THE PHYSICS OF GRAPHENE

Leading graphene research theorist Mikhail I. Katsnelson systematically presents the basic concepts of graphene physics in this fully revised second edition. The author illustrates and explains basic concepts such as Berry phase, scaling, Zitterbewegung, Kubo, Landauer and Mori formalisms in quantum kinetics, chirality, plasmons, commensurate–incommensurate transitions, and many others. Open issues and unsolved problems introduce the reader to the latest developments in the field. New achievements and topics presented include the basic concepts of van der Waals heterostructures, many-body physics of graphene, electronic optics of Dirac electrons, hydrodynamics of electron liquid, and the mechanical properties of one-atom-thick membranes. Building on an undergraduate-level knowledge of quantum and statistical physics and solid-state theory, this is an important graduate textbook for students in nanoscience, nanotechnology, and condensed matter. For physicists and material scientists working in related areas, this is an excellent introduction to the fast-growing field of graphene science.

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THE PHYSICS OF GRAPHENE

SECOND EDITION

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In memory of my teacher Serghey Vonsovsky and my friend Sasha Trefilov
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Preface to the second edition

First of all, I still agree with everything that I wrote in the preface to the first edition; however, I probably need to add a few words on the differences between the second edition and the first.

As you can see, I have changed the title. In 2011 when I finished Graphene: Carbon in Two Dimensions, there were no other books on graphene, and the accuracy of the title was probably not so important. I would also like to emphasize what is special about this book and in what respect it is different from the many others that have appeared in the market in the meantime. To my knowledge, this is the only book on graphene (yet) that focuses completely on fundamental issues of physics and completely ignores all aspects of fabrication, devices, applications, chemistry, etc. Hopefully, the new title, The Physics of Graphene, stresses this point clearly enough and helps potential readers to avoid any disappointment if they do not find something in the book which, in their mind, should be in a book on graphene. Of course, I do not mean that these aspects are not important; I just believe that I am not the proper person to write about them and that other people can do that much better.

In the field of graphene, eight years is a very long period of time, when many things have happened. To my surprise, when I started to work on the new edition, I did not find anything that should be eliminated from the book because it turned out to be fundamentally wrong or irrelevant for further development. Of course, there were some inaccuracies and mistakes, which hopefully have been fixed now, but even so, I think all old issues remain interesting and important. At the same time, many new concepts and facts have appeared that should be reflected in the new book. Therefore I have added three new chapters: Chapters 13 and 14 introduce the basic physics of an important new concept, van der Waals heterostructures, and Chapter 15 gives a very brief summary of our progress in understanding many-body effects in graphene. Eight years ago we had the feeling that a single-particle picture of noninteracting Dirac fermions explained everything; this
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is no longer the case. Huge progress in the quality of graphene samples has opened a way to essentially observe many-body features of the electronic spectrum near the neutrality point.

My work on these subjects was essentially based on a collaboration with Nikita Astrakhantsev, Viktor Braguta, Annalisa Fasolino, Andre Geim, Yura Gongostyrev, Sasha Lichtenstein, Kostya Novoselov, Marco Polini, Burkhard Sachs, Guus Slotman, Misha Titov, Maksim Ulybyshev, Merel van Wijk, Tim Wehling, and Shengjun Yuan. Many thanks!

New material has also been added to the old chapters. The most important new points are:

(1) We now understand the physics and mathematics of chiral tunneling in single- and bilayer graphene much better, therefore Chapter 4 has been expanded. These new results were obtained in collaboration with many people, and I especially thank Koen Reijnders and Victor Kleptsyn.

(2) I have added a new section to Chapter 5 on a spectral flow of Dirac operator in multiconnected graphene flakes. Topological aspects of condensed matter physics have become really hot of late, and this provides a nice and fresh new example. This piece is based on our work with Vladimir Nazaikinskii, to whom I also give thanks.

(3) Chapter 9 was essentially rewritten. I have added new material on mechanical properties, which is based on our work with Jan Los and Annalisa Fasolino, and on thermal expansion of graphene. I thank Igor Burmistrov, Igor Gornyi, Paco Guinea, Valentin Kachorovskii, and Sasha Mirlin for collaboration and useful discussions of this subtle issue. I also thank Achille Mauri who found some inaccuracies in the old Chapter 9 and helped to fix them.

(4) In Chapter 11, I have added a discussion of edge scattering, which is based on our work with Vitaly Dugaev, to whom I am very thankful for his collaboration. Hydrodynamics of electronic liquid in graphene is a very fresh and popular subject now, and I cannot avoid it. When I wrote this part, discussions with Misha Titov and Marco Polini were very helpful.

(5) We now know much more about magnetism and spin-orbit effects in graphene and related two-dimensional materials, therefore Chapter 12 has also been updated. The common work with Andre Geim, Irina Grigorieva, Sasha Lichtenstein, Vladimir Mazurenko, and Sasha Rudenko provided essential insights on my new understanding of the subject.

I would like to repeat all of my acknowledgments from the preface to the first edition. Without all of these old and new collaborations and, of course, without full support from my wife Marina, this book would be impossible.
Preface to the first edition

I do not think that I need to explain, in the preface to a book that is all about graphene, what graphene is and why it is important. After the Nobel Prize for physics in 2010, everybody should have heard something about graphene. I do need, however, to explain why I wrote this book and what is special about it.

I hope it will not be considered a disclosure of insider information if I tell you that Andre Geim is a bit sarcastic (especially with theoreticians). Every time I mentioned that I was somewhat busy writing a book on graphene, he always replied “Go to Amazon.com and search for ‘graphene’.” Indeed, there are many books on graphene, many more reviews, and infinitely many collections of papers and conference proceedings (well, not really infinitely many . . . in the main text I will use the mathematical terminology in a more rigorous way, I promise). Why, nevertheless, has this book been written and why may it be worthwhile for you to read it?

