Introduction to Quantum Field Theory

This textbook offers a detailed and uniquely self-contained presentation of quantum and gauge field theories. Writing from a modern perspective, the author begins with a discussion of advanced dynamics and special relativity before guiding students steadily through the fundamental principles of relativistic quantum mechanics and classical field theory. This foundation is then used to develop the full theoretical framework of quantum and gauge field theories. The introductory, opening half of the book allows it to be used for a variety of courses, from advanced undergraduate to graduate level, and students lacking a formal background in more elementary topics will benefit greatly from this approach. Williams provides full derivations wherever possible and adopts a pedagogical tone without sacrificing rigor. Worked examples are included throughout the text and end-of-chapter problems help students to reinforce key concepts. A fully worked solutions manual is available online for instructors.

Anthony G. Williams is Professor of Physics at Adelaide University, Australia. He has worked extensively in the areas of hadronic physics and computational physics, studying quark and gluon substructure. For this work, he was awarded the Walter Boas Medal by the Australian Institute of Physics in 2001 and elected Fellow of the American Physical Society in 2002. In 2020, he became the deputy director of the Centre for Dark Matter Particle Physics of the Australian Research Council.

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Classical Mechanics to Gauge Field Theories

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Preface

The Standard Model of particle physics unites three of the four known forces of nature into a single, elegant theory. It is arguably the most successful physical theory devised to date. The accuracy with which the Standard Model can reproduce the measurements of precise experiments is remarkable. It describes the electromagnetic, weak and strong interactions, but it does not account for the gravitational interaction. We understand that the Standard Model is not the final word, but the path to physics beyond the Standard Model is not yet clear. We therefore set the Standard Model as our end point here.

The primary purpose of this book is to provide a single coherent framework taking us from the postulates of special relativity, Newton's laws and quantum mechanics through to the development of quantum field theory, gauge field theories and the Standard Model. The presentation is as self-contained as possible given the need to fit within a single volume. While building an understanding of quantum field theory, gauge field theories and the Standard Model is the final goal here, we have attempted to include all essential background material in a self-contained way.

There is an emphasis on showing the logically flowing development of the subject matter. In order to achieve this we have attempted to provide proofs of all key steps. Where appropriate these have been separated out from the main text in boxes. The reason for this is that the proofs sometimes require a careful mathematical discussion, and this can distract from the flow of the physical arguments. Students seeing this material for the first time can overlook the more difficult boxed proofs on a first reading while concentrating on the physics. They can be comfortable knowing that the particular result can be proved and then can come back and absorb that proof later as desired. Problem sets are provided at the end of each chapter to build familiarity and understanding of the material and practice in its application.

A word of encouragement to students: Much knowledge that is worthwhile is not easily won. If some piece of mathematics initially seems too challenging, then absorb the physical consequence of the result, move on, and come back to the maths later. Discussing with others is *always* helpful. This author remembers well when he did not understand what is written in these pages and the challenge of needing to overcome that. The hope is that enough handholds have been provided that with some effort and through discussions with others any student of quantum field theory can follow the arguments presented.

Conventions and notation: The notation and conventions that we typically follow are those of Peskin and Schroeder (1995) and Schwartz (2013). The conventions in Bjorken and Drell (1964), Bjorken and Drell (1965) and Greiner (2000) are very similar to each other and differ from our notation in the normalization choices for field operators and Dirac spinors and in their not including a factor of i in the definition of Feynman propagators. Itzykson and Zuber (1980) and Sterman (1993) use different normalization conventions again. We and the above books use the 'West coast' metric, $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, whereas Brown (1992), Srednicki (2007) and Weinberg (1995) use the 'East coast' metric, $g^{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. Here we denote the electric charge as q as done in

Aitchison and Hey (2013) to avoid confusion. The electron charge is then $q = -e \equiv -|e|$, where for us e will always mean the magnitude of the charge of the electron. Some texts choose e negative and some have mixed or inconsistent usage of the sign. Comparing the covariant derivative with ours, $D^\mu = \partial^\mu + iqA^\mu$, will immediately reveal the choice in each text. One needs to carefully compare equations when moving between texts, keeping these issues in mind. To know that one has the correct sign for q in D^μ one can relate it back to the Lorentz force equation in Eq. (4.3.67) or to the Maxwell equations $\partial_\mu F^{\mu\nu} = j^\nu$ as we and Aitchison and Hey (2013) have done.

