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# 1

# Newton’s Third Law: Forces Belong to Interactions

The most misunderstood and yet the most important of Newton’s three laws is the third law. For it introduces the idea of interaction. Without that idea, Newton’s first law (Chapter 3), which requires removing interactions, makes no sense. And without the first law, you don’t know when you can use Newton’s second law (Chapter 4) – the heart of mechanics. For want of an interaction, the kingdom of physics is lost!

Like me, you may have heard or learned the third law in the action–reaction form: “For every action, there is an equal and opposite reaction.” That form confused me for 20 years and might do the same to you. Here is a clearer form inspired by the work of the physics educator Cornellis Hellingman [7].

> ***Newton’s Third Law***. A force is one side of an interaction between two bodies A and B. The interaction acts equally strongly in the two opposite directions, from A to B and from B to A.

This interaction form reminds us, whenever we encounter a force, to look for the interaction to which it belongs and therefore for the two interacting bodies.

We can express the same law in mathematical notation. If the two bodies are A and B, then one force is $\mathbf{F}_{\text{A on B}}$: the force on body B due to its interaction with body A – or, more concisely, the force of A on B. The other force is $\mathbf{F}_{\text{B on A}}$: the force on body A due to its interaction with body B – the force of B on A. The boldface type for **F** indicates that **F** is a vector, meaning that it has magnitude and direction. (In handwriting, where boldface is hard to make, you’ll see an italic letter with an arrow: $\vec{F}$.) Newton’s third law then says that

$$\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}. \tag{1.1}$$

The bare minus sign, meaning multiplication by $-1$, ensures that the two forces have opposite directions and equal magnitudes. The magnitude, in the internationally standard metric system (the SI system), is measured in newtons. In one of history's jokes, this unit was never used by Newton. In Section 1.4, you develop a feel for this unit as we estimate the magnitudes of diverse forces. But first you learn how to use the third law (Section 1.1), learn how to classify forces (Section 1.2), and meet the forces most important in the world around us (Section 1.3). In the final section (Section 1.5), you learn why you should avoid two familiar forces whose use almost inevitably generates confusion.

## 1.1 Using the Third Law

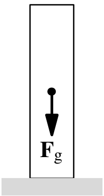

Figure 1.1 Standing on the ground. You stand on the ground and are pulled downward by the gravitational force $\mathbf{F}_g$. What's $\mathbf{F}_g$'s third-law counterpart force?

Here is an everyday example of Newton's third law and of how it reveals a hidden and surprising aspect of the world. The situation: You stand on the ground, and the earth pulls you with a gravitational force $\mathbf{F}_g$ (Figure 1.1). Although the pull acts on each particle within you, this distributed set of forces is, for most purposes, equivalent to a single force acting at your center of mass – at the dot in the diagram. (Problem 5.8 explores a case where this equivalence isn't valid.)

Now try to answer the following question marked with a triangle. As I mention in the Preface (on p. ix), the rightward-pointing triangle indicates a question that I'd ask you in a one-to-one tutorial on Newton's laws. Even in this less personal written format, do make a decent attempt to answer the question; then compare your answer with the full explanation that follows.

► *What's the third-law counterpart force of the gravitational force?*

Most students and many teachers answer this question incorrectly (see, for example, the research by Hellingman [7] and by Terry and Jones [24]). As a student and for many years as a teacher, I would have been among them and would have answered, "the upward force of the ground on me." For I would have used the action–reaction form of Newton's third law and reasoned as follows.

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Gravity is trying to pull you into the ground, so the gravitational force pulling you down into the ground must be the action. The ground complains: “Wait! Bowed low by the weight of the world though I may be, I am still a solid object. You shall not pass through me!” In self-defense, it reacts by pushing you upward. This upward force must be the reaction. Thus, it’s exactly as strong as the gravitational force. (This conclusion is almost unavoidable in the Commonwealth countries, where any upward force from the ground is called a “reaction force.”)

