Uncovering Quantum Field Theory and the Standard Model

This textbook provides an accessible introduction to quantum field theory and the Standard Model of particle physics. It adopts a distinctive pedagogical approach with clear, intuitive explanations to complement the mathematical exposition.

The book begins with basic principles of quantum field theory, relating them to quantum mechanics, classical field theory, and statistical mechanics, before building toward a detailed description of the Standard Model. Its concepts and components are introduced step by step, and their dynamical roles and interactions are gradually established. Advanced topics of current research are woven into the discussion, and key chapters address physics beyond the Standard Model, covering subjects such as axions, technicolor, and Grand Unified Theories.

This book is ideal for graduate courses and as a reference and inspiration for experienced researchers. Additional material is provided in appendices, while numerous end-of-chapter problems and quick questions reinforce the understanding and prepare students for their own research.

Wolfgang Bietenholz is Professor of Physics at the National Autonomous University of Mexico (UNAM). His research focuses on lattice field theory with applications to elementary particles and condensed matter. He also writes articles on physics, mathematics, and the history of science for a broader audience. He studied at ETH Zürich and the University of Bern. He previously held positions at Centro Brasileiro de Pesquisas Físicas in Rio de Janeiro, the Massachusetts Institute of Technology (MIT), Deutsches Elektronen-Synchrotron (DESY), Nordic Institute for Theoretical Physics, and the Humboldt University in Berlin.

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"Wolfgang Bietenholz and Uwe-Jens Wiese have done an impressive job of writing an introduction to quantum field theory and the standard model of particle physics that incorporates modern viewpoints throughout. They manage to cover a great deal of material in a manageable space. Students will come away with an excellent grounding in a difficult and multifaceted subject, and the book will also be a valuable resource for experienced researchers."

Prof. Edward Witten, Institute for Advanced Studies, Princeton

"Bietenholz and Wiese have succeeded in writing a quantum field theory textbook that takes a radically different path from those before it whilst still covering the field in a manner appropriate for a graduate course. The presentation is clear and is interestingly modulated by the authors' backgrounds in lattice approaches to field theory, explaining difficult concepts in concise and interesting ways. Throughout the book are scattered gems of topics that you will not find in existing mainstream texts, where the authors eloquently delve into these selected topics in deeper detail."

Prof. William Detmold, Massachusetts Institute of Technology

"This is a remarkable book! As an introduction to quantum field theory it impresses with its clarity and depth. It also provides great insight into the standard model, in particular its nonperturbative features and connections to condensed matter physics. This book will be an invaluable source of information for students and researchers alike."

Prof. Wilfried Buchmüller, DESY and University of Hamburg

"This is a marvelous book that manages to synthesize the developments over more than fifty years in quantum field theory, statistical mechanics, the Standard Model of particle physics, critical phenomena, and the renormalization group. The authors have a deep understanding of the subject and manage to present the material with explanations and insight rather than heavy use of equations. It serves as an excellent complete textbook for graduate courses but also as a reference book for researchers in the field."

Prof. Poul H. Damgaard, Niels Bohr Institute

"A modern and empowering text on quantum field theory. The book is very successful in engaging its readers right from the start, with the main relevant topics covered in a solid and motivating way, highlighting open questions, and providing interesting historical remarks and exercises. The authors' approach is novel and stimulating and the book will be very valuable for students."

Prof. Tereza Mendes, University of São Paulo

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From Fundamental Concepts to Dynamical Mechanisms

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Preface

Intention of this Book

This book is an introduction to *quantum field theory* and the *Standard Model* of particle physics, which is a relativistic quantum field theory that incorporates the basic principles of quantum physics and special relativity. Quantum field theory provides a systematic, universal framework that allows us to understand the local interactions of field degrees of freedom attached to the different points of space at all times. Quantum field theories are of central importance in many areas of physics, ranging from condensed matter physics, atomic, molecular, and optical physics, to nuclear and particle physics. At the most fundamental level, quantum field theory describes the interactions between elementary particles, which are nothing but quantized field excitations, in the framework of the Standard Model.