Further reading: Due to the breadth of material covered in this single volume there has been a need to focus on the essentials of quantum and gauge field theories. By construction there is more than enough material for any year-long course in quantum and gauge field theories; however, there are topics and applications that could not be included here. A reader who has understood this text will be able to pick up other texts on quantum field theory and readily understand them. Both Peskin and Schroeder (1995) and Schwartz (2013) contain similar conventions and notation, so by design it is straightforward to supplement this book with examples and applications from each of these texts. Srednicki (2007), Weinberg (1995) and Weinberg (1996) provide additional applications and detail using the 'East coast' spacetime metric. For the advanced reader Weinberg's books contain insights and gems of knowledge not readily found elsewhere. A very accessible overview of quantum field theory is given in Zee (2010). There are many other very worthwhile texts and a partial list of these is given at the beginning of Chapter 6.

Organization of the Book

The first four chapters of the book lay out the foundations and structures required for the following chapters. It opens with a discussion of special relativity and Lorentz and Poincaré invariance in Chapter 1, which begins with Einstein's postulates and ends with a discussion of the representations of the Poincaré group used to classify fundamental particles. In Chapter 2 the treatment of classical point mechanics begins with Newton's laws and is developed into analytic mechanics in both its Lagrangian and Hamiltonian formulations. The normal modes of small oscillations of classical systems are treated. The analytic mechanics approach is subsequently extended to include special relativity in the special case of a single particle in an external potential. The discussion of electromagnetism in Sec. 2.7 begins from the traditional form of Maxwell's equations and moves to the relativistic formulation of electromagnetism and the consequent need to understand gauge transformations and gauge fixing. The relation of different unit systems in electromagnetism is given. Chapter 2 ends with a discussion of the Dirac-Bergmann algorithm needed to treat Hamiltonian descriptions of singular systems, which will become relevant for understanding the canonical quantization of gauge fields and fermions. The extension of classical point mechanics to classical relativistic fields is developed in Chapter 3, where the concepts of classical mechanics are extended to an infinite number of degrees of freedom. This was necessary because there is no consistent analytic mechanics formulation of a relativistic system with a finite number of degrees of freedom. We later come to understand that the quanta resulting from the quantization of the normal modes of these relativistic classical fields are the fundamental particles of the corresponding relativistic quantum field theory. A common unifying thread through all of these developments in Chapters 2 and 3 is Hamilton's principle of stationary action. Chapter 4 is devoted to the development of relativistic quantum mechanics. It begins with a review of the essential elements of quantum mechanics, including the derivation of the Feynman path integral approach to quantum mechanics. This chapter has detailed discussions of the Klein-Gordon equation, the Dirac equation, their interaction with external fields and their symmetries.

In the second part of the book we begin in Chapter 5 with a brief history and overview of particle physics to motivate the subsequent chapters. This historical perspective demonstrates the essential role of experimental discovery in driving the direction and development of the field. In Chapter 6 the formulation of free field theory is presented with explicit constructions given for scalar, charged scalar, fermion, photon and massive vector boson fields. In Chapter 7 we discuss the interaction picture, scattering cross-sections and Feynman diagrams and show how to evaluate tree-level diagrams for several theories of interest. In Chapter 8 we first consider the discrete symmetries of charge conjugation (C), parity inversion (P) and time reversal (T) and prove the CPT theorem. Then we turn to the essential elements of renormalization and the renormalization group including dimensional regularization and renormalized perturbation theory. This is followed by a discussion of spontaneous symmetry breaking, Goldstone's theorem and the Casimir effect. Finally, in Chapter 9 the extension of electromagnetism and quantum electrodynamics to nonabelian gauge theories is given. The example of quantum chromodynamics and its relation the strong interactions is then discussed and the lattice gauge theory approach to studies of nonperturbative behavior is introduced. The chapter concludes with a discussion of quantum anomalies and then finally with the construction of the Standard Model of particle physics.