Although the conclusion is right, the reasoning is wrong – the worst combination of right and wrong because the rightness of the conclusion obscures the fundamental error in the reasoning. In our ends-justify-the-means age, the rightness may seem like sufficient justification. However, the same reasoning in many other situations easily leads to wrong conclusions about the upward force from the ground – for example, when someone pushes you downward, when you land after a jump, or when you stand in an accelerating elevator.

Fortunately, you cannot fall into these traps when you use the interaction form of Newton’s third law. It’s embodied in the following procedure.

1. *Determine what two bodies interact to produce the given force.* Here, the given force is the gravitational force on you. Therefore, the two bodies that interact are you and the earth.

2. *Classify the interaction.* The choice, as you soon learn in Section 1.2.1, is between a gravitational and an electromagnetic interaction. This interaction is gravitational.

3. *Describe the given force as one side of this interaction*. In words, it’s the gravitational force of the earth on you. In symbols, it’s $\mathbf{F}_{\text{earth on you}}$.

4. *Describe its third-law counterpart force as the other side of the interaction.* You simply reverse the two bodies’ roles. Here, “the gravitational force of *the earth* on *you*” becomes “the gravitational force of *you* on *the earth*”: $\mathbf{F}_{\text{you on earth}}$.

5. *Remind yourself of how strong the counterpart force is and in what direction it points.* The two forces that constitute the gravitational (or any) interaction are equal and opposite ($\mathbf{F}_{\text{A on B}} = -\mathbf{F}_{\text{B on A}}$), so your gravitational force on the earth has the same strength as the earth’s gravitational force on you and points in the opposite direction.

   Thus, through the gravitational interaction, you pull upward on the earth. I still marvel at this hidden force, revealed by applying the third law. Who would have thought that a mere human could pull the mighty earth?

To summarize this long answer to the triangle question: (1) Forces belong to interactions. (2) The third-law counterpart force of the gravitational force on you is $\mathbf{F}_{\text{you on earth}}$, which is the gravitational force on the earth *from* you. The counterpart is not $\mathbf{F}_{\text{ground on you}}$, which belongs to an entirely different interaction.

Interaction was Newton’s own view of the third law [2, pp. 568–569]:

> For all action is mutual… It is not one action by which the Sun attracts Jupiter, and another by which Jupiter attracts the Sun; but it is one action by which the Sun and Jupiter mutually endeavor to come nearer together (by the Third Law of Motion).

Rather than “action–reaction” with its easy causal-sequence misinterpretation, “interaction” is the heart of the third law. The distinction is illustrated in the following dialogue shared with me by Joshua Roth from his many years of teaching physics in Arlington High School in Massachusetts. A student, giving an example of a third-law pair, had suggested the causal sequence: “Action: I punch you in the face. Reaction: You punch me back.” Roth: “No! That’s the Mosaic law and maybe justice, but it’s not Newton’s third law. Action: You punch me in the face. Reaction according to Newton’s law: You break your wrist.” Or: “[W]e cannot touch without being touched” [8, p. 81].

## 1.2 Classifying Forces

Force is the star of the Newtonian drama. You can help yourself understand the play by enriching your force vocabulary. Therefore, we next discuss three ways to classify forces: as one of four fundamental interactions (Section 1.2.1), as active or passive (Section 1.2.2), and as short or long range (Section 1.2.3).

This seemingly tedious classification process may raise in you the following question marked with a leftward-pointing triangle. As I mention in the Preface (on p. ix), that triangle indicates a question that students ask or should ask me. Make a decent stab at an answer and compare your answer with the full explanation that follows.

◀ *Why go through all this effort to classify forces?*

I offer you an analogy from my learning piano. I just couldn’t learn a difficult passage, the last line of Händel’s Gavotte in G (Figure 1.2). My teacher showed me several ways to play and think about that line: by connecting the notes in the left hand into groups of notes (chords) while playing the right hand’s melody, by connecting the right hand’s notes while playing the left hand’s melody, and by singing the right hand's melody while playing the left hand. After I tried these approaches, the line came to make sense, and I could play it as written. You are playing one of the hardest passages in physics, the concept of force, so take aid from and find comfort in all ways of reflecting on forces!

