

## A Student's Guide to Newton's Laws of Motion

Newton's laws of motion, which introduce force and describe how it affects motion, are the gateway to physics – yet they are often misunderstood due to their many subtleties. Based on the author's twenty years of teaching physics and engineering, this intuitive guide to Newton's laws of motion corrects the many misconceptions surrounding this fundamental topic. Adopting an informal and pedagogical approach and a clear, accessible style, this concise text presents Newton's laws in a coherent story of force and motion. Carefully scaffolded everyday examples and full explanations of concepts and equations ensure that all students studying physics develop a deep understanding of Newton's laws of motion.

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To John William Warren (1923–2016),  
Senior Lecturer in Physics and Reader in Physics Education  
at Brunel University, London,  
whose works set me on  
the path to understanding  
Newton's enchanting laws of motion

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## Preface

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Newton's three laws of motion, the basis of almost all science and engineering, are one of the great achievements of human culture. Using them, we explain, predict, and plan the motion of bodies in the natural and in our human-created worlds. Doing so requires knowing how forces affect motion – knowledge embodied in Newton's second law. Your fluency with this law is the ultimate goal of this book. But first you must know when this law is valid – knowledge provided by Newton's first law. And understanding the first law requires a prior idea, interaction – embodied in Newton's third law.

Thus, you will meet the three laws in the following order: (1) the third law, to introduce interaction; (2) the first law, to describe when the second law can even be used; (3) and finally the second law, to describe what forces do.

But, wait! Before studying the effect of force (the second law) or the idea of interaction (the third law), don't you need to know what force is? No one has answered that question fully. Fortunately, we can understand and use Newton's laws without a solution to that philosophical conundrum. All that we need to know is that a force is a push or a pull. Thus, a force has a strength, formally known as its magnitude, and a direction. Mathematically, force is a vector.

Now you are ready for Newton's laws. To help you learn them, I have embedded throughout this book three types of questions. Questions preceded by a rightward-pointing triangle ( $\blacktriangleright$ ) are from me to you. They are what I would ask you in a one-to-one tutorial on Newton's laws. Questions preceded by a leftward-pointing triangle ( $\blacktriangleleft$ ) are from you to me. They are questions that students have asked or should ask me. For both types of triangle questions, but especially for the questions from me to you ( $\blacktriangleright$ ), try to answer the question before reading on for my explanation. In that way, you will learn Newton's laws more quickly. (When my explanation is lengthy and the answer itself easy to miss, I point out the answer to the triangle question explicitly.)

The third type of question is end-of-chapter problems. Like traditional homework problems, they ask you to apply the ideas that you have learned so far (including in earlier chapters!). Their solutions are available online. As with the triangle questions, try your hand before studying my solution, but do use my solutions as worked examples – one of the most effective ways to learn [20].

Newton's laws are subtle. I have been studying them for over 30 years and teaching them for over 20 years. Only now do I understand many of their subtleties. This book will help you learn in weeks what I learned over decades, an attempt to fulfill the purpose of teaching described by the physicist Edwin Jaynes: to implant a way of thinking so that you, the student, can “learn in one year what the teacher learned in two” [10]. And you can. For I took detours, covered up deep misunderstandings with symbol manipulation and formalism, and fell into many conceptual traps – traps arising partly from what the physics educator J. W. Warren describes as the “incredible confusion of approach” [25, p. 45]. In the following chapters, we journey quickly and directly to the heart of this fascinating subject.

*On y va!*

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