effects in Stock Markets	67
3	Tests of Stochastic Dominance: Further Results	73
3.1	Stochastic Dominance Tests with Improved Power	73
	3.1.1 The Contact Set Approach	73
	3.1.2 The Selective Recentering Approach	80
	3.1.3 Remarks	83
	3.1.4 A Numerical Example	85
3.2	Program Evaluation and Stochastic Dominance	87
	3.2.1 Distributional Treatment Effects	87
	3.2.2 Counterfactual Policy Analysis	98
3.3	Some Issues of Stochastic Dominance Tests	104
	3.3.1 Stochastic Dominance Tests with Unbounded Supports	104
	3.3.2 Classification of Stochastic Dominance Relations	106
	3.3.3 Large Deviation Approximation	108
3.4	Empirical Examples	109
	3.4.1 Distributional Treatment Effects of Veteran Status	109
	3.4.2 Returns to Schooling: Quantile Treatment Effects	110
4	Stochastic Dominance with Covariates	113
4.1	Introduction	113
4.2	Conditional Stochastic Dominance at Fixed Values of Covariates	114
	4.2.1 Quantile-Based Tests	114
	4.2.2 Cumulative Distribution Function-Based Tests	117
4.3	Conditional Stochastic Dominance at All Values of Covariates	119
	4.3.1 The Poissonization Approach	119
	4.3.2 The Least Concave Majorant Approach	122
	4.3.3 The Strong Approximation Approach	124
	4.3.4 The Unconditional Moment Representation Approach	127
4.4	Stochastic Monotonicity	130
4.5	Empirical Examples	134
	4.5.1 Testing for Conditional Treatment Effects	134
	4.5.2 Testing for a Strong Leverage Hypothesis	136
5	Extensions of Stochastic Dominance	141
5.1	Multivariate Stochastic Dominance	141
5.2	Analysis of Economic Inequality and Poverty	143
	5.2.1 Lorenz Dominance	143

Contents	ix
5.2.2 Poverty Dominance	146
5.2.3 Initial Dominance	147
5.3 Analysis of Portfolio Choice Problems	149
5.3.1 Marginal Conditional Stochastic Dominance	149
5.3.2 Stochastic Dominance Efficiency	151
5.3.3 Convex Stochastic Dominance and Stochastic Dominance Optimality	154
5.4 Weaker Notions of Stochastic Dominance	159
5.4.1 Almost Stochastic Dominance	159
5.4.2 Approximate Stochastic Dominance	162
5.4.3 Infinite-Order Stochastic Dominance	166
5.5 Related Concepts of Stochastic Dominance	169
5.5.1 Density Ratio Dominance	169
5.5.2 Uniform Stochastic Ordering	172
5.5.3 Positive Quadrant Dependence	176
5.5.4 Expectation Dependence Dominance	178
5.5.5 Central Dominance	180
5.5.6 Spatial Dominance	186
6 Some Further Topics	187
6.1 Distributional Overlap Measure	187
6.2 Generalized Functional Inequalities	189
6.3 Distributions with Measurement Errors	192
6.4 Stochastic Dominance Tests with Many Covariates	195
6.5 Robust Forecasting Comparisons	198
7 Conclusions	200
Appendix A Basic Technical Tools	202
A.1 A Probability Background	202
A.2 Empirical Processes	205
A.2.1 Independent Observations	205
A.2.2 Dependent Observations	209
A.3 Poissonization Methods	213
Appendix B Computer Code for Stochastic Dominance Tests	218
B.1 Introduction	218
B.2 Supremum-Type Tests	221
B.2.1 McFadden Test	221
B.2.2 Barrett–Donald Test	222
B.2.3 Linton–Maasoumi–Whang Test	225
B.2.4 Donald–Hsu Test	227
B.3 Integral-Type Test	230
B.3.1 Hall–Yatchew Test	230

x	Contents	
	B.3.2 Bennett Test	232
	B.3.3 Linton–Song–Whang Test	235
	B.4 Stochastic Dominance with Covariates: Lee–Whang Test	237
	<i>Bibliography</i>	242
	<i>Index</i>	259

