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978-1-108-46402-4 — Cambridge International AS & A Level Mathematics: Mechanics Practice Book
Series: Cambridge International AS and A Level
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Chapter 1
Velocity and acceleration

- Work with scalar and vector quantities for distance and speed.
- Use equations of constant acceleration.
- Sketch and read displacement–time graphs and velocity–time graphs.
- Solve problems with multiple stages of motion.

1.1 Displacement and velocity

WORKED EXAMPLE 1.1

A plane flies from Warsaw to Athens, a distance of 1600 km, at an average speed of 640 km h^{-1}. How long does the flight take?

Answer

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

State the equation to use.

$$\text{so time} = \frac{\text{distance}}{\text{speed}}$$

Rearrange the equation to make time the subject.

$$= \frac{1600}{640}$$

$$= 2.5$$

Use consistent units, substitute values into the equation.

Flight takes 2 hours 30 minutes.

Convert the decimal answer into hours and minutes.

EXERCISE 1A

1 How long will an athlete take to run 1500 metres at 7.5 m s^{-1}?

2 A train maintains a constant velocity of 60 m s^{-1} due south for 20 minutes. What is its displacement in that time? Give the distance in kilometres.

3 Some Antarctic explorers walking towards the South Pole expect to average 1.8 kilometres per hour. What is their expected displacement in a day in which they walk for 14 hours?

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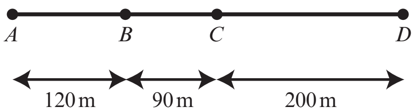
Questions 4 and 5 refer to the four points, A, B, C and D, which lie in a straight line with distances between them shown in the diagram. The displacement is measured from left to right.



4 Find:

a **i** the displacement from D to A

ii the displacement from D to B

b **i** the distance from D to B

ii the distance from C to A

c **i** the total displacement when a particle travels from B to C and then to A

ii the total displacement when a particle travels from C to D and then to A.

TIP

Remember displacement is a vector quantity, and distance is a scalar quantity.

PS **5** **a** **i** A particle travels from A to C in 23 seconds and then from C to B in 18 seconds. Find its average speed and average velocity.

ii A particle travels from B to D in 38 seconds and then from D to A in 43 seconds. Find its average speed and average velocity.

b **i** A particle travels from B to D in 16 seconds and then back to B in 22 seconds. Find its average speed and average velocity.

ii A particle travels from A to C in 26 seconds and then back to A in 18 seconds. Find its average speed and average velocity.

TIP

Remember speed is a scalar quantity, and velocity is a vector quantity.

6 Here is an extract from the diary of Samuel Pepys for 4 June 1666, written in London.

'We find the Duke at St James's, whither he is lately gone to lodge. So walking through the Parke we saw hundreds of people listening to hear the guns.'

These guns were at the battle of the English fleet against the Dutch off the Kent coast, a distance of between 110 and 120 km away. The speed of sound in air is $344\,\mathrm{m\,s^{-1}}$. How long did it take the sound of the gunfire to reach London?

7 Light travels at a speed of $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$. Light from the star Sirius takes 8.65 years to reach the Earth. What is the distance of Sirius from the Earth in kilometres?

TIP

Consider how many seconds there are in 8.65 years.

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1.2 Acceleration

WORKED EXAMPLE 1.2

A skateboarder travels down a hill in a straight line with constant acceleration. She starts with speed $1.5\,\text{m s}^{-1}$ and finishes with speed $9.5\,\text{m s}^{-1}$. The length of the hill is 22 m.

a Find the time taken.

b Find the acceleration of the skateboarder.

Answer

a $s = 22, u = 1.5, v = 9.5$ — Begin by listing the information given.

$s = \frac{1}{2}(u + v)t$ so $22 = \frac{1}{2}(1.5 + 9.5)t$ — State the equation to be used and substitute in the known values.

$t = 4$ — Rearrange the equation to find the time.

Time taken = 4 seconds — Include the units in the final answer.

b $a = \frac{v - u}{t}$ — Choose the equation to be used.

$= \frac{9.5 - 1.5}{4}$ — Substitute in the known values.

$= 2$

Acceleration = $2\,\text{m s}^{-2}$ — Include the units in the final answer.

EXERCISE 1B

1 Write the following quantities in the specified units, giving your answers to 3 significant figures.

a **i** $3.6\,\text{km h}^{-1}$ in m s^{-1} **ii** $62\,\text{km h}^{-1}$ in m s^{-1}

b **i** $5.2\,\text{m s}^{-1}$ in km h^{-1} **ii** $0.26\,\text{m s}^{-1}$ in km h^{-1}

c **i** $120\,\text{km h}^{-2}$ in m s^{-2} **ii** $450\,\text{km h}^{-2}$ in m s^{-2}

d **i** $0.82\,\text{m s}^{-2}$ in km h^{-2} **ii** $2.7\,\text{m s}^{-2}$ in km h^{-2}

TIP

Use velocities not speeds.