How to Use This Book

This book is intended to be suitable for first-time students as well as for readers more experienced in the field. Some suggestions on how to use the material covered to build various courses are the following.

(i) For a lecture course on **Special Relativity**: Secs. 1.1, 1.2 and some or all of the material from 1.4. Some aspects of Sec. 1.3 could be included for an advanced class. Careful selections of key results from Sec. 2.6 on relativistic kinematics and Sec. 2.7.1 on the relativistic formulation of Maxwell's equations might also be considered.

(ii) For a lecture course on **Classical and/or Analytic Mechanics**: Secs. 2.1–2.7 form a sound basis. As needed, subsets of this material can be used depending on the length of the course. For an advanced class, selections of material on analytic relativistic mechanics and/or Sec. 2.9 on constrained Hamiltonian mechanics could also be included. Working through and summarizing these two advanced sections could also be assigned to students as undergraduate research or reading projects;

(iii) For a course on **Advanced Dynamics and Relativity**: Core material from Secs. 1.1, 1.2 and Secs. 2.1–2.8 form the basis of a one-semester course that I taught over many years.

(iv) For a course on **Relativistic Classical Field Theory**: Secs. 3.1–Sec. 3.3.1 are suitable for a lecture course. For a shorter course a focus on the key results and their proofs would be sufficient. For an advanced class in a longer course Sec. 3.3.2 could be included in the lectures, but this advanced material is also a candidate for a reading topic or small research project.

(v) For a course on **Relativistic Quantum Mechanics**: Secs. 4.2–4.6.5 contain all necessary course material. The review of quantum mechanics in Sec. 4.1 could be treated as assumed knowledge or some elements could be chosen for inclusion. Relevant sections of Sec. 4.1 could be assigned as reading topics in this course as well as in other courses.

(vi) For a lecture course on **Relativistic Quantum Mechanics and Particle Physics**: Material on the Klein-Gordon and Dirac equations in Secs. 4.3, 4.4 and 4.5.1 can be combined with selected elements from Chapter 5 such as the Cabibbo-Kobayashi-Maskawa (CKM) matrix, neutrino mixing, the quark model of strongly interacting particles and representations of group theory.

(vii) For a one-semester course on **Relativistic Quantum Field Theory**: Chapter 6 would form the core of such a course with an emphasis on Secs. 6.2–6.4. Technical sections such as the derivation of the functional integrals for fermions and photons could be abbreviated or omitted and the second part of Sec. 6.3.6 on the derivation of the Dirac fermion canonical anticommutation relations could be mentioned but not explicitly covered. Some of the important results in Secs. 7.1–7.6, including the Feynman rules and example tree-level cross-section calculations, could be summarized and included as course length allows.

(viii) For a one-semester course on **Gauge Field Theories**: The remainder of Chapter 7 not covered above and the core material in Chapters 8 and 9 on renormalization, gauge field theories, Goldstone's theorem, quantum chromodynamics (QCD), anomalies and the Standard Model.

(ix) For a full-year course on **Quantum and Gauge Field Theories**: Combine the material in the above two suggested courses and choose the division of material between semesters to best suit the pace of the lectures and the desired emphasis of the course.

Corrections to This Book

Despite the best efforts of all involved, there will be remaining errors in this book for which I am solely responsible. The current list of corrections along with the names of those who suggested them can be found at:

www.cambridge.org/WilliamsQFT

It would be greatly appreciated if anyone finding additional errors could please report them using the relevant corrections link provided on this website.

Acknowledgments

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