Figures

1.1	X_1 first-order stochastically dominates X_2	<i>page 2</i>
1.2	X_1 does not first-order stochastically dominate X_2 , but X_1 second-order stochastically dominates X_2	4
1.3	PDFs and CDFs for the simulation design	10
3.1	CDFs of the first and second designs	86
3.2	Empirical CDFs of earnings of veterans and nonveterans	110
3.3	Estimated distributions of potential earnings for compliers	111
3.4	Estimates of schooling coefficients	112
4.1	Conditional mean of CSAT scores (control: Income)	136
4.2	Plot of $\hat{m}_T(y, g, \hat{\pi}^+)$ for MSFT	140
B.1	CDFs of Designs 1, 2, 3, and 4	219
B.2	CDFs of Designs 7, 8, 9, and 10	221

Tables

1.1	Mean-variance and SSD criteria	<i>page</i> 11
2.1	Canadian family income distributions: descriptive statistics	66
2.2	Stochastic dominance in Canadian before tax family income	66
2.3	Monday effects: OLS estimates (EWX)	71
2.4	Monday effects: quantile regression estimates (EWX)	71
2.5	Monday effects: median of p-values of SD tests	72
3.1	SD tests with power improvements: Rejection probabilities (Size)	86
3.2	SD tests with power improvements: Rejection probabilities (Power)	86
6.1	Rejection probabilities of the BD test under measurement errors	193
B.1	Rejection probabilities of the McFadden test	222
B.2	Rejection probabilities of the Barrett–Donald test	224
B.3	Rejection probabilities of the Linton–Maasoumi–Whang test	227
B.4	Rejection probabilities of the Donald–Hsu test	230
B.5	Rejection probabilities of the Hall–Yatchew test	232
B.6	Rejection probabilities of the Bennett test	235
B.7	Rejection probabilities of the Linton–Song–Whang test	237
B.8	Rejection probabilities of the Lee–Whang test	240

Preface

Stochastic dominance (SD) is an ordering rule of distribution functions. Since the work of Hadar and Russell (1969), Hanoch and Levy (1969), and Rothschild and Stiglitz (1970), the concept has been theoretically explored and empirically implemented in various areas including economics, finance, insurance, medicine, and statistics. The stochastic dominance rule is based on the expected utility paradigm and gives a uniform ordering of prospects that does not depend on specific utility (or social welfare) functions. For example, it can produce a majority assessment (valid over a large class of utility functions) on investment strategies, welfare outcomes (income distributions or poverty levels), and program evaluation exercises. In contrast, the traditional strong orders, based on specific indices of inequality or poverty in welfare, mean-variance analysis in finance, or performance indices in program evaluation, do not achieve consensus. In addition, the stochastic dominance rule does not require restrictive parametric assumptions on the distributions of the prospects.

Since the early work of McFadden (1989), a substantial body of literature has been developed on nonparametric inference on stochastic dominance and its related concepts. The stochastic dominance relation corresponds to an inequality restriction between (functionals of) nonparametric distribution functions. Not only is its statistical inference complicated, but it is also multifarious; since the concept of stochastic dominance itself has many variants in different contexts, it calls for separate inference procedures.

The purpose of this book is to provide an overview of the literature in a unified framework, with a focus on recent developments. There are excellent books on stochastic dominance, including Levy (2016) and Sriboonchita, Wong, Dhompongsa, and Nguyen (2010). There are also several surveys: Levy (1992), Maasoumi (2001), Davidson (2010), and Guo (2013). However, the main focus of the two books mentioned is on the theoretical aspects of stochastic dominance rules in finance, rather than on statistical inference for the rules. Also, the coverage of the surveys is somewhat limited. In contrast, this book

provides up-to-date and comprehensive coverage of the inference methods proposed in the literature. This book also cites and summarizes several empirical studies that employ the inference methods, so that the reader can relate the econometric methods to real applications.