The Standard Model of particle physics is one of the greatest achievements of science in the second half of the twentieth century, and – in light of its high-precision predictions – of all history. It summarizes all we know today about the fundamental structure of matter, forces, and symmetries, by describing the *electromagnetic, weak*, and *strong* interactions of *Higgs particles, leptons*, and *quarks,* mediated by *photons, W- and Z-bosons*, and *gluons.* It describes further interactions by Yukawa couplings between the Higgs field and the fermion fields, and by the self-interaction of the Higgs field. It is our goal to explain these topics sufficiently well, such that a deep understanding becomes possible. Achieving profound insight into a complex subject such as quantum field theory takes time, but leaves us with a sense of empowerment and an urge to progress to more advanced topics. Empowering the curious reader and encouraging him or her to think about Nature's biggest puzzles at a deep level is a major goal of this book.

This book provides a detailed description of those features of the Standard Model and some subjects beyond it that should be of general interest to any physicist of the twenty-first century, irrespective of his or her specialization. It concentrates on the model's symmetries, on its hierarchies of energy scales and the related puzzles, on its predictive power and its limitations, as well as on its possible extensions to even higher energies. Our goal is to expose the structure of the Standard Model in a language that is accessible to physicists with just a basic background in special relativity, quantum mechanics, statistical mechanics, and classical – but not necessarily quantum – field theory. This book introduces quantum field theory at a level that is of interest for applications not only in particle but also in nuclear and condensed matter physics.

The enormous importance and robustness of the Standard Model, which has maintained its validity in the era of the Large Hadron Collider (LHC) at CERN (near Geneva, Switzerland), has intrigued many physicists. Irrespective of whether or not an extended Standard

Model (perhaps with additional right-handed neutrino fields) may be valid all the way up to the Planck scale, the Standard Model will stay with us as the most fundamental description of Nature up to the TeV energy scale. Familiarizing the interested reader, either with a particle physics or with a more general physics orientation, with its fascinating dynamical structures is a main intention of this book. The broad relevance of the Standard Model is also reflected in the structure of M.Sc. and Ph.D. programs. While all physicists should become familiar with the basic features of the Standard Model, many physicists will want to gain a deeper understanding of its fundamental structures and the underlying dynamical mechanisms. The present book fulfills this purpose.

Victor Weisskopf's teaching at MIT was characterized by his motto:

> It is better to uncover a little than to cover a lot.

As the title of this book suggests, it is our intention to uncover some fundamental structures of quantum field theory and the Standard Model, which are usually not strongly emphasized in most of the textbook literature. Exposing some deeper layers of the subject, whose understanding is sometimes obstructed by the involved mathematical structure of quantum field theory or by the rich particle phenomenology of the Standard Model, is a central intention of the book. Lectures based on this book should be able to follow Weisskopf's motto. In particular, we aim at facilitating a deep understanding of a subject by going through each individual step of an extensive explanation. While the variety of subjects discussed in the book is not small, each one is presented in sufficient detail, in order to clarify it at a deep level. In this way, somewhat like in archeology, we aim at uncovering some hidden layers, which are obstructed from the more common perspective of large parts of the textbook literature.

Subjects of this Book

The book begins with an overview of the fundamental concepts underlying quantum field theory. A basic understanding of quantum field theory is facilitated by relating it to quantum mechanics and to classical field theory, as well as (in its Euclidean functional integral formulation) to classical statistical mechanics. Canonical quantization as well as the perturbative quantization applying *dimensional regularization*, and the non-perturbative *lattice regularization* are then introduced, using scalar field theory as a simple framework. In this context, we address the very nature of *"particles"* in quantum field theory, which arise as quantized wave excitations of the corresponding quantum fields.