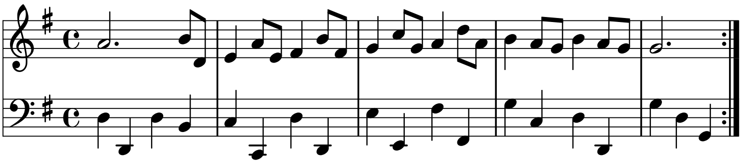

Figure 1.2 The last line of Händel's Gavotte in G (HWV 491). The right hand plays the notes on upper staff, and the left hand plays the notes on the lower staff.

### 1.2.1 The Four Interactions in Nature

Nature, as far as is known to science, uses four types of interactions and therefore four types of forces.

1. *Gravitational interaction.* This interaction acts between any two bodies, anywhere in the universe.
2. *Electromagnetic interaction.* This interaction acts only between charged bodies. It includes the electrostatic interaction between two charges and the magnetic interaction between moving charges (including magnets).
3. *Strong nuclear interaction.* This interaction acts between and holds together the quarks that make up protons and neutrons.
4. *Weak nuclear interaction.* This interaction acts between protons, neutrons, and electrons and can turn protons into neutrons and vice versa. It lies behind radioactive decay and nuclear fission. As its name suggests, it's much weaker than the strong nuclear interaction.

The last two interactions, the strong and weak nuclear interactions, have a minuscule range, about 1 femtometer ($10^{-15}$ meters) or roughly the size of a nucleus. In the world around us, they have no direct effect. (Their indirect effect, however, is essential: Without them, protons and neutrons would not hold together, and there would be no atoms.) So, in learning and when using Newton's laws, we can ignore them.

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With that simplification, classify every interaction – and the two forces that constitute it – as either gravitational or electromagnetic. The consequence:

If a force is neither gravitational nor electromagnetic, it doesn’t exist.

Gravitational forces and interactions, because they join every pair of bodies in the universe, are surprisingly generic. In contrast, electromagnetic forces are diverse. They include contact forces between touching bodies, covalent bonds within molecules (for example, between hydrogen and oxygen atoms in water), hydrogen bonds between polar molecules (for example, between water molecules), ionic bonds within solids (for example, between sodium and chlorine ions in table salt), and Van der Waals bonds between nonpolar atoms or molecules (for example, between helium atoms or nitrogen molecules).

The classification into gravitational, electromagnetic, or nothing helps prevent a common and dangerous misconception.

► *Imagine a passenger sitting in a car or train wagon going around a turn. Is there an outward force on the passenger? If so, what’s it called?*

You might suspect that there is an outward force and, if you had a proper education in Latin, that it’s called the centrifugal force (“centrifugal” means away from the center). As a student, even lacking Latin, I would have agreed.

However, now you and I both know how to classify forces and interactions into one of four types. How then fares the alleged centrifugal force? Is it one of the two nuclear forces, either strong or weak? No. For as I mentioned on p. 5, no force in everyday life is a nuclear force. These forces act over too short a range (the size of a nucleus). Because “too short a range” is always the answer to a question about their relevance, I now really forget about the nuclear forces for the rest of the book.

Is the outward force a gravitational force? No. For no planet lies outside the vehicle’s door and pulls the passenger toward it. Is it an electromagnetic force? If it is, what charges (or magnets) would be responsible for it? Perhaps they are in the door of the vehicle? But have you ever felt the door of a vehicle pulling you outward and toward it? I haven’t. The door can only push the passenger inward and away from it.

Perhaps, instead, the outward force is due to the seat – that is, the seat acts on the passenger with an outward force. But how could the seat manage this feat? Its deformed part is its outward end, which gets compressed and thus pushes the passenger inward rather than outward. The mistake here, discussed further in Section 1.5.1, is confusing the force of the seat *on the passenger* (an inward force) with the force of the passenger *on the seat* (an outward force).

Indeed, and in answer to the triangle question, *no outward force acts on the passenger.* Thus, the alleged centrifugal force is none of the four forces in nature. It does not exist. (Why then do the passenger and vehicle move in a circle? This deep question, which involves all of Newton's laws, gets the longer answer that it deserves in Section 7.3.2.)