This book is intended for graduate students and researchers in economics and other sciences who are interested in applications of the stochastic dominance rules. It is also useful to theoretical econometricians as a reference, because it shows several examples of using modern econometric tools to compare functional variables and suggests some open questions. It is expected that the reader has a good background in econometrics at the level of, for example, Greene (2012) or Hayashi (2000). Some knowledge of nonparametric inference methods and probability theory would be desirable but is not essential.

I am grateful to many people who helped me in the writing of this book in one way or another. First of all, I would like to thank my advisors, Professors Don Andrews and Peter Phillips at Yale, for their encouragement and support in the early 1990s. My special thanks also go to Oliver Linton, who introduced me to the fascinating topic of stochastic dominance, and who has been working together with me on various subjects over the past decades. I am also grateful to Gordon Anderson and Kyungchul Song, who read an earlier version of the entire manuscript and gave me very useful comments. I also owe a great intellectual debt to Xiaohong Chen, Doo-Bong Han, Yuichi Kitamura, Sokbae Lee, Haim Levy, Essie Maasoumi, Taisuke Otsu, Joon Park, Thierry Post, and Myung Hwan Seo. Parts of this book were written while I was visiting the Cowles Foundation at Yale, whose hospitality is gratefully acknowledged. Deborah Kim (now a graduate student at Northwestern) wrote the MATLAB code in Appendix B, which I deeply appreciate. I also thank Yeongmi Jeong, MinKyung Kim, Sue-Youl Kim, and Jaewon Lee (now a graduate student at Yale) for their excellent research assistance. Thanks also go to Jooyoung Cha, Danbi Chung, Jae Yu Jung, Eunsun Kim, Hyunkyeong Lim, and Siwon Ryu for their careful proofreading. This book was financially supported by the Korea Research Foundation Grant funded by the Korean Government (NRF-2011-342-B00004).

Last, but not least, my warmest thanks go to my parents, my wife Mi Kyung, and my daughters Sun Young and Soo Young, for their love and support. They have been the most important source of my personal strength throughout my academic life.

Abbreviations and Notation

CDF	cumulative distribution function
CDTE	conditional distributional treatment effect
CLT	central limit theorem
EL	empirical likelihood
FSD	first-order stochastic dominance
i.i.d.	independent and identically distributed
LCM	least concave majorant
LFC	least favorable case
PDF	probability density function
QTE	quantile treatment effect
SD	stochastic dominance
SDE	stochastic dominance efficiency
SM	stochastic monotonicity
SMM	studentized maximum modulus
SSD	second-order stochastic dominance
TSD	third-order stochastic dominance
WLLN	weak law of large numbers
\succeq_s	stochastic dominance at order s
$F^{(s)}(x)$	integrated CDF at order s
$\bar{F}^{(s)}(x)$	integrated empirical CDF at order s
$D_{1,2}^{(s)}(x)$	difference $F_1^{(s)}(x) - F_2^{(s)}(x)$ between integrated CDFs
$\bar{D}_{1,2}^{(s)}(x)$	difference $\bar{F}_1^{(s)}(x) - \bar{F}_2^{(s)}(x)$ between integrated empirical CDFs
$Q^{(s)}(x)$	integrated quantile function at order s
$[x]_+$	$\max\{x, 0\}$ (i.e., maximum of x and 0)
$[x]_-$	$\min\{x, 0\}$ (i.e., minimum of x and 0)
$a \vee b$	$\max\{a, b\}$ (i.e., maximum of a and b)
$a \wedge b$	$\min\{a, b\}$ (i.e., minimum of a and b)
$sgn(x)$	sign of x
\rightarrow_p	convergence in probability

xvi **Abbreviations and Notation**

\Rightarrow	convergence in distribution or weak convergence
$x := y$	equality by definition (i.e., x and y are equal by definition)
$X \stackrel{d}{=} Y$	equality in distributions (i.e., X and Y have the same distribution)
A^\top	transpose of matrix A
$f'(x)$	first-derivative of f with respect to x
$f''(x)$	second-derivative of f with respect to x
\mathbb{Z}	standard normal random variable, $N(0, 1)$
$U(a, b)$	uniform distribution on (a, b)