Of course, this is a personal view of the field. I do love it, and it has been my main scientific activity during the last seven years, from 2004 when graphene started to be the subject of intensive and systematic investigations. Luckily, I was involved in this development almost from the very beginning. It was a fantastic experience to watch a whole new world coming into being and to participate in the development of a new language for this new world. I would like to try to share this experience with the readers of this book.

The beauty of graphene is that it demonstrates in the most straightforward way many basic concepts of fundamental physics, from Berry’s phase and topologically protected zero modes, to strongly interacting fluctuations and scaling laws for two-dimensional systems. It is also a real test bed for relativistic quantum phenomena such as Klein tunneling or vacuum reconstruction – “CERN on one’s desk.” I was not able to find a book that focused on these aspects of graphene, namely on its role in our general physical view of the world. I have tried to write such a book myself. The price is that I have sacrificed all practical aspects of graphene science and
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technology, so you will not find a single word here about the ways in which graphene is produced, and there is hardly anything about its potential applications. Well, there is a lot of literature on these subjects. Also, I have said very little about the chemistry of graphene, which is an extremely interesting subject in itself. It certainly deserves a separate book, and I am not chemist enough to write it.

The field is very young, and it is not easy to know what will not be out of date in just a couple of years. My choice is clear from the contents of this book. I do believe that it represents the core of graphene physics, which will not be essentially modified in the near future. I do not mean that this is the most interesting part; moreover, I am sure that there will be impressive progress, at least, in two more directions that are hardly mentioned in the book: in the many-body physics of graphene and in our understanding of electron transport near the neutrality point, where the semiclassical Boltzmann equation is obviously inapplicable. I think, however, that it is a bit too early to cover these subjects in a book, since too many things are not yet clear. Also, the mathematical tools required are not as easy as those used in this book, and I think it is unfair to force the reader to learn something technically quite complicated without a deep internal confidence that the results are relevant for the real graphene.

The way the book has been written is how I would teach a course with the title “Introduction to the Theory of Graphene.” I have tried to make a presentation that is reasonably independent of other textbooks. I have therefore included some general issues such as Berry’s phase, the statistical mechanics of fluctuating membranes, a quick overview of itinerant-electron magnetism, a brief discussion of basic nonequilibrium statistical mechanics, etc. The aims were, first, to show the physics of graphene in a more general context and, second, to make the reading easier.

It is very difficult to give an overview of a field that has developed so quickly as has that of graphene. So many papers appear, literally every day, that keeping permanently up to date would be an enterprise in the style of ancient myths, e.g., those of Sisyphus, the Danaïdes, and some of the labors of Hercules. I apologize therefore for the lack of many important references. I tried to do my best.

I cannot even list all of the scientific reviews on the basic physics of graphene that are available now (let alone reviews of applications and of popular literature). Let me mention at least several of them, in chronological order: Katsnelson (2007a), Geim and Novoselov (2007), Beenakker (2008), Castro Neto et al. (2009), Geim (2009), Abergel et al. (2010), Vozmediano, Katsnelson, and Guinea (2010), Peres (2010), Das Sarma et al. (2011), Goerbig (2011), and Kotov et al. (2012). There you can find different, complementary views on the field (with the possible exception of the first one). Of course, the Nobel lectures by Geim (2011) and Novoselov (2011) are especially strongly recommended. In particular, the lecture by Andre Geim contains a brilliant presentation of the prehistory and
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history of graphene research, so I do not need to discuss these unavoidably controversial issues in my book.

I am very grateful to Andre Geim and Kostya Novoselov, who involved me in this wonderful field before it became fashionable (otherwise I would probably never have dared to join such a brilliant company). I am especially grateful to Andre for regular and lengthy telephone conversations; when you have to discuss a theory using just words, without formulas and diagrams, and cannot even make faces, after several years it does improve your understanding of theoretical physics.

It is impossible to thank all my other collaborators in the field of graphene in a short preface, as well as other colleagues with whom I have had fruitful discussions. I have to thank, first of all, Annalisa Fasolino, Paco Guinea, Sasha Lichtenstein, and Tim Wehling for especially close and intensive collaboration. I am very grateful to the former and current members of our group in Nijmegen working on graphene: Misha Akhukov, Danil Boukhvalov, Jan Los, Koen Reijnders, Rafa Roldan, Timur Tudorovskiy, Shengjun Yuan, and Kostya Zakharchenko, and to my other collaborators and coauthors, especially Mark Auslender, Eduardo Castro, Hans De Raedt, Olle Eriksson, Misha Fogler, Jos Giesbers, Leonya Levitov, Tony Low, Jan Kees Maan, Hector Ochoa, Marco Polini, Sasha Rudenko, Mark van Schilfgaarde, Andrey Shytov, Alyosha Tsvelik, Maria Vozmediano, Oleg Yazyev, and Uli Zeitler.

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Of course, the role of my wife Marina in this book amounts to much more than her help with the manuscript. You cannot succeed in such a long and demanding task without support from your family. I am very grateful for her understanding and full support.

The book is dedicated to the memory of two people who were very close to me, my teacher Serghey Vonsovsky (1910–1998) and my friend Sasha Trefilov (1951–2003). I worked with them for about 20 years, and they had a decisive influence on the formation of my scientific taste and my scientific style. I thought many times during these last seven years how sad it is that I cannot discuss some of the new and interesting physics about graphene with them. Also, in a more technical sense, I would not have been able to write this book without the experience of writing my previous books, Vonsovsky and Katsnelson (1989) and Katsnelson and Trefilov (2002).