Other fields are introduced one by one, starting with *Abelian gauge fields* and then moving on to fermion fields. We put an emphasis on *Weyl fermions*, which are basic building blocks of the Standard Model, and then relate them to *Dirac and Majorana fermions.* We discuss the chiral symmetries of fermions, both in continuum and in lattice quantum field theory. We then move on to *non-Abelian gauge fields.*

In order to elucidate the interplay of its various dynamical ingredients, the Standard Model is then constructed step by step, starting from the Higgs sector, proceeding further to the electroweak and strong gauge fields, and finally adding fermionic lepton and quark matter fields. Important dynamical mechanisms, such as the spontaneous breakdown of a global symmetry, the *Higgs mechanism* describing the "spontaneous breaking" of a gauge symmetry, as well as the quantum-induced anomalous breakdown of symmetries, and the requirement of *gauge anomaly cancellation* are each addressed at the appropriate stage of the construction. For example, the consistency of the Standard Model as a quantum field theory, which is tied to the cancellation of perturbative and non-perturbative anomalies, is discussed in detail and is related to the issue of charge quantization.

The book concentrates on the fundamental structures of the Standard Model, its symmetries and their various realizations, its basic dynamical mechanisms, and some of its less obvious beautiful structures, more than on particle phenomenology or advanced perturbative techniques. The book also emphasizes non-perturbative physics and uses the lattice regularization when appropriate, but it is by no means focused on lattice field theory (when it is possible, we use continuum notation).

The lattice regularization is more physical than dimensional regularization (by analytic continuation of the space–time dimension). In particular, it also arises naturally in condensed matter physics (in the form of spatial crystal lattices); thus it provides a bridge between the different disciplines. Most important, the lattice regularization leads to *non-perturbative* insights into essential dynamical mechanisms including the *confinement of quarks and gluons* and the spontaneous breakdown of the quarks' *chiral symmetry*.

Along with its impressive success, the Standard Model also gives rise to puzzles and open questions, which may hint to new physics. The discussion of theories that go beyond the Standard Model, such as *technicolor*, *axion models*, and *Grand Unified Theories*, is embedded in the construction of the Standard Model itself. Such ideas are addressed as soon as it is suitable; certain aspects of these theories are discussed even before addressing the theory of the strong interaction – Quantum Chromodynamics (QCD).

Special attention is given to the *number of quark colors* N_c (which is 3 in the real world) as a discrete parameter of the Standard Model. Via anomaly cancellation, it affects the electric quark charges, with consequences, *e.g.*, for the electromagnetic decay of the neutral pion, which is not addressed correctly in large parts of the textbook literature.

Again in contrast to most standard textbooks, this book puts an emphasis on *topological aspects* of the Standard Model and its low-energy effective theories. Topological effects are related to anomalies and play an important role in the dynamics of pions and the other Nambu–Goldstone bosons of the strong interaction, as well as for the *$U(1)_A$-problem*, the *strong CP-problem,* and for the generation of the *baryon asymmetry.* The topology of quantum fields is again of general interest, with numerous links to condensed matter physics.

Finally, the book supplements nine appendices, which address, for instance, the historical development of experimental and theoretical high-energy physics, as well as units, energy scales, and fundamental parameters in particle physics. Other appendices describe Minkowski space–time, the Lorentz-covariant formulation of classical electrodynamics, as well as the Monte Carlo method and second-order phase transitions, in order to facilitate a smooth transition beyond B.Sc. knowledge. More advanced mathematical topics including group theory and homotopy theory, which are again of general interest beyond particle physics, are addressed in two more appendices.

Structure of this Book

This book has emerged from numerous lectures at the Master and Ph.D. student level, held over three decades at the Rheinisch-Westfälische Technische Hochschule (RWTH) Aachen, the Humboldt University in Berlin, the University of Bern, the Massachusetts Institute of Technology (MIT), Potsdam University, the Universidad Nacional Autónoma de México (UNAM), and at Wuppertal University.