As a useful rule, do not mention the centrifugal force! Like most rules, it has an exception, discussed in Section 8.1. Until then, keep to the rule, avoid a widespread source of confusion, and greatly increase your chances of using Newton's laws correctly.

### 1.2.2 Active versus Passive Forces

The second force classification is into active versus passive forces. This classification, unlike the classification into four fundamental kinds of force (Section 1.2.1), isn't inherent in nature. Rather, it's a human choice.

1. *Active forces* are known gravitational and electromagnetic forces and pushes and pulls made intentionally by an animate being (be it a person, raccoon, or bear). Examples include the gravitational force on you or me and my push on a heavy box that remains sitting on the floor.
2. *Passive forces*, in contrast, arise and adjust themselves in response to active forces. One example is the force of the ground on you while you stand on the ground (Section 1.1). This force adjusts itself in response to the gravitational force on you as that active force tries to pull you into the ground. A second example is the force of friction preventing my moving that heavy box. This force adjusts itself in response to my active force on the box. As I push harder, the friction force grows in magnitude. If I am strong enough, the friction force can no longer adjust itself to match, and the box starts moving. This change illustrates a characteristic of passive forces, that they can adjust themselves in magnitude or direction only within limits.

This classification, like most human-created ones, isn't airtight. For example, when I hold a book over my head, the active force on the book is the downward gravitational force on it, but what kind of force is my upward force on it? Is it active because I'm animate? Or is it passive because I adjust how hard I push based on the gravitational force (pushing harder on a bigger book)? I would call it a passive force, but either choice fulfills the main purpose of this classification, which is to prevent us from overlooking a force from an inanimate object (like a door or a floor).

A puzzling feature of passive forces, especially given that they are produced (almost always) by inanimate object, is how they know their strength. How, for example, does the friction force on that heavy box (p. 7) adjust itself to prevent the box from moving even as I, frustrated by the box’s obstinacy, increase how hard I push? This deep question implicates three subtle ideas: spring forces (Section 1.3.2), Newton’s second law (Chapter 4), and acceleration (Section 6.2). Thus, its answer comes after their development (Section 7.1.6).

Until then, keep in mind the main reason for the idea of passive forces. It gives us a category, and therefore a name, for the forces exerted by inanimate objects. This category reminds us that inanimate objects can exert forces. These forces then become harder to overlook.

As a second benefit, the classification names an important connection between forces, one otherwise easily mislabeled as Newton’s third law. When I stand on the ground and the gravitational force – here, the action and the active force – pulls me downward, the ground reacts by pushing me upward. This reaction, as you learned in Section 1.1, is *not* a third-law counterpart to the action (which is why I loathe the action–reaction name for the third law). However, this reaction *is* the passive force arising in response to the active force. When one force leads to another, the two forces cannot constitute a third-law pair; almost always, they comprise an active force and its corresponding passive force.

### 1.2.3 Long-Range versus Short-Range Forces

The third and final classification is into long-range forces, also known as body or volume forces, versus short-range forces, also known as contact or surface forces. For gravitational forces, this classification is easy. They are always long range: Gravitational forces are so weak that they require large sources, such as asteroids, moons, or planets, to have significant effects.

In contrast, electromagnetic forces, being relatively strong, don’t require large sources of charge to have an appreciable effect. Thus, they can be either short or long range. One long-range example is the electromagnetic force on electrons in your retina due to jiggling electrons in the sun; that force is how you see the sun. Another example is the force on a compass needle due to electrons circulating in the earth’s core. In contrast, the force between my finger and a pen, though also electromagnetic, is a short-range force (a contact force). As you learn when you meet spring forces (Section 1.3.2), this contact force is due to electrostatic repulsion between the outermost electrons in my skin cell’s molecules and the outermost electrons in the pen’s molecules.

In this book, with its focus on mechanics rather than electromagnetism, all electromagnetic forces will be short-range, contact forces. Thus, the classification into gravitational and electromagnetic forces will parallel the classification into long-range and short-range forces.

