Introduction to the Finite Element Method and Implementation with MATLAB®

Connecting theory with numerical techniques using MATLAB®, this practical textbook equips students with the tools required to solve finite element problems. This hands-on guide covers a wide range of engineering problems through nine well-structured chapters including solid mechanics, heat transfer, and fluid dynamics; equilibrium, steady state and transient; and 1-D, 2-D, and 3-D problems. Engineering problems are discussed using case-study examples, which are solved using a systematic approach, both by examining the steps manually and by implementing a complete MATLAB® code. This topical coverage is supplemented by discourse on meshing, with a detailed explanation and implementation of 2-D meshing algorithms. Introducing theory and numerical techniques alongside comprehensive examples, this text increases engagement and provides students with the confidence needed to implement their own computer codes to solve given problems.

Dr. Gang Li is a professor and D. W. Reynolds Emerging Scholar of Mechanical Engineering at Clemson University. He was an awardee of the National Science Foundation Early Career Award. He is an associate editor of the *Journal of Computational Electronics* and serves on the ASME Committee on Computing in Applied Mechanics. Dr. Li's scholarly articles on computational mechanics and finite element method frequently appear in leading journals.

An enormously accessible, didactic, and comprehensive text treating key engineering topics and providing the reader with the necessary elements of linear algebra, numerical methods, and meshing techniques, as well as numerous programming examples using MATLAB®. Professor Li's book can be used by teachers in the classroom for final-year undergraduate and graduate students, and by anyone else interested in learning the theory and computational implementation of the finite element method.

Gabriel Potirniche, University of Idaho

From one of the experts in the field, this book on the finite element method is a comprehensive and thorough guide for graduate and senior undergraduate students. The book is engaging not just in content but also in delivery. Its focus on step-by-step explanation and implementation is particularly useful for helping students to connect the theory and practice. The reusable MATLAB® functions and programs that are integrated with the theoretical content reinforce the important components of FEA and provide a unique learning experience. Detailed description of numerical analysis and meshing techniques is also a major plus since these topics are barely covered in existing FEA textbooks. This introductory FEA book is suitable for students of all engineering majors.

Narayana Aluru, University of Illinois

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To my family

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Preface

With unprecedented computing power, data transfer speed, and data processing capacity, today's engineering is experiencing a revolution of the digital age. What we are witnessing today is the explosive trend of digitization and computation of everything: from atoms to the Milky Way, from 3-D printing to brain engineering, from artificial intelligence to virtual reality ... Among other key technologies, physics based computing lies at the core of this revolution, and the Finite Element Method (FEM) is one of the most powerful enabling tools to do that. This is precisely why Finite Element Analysis (FEA) is one of the core courses taught in many Engineering departments, at undergraduate and/or graduate level. While there are many good introductory textbooks to the Finite Element Method, the 12 years I have spent teaching this subject have shown me that the conventional introductory approach covers much of the theory, but doesn't allow for much practical application. At the end of the semester, students could rarely implement a complete computer code to solve a given engineering problem. In addition to this, I found that students were able to learn the subject most effectively when they were required to implement their own finite element codes to solve given engineering problems. This observation gave me the motivation to write this introductory level textbook which emphasizes the connection from the mathematical foundation of the method, to the procedure of the numerical analysis, and then further to the implementation of a computer code. For the past several years I have been implementing this idea in my own teaching, and writing my own notes for my classes. The student learning outcomes and feedback were positive and encouraging. Now that the content of my teaching notes has become sufficiently mature, I feel that I am ready to transform the notes into a textbook.

This book is designed for senior undergraduate and first-year graduate students. As prerequisites, the students are expected to have:

1 A basic knowledge of linear algebra, ordinary and partial differential equations, and vector analysis.
2 A basic knowledge of solid mechanics, heat transfer, and fluid dynamics.
3 Basic skills with a programming language (MATLAB®, C, C++, Java, Python, Fortran, etc.)

The topics of the textbook include: an introduction to the mathematical foundation, solution procedure, and numerical implementation of the finite element method; applications to heat transfer, fluid flow, and structural analysis of solids; introduction to transient and dynamic analysis; analysis strategies using finite elements; introduction to solid modeling, meshing, data structures, and computer algorithms for the development of finite element computer codes in MATLAB.

The textbook aims to enable students to:

1 Gain an in-depth understanding of the mathematical formulation and the numerical implementation of the finite element method for solving engineering problems.
2 Apply numerical algorithms and techniques that are necessary for the numerical implementation of FEM.
3 Develop their own computer codes and obtain Finite Element (FE) solutions for a variety of engineering problems, including static/transient heat transfer, structural, elasticity, and fluid dynamics problems.

Some of the unique features of the book include the following:

- Implementation oriented step-by-step demonstration of the finite element procedure for solving linear solid mechanics, heat transfer, and fluid flow problems.
- Introduction of practical modeling and the numerical techniques required to implement a complete finite element program using MATLAB: data structures and storage, solid modeling, meshing, data visualization, solution of linear system of equations, numerical integration and differentiation, interpolation and approximations.
- The book gives the big picture at the beginning and enables the students to quickly understand the framework of the method. The students will be able to implement the method and use their own codes to solve 1-D elasticity problems in just three weeks. After that, more advanced numerical techniques and more complicated 2-D and 3-D problems can be learned by building upon this established understanding and implemented program.
- MATLAB codes are provided for all numerical examples and exercise problems. All the MATLAB codes can be accessed and downloaded from the book's online resource portal at www.cambridge.org/introtofem.
- Fifteen engineering problems of elasticity, heat transfer and fluid flow and mesh creation examples fully worked and described in step-by-step detail.
- Over 110 end-of-chapter problems for students to practice.

Pedagogically, I would suggest go through Chapter 2 quickly and only discuss the contents that senior undergraduate or first year graduate students may not be familiar with, such as vector and matrix norms, condition number, and variational calculus. After that, I would skip Chapter 3 and directly start the 1-D elasticity problem. The numerical analysis methods are presented in a separate chapter (Chapter 3) to avoid repetition. When the relevant numerical methods are needed in the discussion of the

FEA procedure for various types of physical problems, I will go back to Chapter 3 and discuss the content. This book is organized in a way that it tells a story for each type of engineering problem covered. At the beginning of each story an engineering problem is given. The story expands itself when we go on analyzing and solving the problem step by step. The story ends when the FE solution to the problem is obtained. The math and numerical methods are integrated parts of the storyline. Once the story is done, the students are expected to have gained intimate knowledge of all things related to the FEA of that type of problem. In fact, the students are expected to be able to work the problem manually and solve the problem on paper by using the FEM, with the help of MATLAB just for the algebraic calculations. With that, the computer implementation becomes natural and straightforward. As an added benefit, the students will be able to check their computer implementation by comparing intermediate results obtained from their MATLAB codes with their manual calculations. This dual-path learning process is carried out for types of the most problems covered in the book. The concepts and methods that are common for multiple types of problems are reinforced multiple times over the chapters to enable the students to better retain what they have learned. On the other hand, when we go through the steps of solving a different problem in a new chapter, the new or different content in the story becomes obvious and easy to grasp since the students have previous stories for comparison.

I would like to thank the students and colleagues who have provided their feedback, advice, and criticisms for the improvement of this book. I am also grateful to the editors and staff members at Cambridge University Press, especially Steve Elliot, Lisa Pinto, and Stefanie Seaton, for their help in publishing this manuscript. I would also like to thank Dale and Jackie Reynolds for their generous support through the Dale Reynolds 67 Emerging Faculty Scholar Endowment at Clemson University.