Part I of this book can be covered in a 1-semester introduction to quantum field theory. It begins with an Ouverture that provides an overview of the fundamental concepts underlying quantum field theory. It continues with a concise presentation of the theoretical framework of quantum field theory in the functional integral approach, provided in Chapter 1. Chapters 2–4 discuss scalar field theory, starting at the classical level, and progressing from canonical quantization to the perturbative quantization using the Euclidean functional integral in dimensional regularization. In this context, the nature of "particles" in a quantum field theory is clarified: they are quantized waves, which we temporarily address as "wavicles". Chapter 5 provides an introduction to the renormalization group from a Wilsonian perspective. The quantization of the electromagnetic field, and the characterization of electrically charged particles as "infraparticles" are the subjects of Chapters 6 and 7. Chapters 8–10 discuss Weyl, Dirac, and Majorana fermions, as well as their various symmetries, both in canonical quantization and using a functional integral approach. Part I ends with Chapter 11, in which non-Abelian gauge fields and their quantization are introduced. Part I serves as a prerequisite for studying the other parts of the book.

Part II can be the subject of a 1-semester introduction to the Standard Model. It begins with an Intermezzo that summarizes the fundamental concepts and dynamical mechanisms that underlie the Standard Model. Chapters 12–14 introduce the bosonic Higgs and gauge fields with their dynamics, including the Higgs mechanism, as well as confinement and Coulomb phases. The gauge interactions of lepton and quark fields of the first fermion generation are introduced in Chapter 15. The fermions are endowed with mass by coupling them to the Higgs field in Chapter 16, while Chapter 17 discusses the physics related to the other fermion generations.

Part III corresponds to a course on the strong interaction, from a fundamental Standard Model point of view (Chapters 18–20), from the perspective of effective models (Chapters 21 and 22), and in the framework of systematic low-energy effective theories (Chapters 23 and 24). Part IV, which includes Chapter 25 on the strong CP-problem and Chapter 26 on Grand Unified Theories, addresses selected topics beyond the Standard Model, and can be taught as part of a special topics course. Both Part III and Part IV can be presented in another 1-semester course, relying on the material of Part I and II as a prerequisite. The appendices provide some background for the various chapters of the main text.

All chapters end with a set of exercises. In addition, "Quick Questions", which allow to test the reader's immediate understanding, are embedded in the text. It cannot be stressed enough that theoretical physics can be learned in depth only by working through a large number of problems. This means a lot of work for the student which, however, is well worth investing. The skills that one learns by solving the exercises are very useful when working on actual research problems.

As an additional motivation for the curious reader, the book connects the Standard Model with some deep fundamental questions (inserted in boxes) that we hope to be answered in the course of the twenty-first century. For example, one such question is known as the *hierarchy problem*, *i.e.* the question: *Why is the electroweak scale so much lower than the Planck scale?* The authors encourage the reader to face such fundamental questions and think about them at a deep level. This should serve as a strong motivation to penetrate the subject of the Standard Model in a profound manner. Although quantum field theory is almost a century old, it remains one of the most promising tools that will allow us to push the boundaries of current knowledge further into the unknown.

Readers who find mistakes of any kind are encouraged to kindly report them to us. Outstanding lists of mistakes will be awarded. Relevant corrections will be listed – if necessary – on the book website.

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Notations and Conventions

In the following, we list various notations and conventions to be used throughout this book.