Either classification prevents you from overlooking forces. The short-range forces acting on a body are usually easy to spot: one due to each touching body. But the long-range forces are easier to overlook: out of sight, out of mind. By asking yourself, “On this body, are there also any long-range forces?” you are more likely to spot them too.

## 1.3 Important Forces

Understanding that forces belong to interactions (Section 1.1) is important, as is classifying forces (Section 1.2). However, you need forces to classify. Thus, you next meet the most important forces in the world around us. They, through Newton’s laws, explain the motion of most everyday and heavenly bodies.

### 1.3.1 Gravitational Forces

The gravitational force comes in two seemingly different forms. One form is more famous: Newton’s law of universal gravitation. It states that the gravitational force between mass $m_1$ and mass $m_2$ has magnitude

$$F = \frac{Gm_1m_2}{r^2}, \tag{1.2}$$

where $G$ is Newton’s constant of gravitation, and $r$ is the distance between the two masses (assumed to be points). As for the direction: The force is attractive, pointing from each mass to the other (Figure 1.3).

Specifying a vector, such as force, requires giving its magnitude and direction. Thus, never say that the gravitational force is $Gm_1m_2/r^2$ – which provides only the magnitude. I recommend fanaticism about the distinction between a vector and its magnitude: Confusing these two quantities leads to many further difficulties with Newton’s laws, an already subtle subject. (Even fanatics stray by mistake, so let me know if you find any such mistakes in this book.)

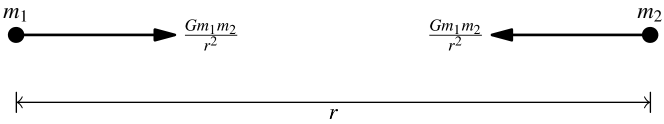

Figure 1.3 Gravitational interaction. Two particles, $m_1$ and $m_2$, separated by a distance $r$, participate in a gravitational interaction. The interaction's two forces have magnitude $Gm_1m_2/r^2$ and are opposite in direction.

If the bodies are not point particles but are spheres with a spherically symmetric density (which is roughly true for most planets), the force law (1.2) still works as long as $r$ is measured between the bodies' centers (Newton invented calculus partly to prove this statement). In other words, a spherically symmetric body acts, for the purposes of the gravitational interaction, as if all its mass were concentrated at its center.

The most important gravitational force, at least for humanity, is the force of the sun on the earth. You could also argue for the gravitational force of the earth on each of us. However, without the sun holding the earth at just the right distance from the sun, giving the earth's surface just the right temperature to support life, none of us would be alive to argue for this alternative.

► *Roughly how large is the gravitational force of the sun on the earth?*

To find out, just put the appropriate values into the force law (1.2). Newton's constant ($G$) is roughly $7 \times 10^{-11}$ crazy SI units, which are meters cubed per kilogram second squared ($\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}$). The earth's mass ($m_1$) is approximately $6 \times 10^{24}$ kilograms. The sun's mass ($m_2$) is almost exactly $2 \times 10^{30}$ kilograms. And the earth–sun distance ($r$) is roughly $1.5 \times 10^{11}$ meters. Then

$$F \approx \frac{\overbrace{7 \times 10^{-11}\,\text{kg}^{-1}\,\text{m}^3\,\text{s}^{-2}}^{G} \times \overbrace{6 \times 10^{24}\,\text{kg}}^{m_1} \times \overbrace{2 \times 10^{30}\,\text{kg}}^{m_2}}{\underbrace{\left(1.5 \times 10^{11}\,\text{m}\right)^2}_{r^2}}. \tag{1.3}$$

Rather than calculating $F$ by breaking out the calculator, which would give us many spurious decimal places of precision and atrophy our intuitive sense for quantities in the world, let's estimate $F$ by hand. Such calculations are best broken into three stages ordered from most to least important: the units, the powers of 10, and the mantissa (the remaining factor). The three stages are then reassembled to form the estimate:

$$F \approx \text{mantissa} \times 10^{\text{exponent}}\ \text{units}. \tag{1.4}$$

1. *Units.* This stage comes first because using the wrong units with even the correct number in front is dangerously wrong (as the sad crash of NASA's Mars Climate Orbiter shows [15]). Here, the units portion is