- Minkowski and Euclidean space–time:

  In Minkowski space–time, we use the metric

$$g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1).$$

Co- and contra-variant vectors describing a space–time point are given by

$$x^\mu = \left(x^0, \vec{x}\right), \quad x_\mu = g_{\mu\nu} x^\nu = (x_0, -\vec{x}), \quad x^0 = x_0 = ct,$$

and the corresponding space–time derivatives take the form

$$\partial_\mu = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right) = \left(\frac{1}{c}\partial_t, \vec{\nabla}\right), \quad \partial^\mu = \left(\frac{1}{c}\partial_t, -\vec{\nabla}\right).$$

Minkowski and Euclidean space–time are related by the Wick rotation $x_4 = \mathrm{i} x_0$. We distinguish the imaginary unit i from a generic index i. In Euclidean space–time, we exclusively use lower indices, with $x_\mu = (x_1, x_2, x_3, x_4)$ and the standard Euclidean metric $g_{\mu\nu} = \delta_{\mu\nu}$. The totally anti-symmetric Levi-Civita symbol $\epsilon_{\mu\nu\rho\sigma}$ obeys $\epsilon_{1234} = 1$.

Both in Minkowski and in Euclidean space–time, we denote a Lagrange density or Lagrangian by $\mathcal{L}$, while a Lagrange function is denoted by L. A Hamiltonian density is denoted by $\mathcal{H}$, and a Hamilton operator by $\hat{H}$. We dress the operators that act in a Hilbert or Fock space with a hat.

- Internal symmetries:

  The generators T^a of a Lie algebra obey

$$[T^a, T^b] = \mathrm{i} f_{abc} T^c, \quad \mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta_{ab}.$$

Examples of internal symmetries are $\mathrm{SU}(2)_I$ isospin, which is generated by $T^a = \tau^a/2$ ($a \in \{1, 2, 3\}$), with the Pauli matrices

$$\vec{\tau} = (\tau^1, \tau^2, \tau^3) = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right),$$

$\mathrm{SU}(3)_c$ color, which is generated by $T^a = \lambda^a/2$, with the Gell-Mann matrices λ^a ($a \in \{1, 2, \dots, 8\}$), which are written down in Appendix F.7, or a general $\mathrm{SU}(N)$ symmetry, which is generated by $T^a = \eta^a/2$ ($a \in \{1, 2, \dots, N^2 - 1\}$).

- Gauge fields:

  Electrodynamics is formulated with the 4-vector potential $A^\mu(x) = \left(\phi(\vec{x}, t), \vec{A}(\vec{x}, t)\right)$. The electromagnetic interaction is governed by the Abelian gauge group $\mathrm{U}(1)_{\mathrm{em}}$. We use Lorentz–Heaviside units in which the potential of a static point charge carrying the elementary electric charge e is given by

$$\phi(\vec{x}) = \frac{e}{4\pi |\vec{x}|}.$$

The 4-vector potential as well as a complex field $\Phi(x) \in \mathbb{C}$, which carries $Q \in \mathbb{Z}$ units of the elementary charge e, transform under gauge transformations as

$$A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x), \quad \Phi'(x) = \exp(\mathrm{i} Q e \alpha(x))\, \Phi(x).$$

The gauge fields of the Standard Model transform under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The Abelian hypercharge gauge field $B_\mu(x) \in \mathbb{R}$ is associated with the symmetry $U(1)_Y$. It couples with the strength g' to a scalar field that carries the weak hypercharge Y, such that

$$B'_\mu(x) = B_\mu(x) - \partial_\mu \varphi(x), \quad \Phi'(x) = \exp(\mathrm{i} Y g' \varphi(x))\, \Phi(x).$$

Non-Abelian vector potentials mediate the weak and the strong gauge interaction with the fundamental coupling constants g and g_s, and with the matrix-valued gauge transformations $L(x) \in SU(2)_L$ and $\Omega(x) \in SU(3)_c$. They are defined by the anti-Hermitian gauge fields W_μ and G_μ,

$$W_\mu(x) = \mathrm{i} g W^a_\mu(x) \frac{\tau^a}{2}, \quad W^a_\mu(x) \in \mathbb{R}, \quad W'_\mu(x) = L(x) \left(W_\mu(x) + \partial_\mu \right) L(x)^\dagger,$$
$$G_\mu(x) = \mathrm{i} g_s G^a_\mu(x) \frac{\lambda^a}{2}, \quad G^a_\mu(x) \in \mathbb{R}, \quad G'_\mu(x) = \Omega(x) \left(G_\mu(x) + \partial_\mu \right) \Omega(x)^\dagger.$$

Occasionally, we will extend the Abelian gauge symmetry $U(1)_Y$ to a non-Abelian symmetry with $R(x) \in SU(2)_R$, or the $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry to a grand unified symmetry with $\Upsilon(x) \in SU(5)$, such that

$$X_\mu(x) = \mathrm{i} g' X^a_\mu(x) \frac{\tau^a}{2}, \quad X^3_\mu(x) = B_\mu(x), \quad X'_\mu(x) = R(x) \left(X_\mu(x) + \partial_\mu \right) R(x)^\dagger,$$
$$V_\mu(x) = \mathrm{i} g_5 V^a_\mu(x) \frac{\eta^a}{2}, \quad V^a_\mu(x) \in \mathbb{R}, \quad V'_\mu(x) = \Upsilon(x) \left(V_\mu(x) + \partial_\mu \right) \Upsilon(x)^\dagger.$$

In the context of the Higgs mechanism, we always write "spontaneous symmetry breaking" in inverted commas, because a gauge symmetry (which reflects a redundancy) can, in fact, not break.

- Fermion fields:

  The spin $\vec{S} = \vec{\sigma}/2$ of elementary fermions is described by the same set of Pauli matrices

$$\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3) = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right).$$

In the context of canonical quantization, fermion fields are described by field operators $\hat{\psi}(\vec{x})$ and $\hat{\psi}(\vec{x})^\dagger$. In the framework of the functional integral, fermions are described by independent, anti-commuting Grassmann fields $\psi(x)$ and $\bar{\psi}(x)$.

In Minkowski space–time, we follow the γ-matrix conventions of the books of Peskin and Schroeder (1997), Srednicki (2007), Zee (2010), Schwartz (2014), and others,

$$\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}, \quad \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3, \quad \{\gamma^\mu, \gamma^5\} = 0,$$

which in the chiral basis amount to

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}.$$

In order to distinguish them from γ-matrices in Euclidean space–time, we arrange things such that γ-matrices in Minkowski space–time occur exclusively with upper indices.

A 4-component Dirac spinor $\psi(x)$ is built from a left- and a right-handed 2-component Weyl spinor, $\psi_L(x)$ and $\psi_R(x)$,

$$\psi(x) = \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix}, \quad \bar{\psi}(x) = \left(\bar{\psi}_R(x), \bar{\psi}_L(x)\right), \quad P_L = \frac{1}{2}\left(1 - \gamma^5\right), \quad P_R = \frac{1}{2}\left(1 + \gamma^5\right),$$
$$\begin{pmatrix} \psi_L(x) \\ 0 \end{pmatrix} = P_L \psi(x), \quad \begin{pmatrix} 0 \\ \psi_R(x) \end{pmatrix} = P_R \psi(x).$$

Left- and right-handed Weyl spinors are associated with the matrices $\bar{\sigma}^\mu$ and σ^μ, respectively,

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^\mu = (\mathbb{1}, \vec{\sigma}), \quad \bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma}).$$

A Majorana spinor results from a Dirac spinor by imposing the constraints

$$\psi_L(x) = -\mathrm{i}\sigma^2 \bar{\psi}_R(x)^\mathsf{T}, \quad \bar{\psi}_L(x) = \psi_R(x)^\mathsf{T} \mathrm{i}\sigma^2.$$

Here, as well as in other places in the book, T denotes transpose.

The γ-matrices in Euclidean space–time result from a Wick rotation. They are Hermitian, $\gamma_\mu^\dagger = \gamma_\mu$, and obey the relations

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}, \quad \gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4, \quad \{\gamma_\mu, \gamma_5\} = 0.$$

In the chiral basis, they take the form

$$\gamma_i = \begin{pmatrix} 0 & -\mathrm{i}\sigma^i \\ \mathrm{i}\sigma^i & 0 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}.$$

Euclidean γ-matrices will be used with lower indices only.

The Euclidean variants of the matrices σ^μ and $\bar{\sigma}^\mu$ again carry lower Lorentz indices only,

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad \sigma_\mu = (-\mathrm{i}\vec{\sigma}, \mathbb{1}), \quad \bar{\sigma}_\mu = (\mathrm{i}\vec{\sigma}, \mathbb{1}).$$

- Discrete symmetries C, P, and T:

  In Euclidean space–time, charge conjugation C, parity P, and time reversal T act on left- and right-handed Weyl fermion fields as

$$\begin{aligned}
&{}^C\psi_R(x) = \mathrm{i}\sigma^2 \bar{\psi}_L(x)^\mathsf{T} = {}^c\psi_L(x), \quad {}^C\bar{\psi}_R(x) = -\psi_L(x)^\mathsf{T}\mathrm{i}\sigma^2 = {}^c\bar{\psi}_L(x), \\
&{}^C\psi_L(x) = -\mathrm{i}\sigma^2 \bar{\psi}_R(x)^\mathsf{T} = {}^c\psi_R(x), \quad {}^C\bar{\psi}_L(x) = \psi_R(x)^\mathsf{T}\mathrm{i}\sigma^2 = {}^c\bar{\psi}_R(x), \\
&{}^P\psi_R(x) = \psi_L(-\vec{x}, x_4), \quad {}^P\bar{\psi}_R(x) = \bar{\psi}_L(-\vec{x}, x_4), \\
&{}^P\psi_L(x) = \psi_R(-\vec{x}, x_4), \quad {}^P\bar{\psi}_L(x) = \bar{\psi}_R(-\vec{x}, x_4), \\
&{}^T\psi_R(x) = \mathrm{i}\sigma^2 \bar{\psi}_R(\vec{x}, -x_4)^\mathsf{T}, \quad {}^T\bar{\psi}_R(x) = \psi_R(\vec{x}, -x_4)^\mathsf{T}\mathrm{i}\sigma^2, \\
&{}^T\psi_L(x) = \mathrm{i}\sigma^2 \bar{\psi}_L(\vec{x}, -x_4)^\mathsf{T}, \quad {}^T\bar{\psi}_L(x) = \psi_L(\vec{x}, -x_4)^\mathsf{T}\mathrm{i}\sigma^2.
\end{aligned}$$

The resulting, discrete transformations for Euclidean Dirac spinor fields are

$$
\begin{aligned}
&{}^{\mathrm{C}}\psi(x) = C\bar{\psi}(x)^{\mathsf{T}}, \quad {}^{\mathrm{C}}\bar{\psi}(x) = -\psi(x)^{\mathsf{T}}C^{-1}, \quad C = \begin{pmatrix} -\mathrm{i}\sigma^2 & 0 \\ 0 & \mathrm{i}\sigma^2 \end{pmatrix}, \\
&{}^{\mathrm{P}}\psi(x) = P\psi(-\vec{x}, x_4), \quad {}^{\mathrm{P}}\bar{\psi}(x) = \bar{\psi}(-\vec{x}, x_4)P^{-1}, \quad P = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \\
&{}^{\mathrm{T}}\psi(x) = T\bar{\psi}(\vec{x}, -x_4)^{\mathsf{T}}, \quad {}^{\mathrm{T}}\bar{\psi}(x) = -\psi(\vec{x}, -x_4)^{\mathsf{T}}T^{-1}, \quad T = \begin{pmatrix} 0 & \mathrm{i}\sigma^2 \\ \mathrm{i}\sigma^2 & 0 \end{pmatrix}.
\end{aligned}
$$

- Higgs field:

  We use three equivalent parametrizations of the Standard Model Higgs field, as a complex doublet,

$$
\Phi(x) = \begin{pmatrix} \Phi^+(x) \\ \Phi^0(x) \end{pmatrix}, \quad \Phi^+(x),\ \Phi^0(x) \in \mathbb{C},
$$

  as a 4-component, real field

$$
\begin{aligned}
&\vec{\phi}(x) = (\phi_1(x), \phi_2(x), \phi_3(x), \phi_4(x)) \in \mathbb{R}^4, \\
&\Phi^+(x) = \phi_2(x) + \mathrm{i}\,\phi_1(x), \quad \Phi^0(x) = \phi_4(x) - \mathrm{i}\,\phi_3(x),
\end{aligned}
$$

  and as a 2×2 matrix, proportional to an SU(2) matrix,

$$
\begin{aligned}
\mathbf{\Phi}(x) &= \begin{pmatrix} \Phi^0(x)^* & \Phi^+(x) \\ -\Phi^+(x)^* & \Phi^0(x) \end{pmatrix} \\
&= \phi_4(x)\,\mathbb{1} + \mathrm{i}\left[\phi_1(x)\tau^1 + \phi_2(x)\tau^2 + \phi_3(x)\tau^3\right].
\end{aligned}
$$

Glossary

The following acronyms are frequently used throughout the book:

QED	Quantum Electrodynamics
QCD	Quantum Chromodynamics
IR	infrared
UV	ultraviolet

Some important physical scales are:

G	Newton's constant
M_{Planck}	Planck scale
Λ_{c}	cosmological constant
Λ_{QCD}	QCD scale
v	vacuum expectation value of the Higgs field

Relevant integer-valued parameters include:

N_c	number of quark colors
N_f	number of quark flavors
N_g	number of fermion generations

Some important physical parameters are:

λ	self-coupling of the Higgs field
g_s	strong $SU(3)_c$ color gauge coupling
g	weak $SU(2)_L$ gauge coupling
g'	$U(1)_Y$ hypercharge gauge coupling
e	unit of the electric charge
θ_W	Weinberg angle
θ	QCD vacuum-angle
θ_{QED}	QED vacuum-angle
θ_C	Cabibbo angle
$f_u, f_d, f_c, f_s, f_t, f_b$	quark Yukawa couplings
$m_u, m_d, m_c, m_s, m_t, m_b$	quark masses
f_e, f_μ, f_τ	charged lepton Yukawa couplings
m_e, m_μ, m_τ	charged lepton masses
F_π	pion decay constant

The fields of the Standard Model, including only the first fermion generation, are:

$\Phi(x), \mathbf{\Phi}(x), \vec{\phi}(x)$	three equivalent forms of the Higgs field
$G_\mu(x)$	$SU(3)_c$ gluon field
$W_\mu(x)$	$SU(2)_L$ electroweak gauge field
$B_\mu(x)$	$U(1)_Y$ weak hypercharge gauge field
$A_\mu(x)$	$U(1)_{em}$ photon field
$Z_\mu(x)$	neutral Z-boson field
$W_\mu^\pm(x)$	charged W-boson field
$e_R(x),\ \bar{e}_R(x)$	right-handed $SU(2)_L$-singlet electron fields
$l_L(x) = \begin{pmatrix} \nu_L(x) \\ e_L(x) \end{pmatrix}, \quad \bar{l}_L(x) = (\bar{\nu}_L(x), \bar{e}_L(x))$	left-handed $SU(2)_L$-doublet lepton fields
$u_R(x), \bar{u}_R(x), d_R(x), \bar{d}_R(x)$	right-handed $SU(2)_L$-singlet quark fields
$q_L(x) = \begin{pmatrix} u_L(x) \\ d_L(x) \end{pmatrix}, \quad \bar{q}_L(x) = (\bar{u}_L(x), \bar{d}_L(x))$	left-handed $SU(2)_L$-doublet quark fields