2 A police car accelerates from $15\,\text{m s}^{-1}$ to $35\,\text{m s}^{-1}$ in 5 seconds. The acceleration is constant. Illustrate this with a velocity–time graph. Use the equation $v = u + at$ to calculate the acceleration. Find also the distance travelled by the car in that time.

3 A marathon competitor running at $5\,\text{m}\,\text{s}^{-1}$ puts on a sprint when she is 100 metres from the finish, and covers this distance in 16 seconds. Assuming that her acceleration is constant, use the equation $s = \frac{1}{2}(u + v)t$ to find how fast she is running as she crosses the finishing line.

4 Starting from rest, an aircraft accelerates to its take-off speed of $60\,\text{m}\,\text{s}^{-1}$ in a distance of 900 metres. Assuming constant acceleration, find how long the take-off run lasts. Hence calculate the acceleration.

TIP

'Rest' means not moving, so the velocity is zero.

5 A train is travelling at $80\,\text{m}\,\text{s}^{-1}$ when the driver applies the brakes, producing a deceleration of $2\,\text{m}\,\text{s}^{-2}$ for 30 seconds. How fast is the train then travelling, and how far does it travel while the brakes are on?

PS 6 A balloon at a height of 300 m is descending at $10\,\text{m}\,\text{s}^{-1}$ and decelerating at a rate of $0.4\,\text{m}\,\text{s}^{-2}$. How long will it take for the balloon to stop descending, and what will its height be then?

7 A train goes into a tunnel at $20\,\text{m}\,\text{s}^{-1}$ and emerges from it at $55\,\text{m}\,\text{s}^{-1}$. The tunnel is 1500 m long. Assuming constant acceleration, find how long the train is in the tunnel for, and the acceleration of the train.

PS 8 A cyclist riding at $5\,\text{m}\,\text{s}^{-1}$ starts to accelerate, and 200 metres later she is riding at $7\,\text{m}\,\text{s}^{-1}$. Find her acceleration, assumed constant.

1.3 Equations of constant acceleration

WORKED EXAMPLE 1.3

A train is travelling at $55\,\text{m}\,\text{s}^{-1}$. The driver needs to reduce the speed to $35\,\text{m}\,\text{s}^{-1}$ to pass through a junction. The deceleration must not exceed $0.6\,\text{m}\,\text{s}^{-2}$. How far ahead of the junction should the driver begin to slow down the train?

Answer

Using the maximum deceleration:

$u = 55,\ v = 35,\ a = -0.6$ — Begin by listing the information given. As we have deceleration, the acceleration is a negative value.

$v^2 = u^2 + 2as$ so $1225 = 3025 - 1.2s$ — State the equation to be used and substitute in the known values.

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$s = 1500$ — Rearrange to find the distance.

The driver should start to slow down at least 1500 m ahead of the junction. — Include the units in the final answer and clarify the answer in the context of the question.

EXERCISE 1C

TIP

Decide which variable to eliminate.

P **1** Use the formulae $v = u + at$ and $s = \frac{1}{2}(u + v)t$ to prove that $s = ut + \frac{1}{2}at^2$.

P **2** **a** Use the formulae $s = ut + \frac{1}{2}at^2$ and $v = u + at$ to derive the formula $s = vt - \frac{1}{2}at^2$.

b A particle moves with constant acceleration $3.1\,\text{m s}^{-2}$. It travels 300 m in the first 8 seconds. Find its speed at the end of the 8 seconds.

P **3** Use the formulae $v = u + at$ and $s = ut + \frac{1}{2}at^2$ to derive the formula $v^2 = u^2 + 2as$.

4 An ocean liner leaves the harbour entrance travelling at $3\,\text{m s}^{-1}$, and accelerates at $0.04\,\text{m s}^{-2}$ until it reaches its cruising speed of $15\,\text{m s}^{-1}$.

a How far does it travel in accelerating to its cruising speed?

b How long does it take to travel 2 km from the harbour entrance?

5 A boy kicks a football up a slope with a speed of $6\,\text{m s}^{-1}$. The ball decelerates at $0.3\,\text{m s}^{-2}$. How far up the slope does it roll?

6 A cyclist comes to the top of a hill 165 metres long travelling at $5\,\text{m s}^{-1}$, and free-wheels down it with an acceleration of $0.8\,\text{m s}^{-2}$. Write expressions for his speed and the distance he has travelled after t seconds. Hence find how long he takes to reach the bottom of the hill, and how fast he is then travelling.

PS **7** A particle reduces its speed from $20\,\text{m s}^{-1}$ to $8.2\,\text{m s}^{-1}$ while travelling 100 m. Assuming it continues to move with the same constant acceleration, how long will it take to travel another 20 m?

PS **8** A particle moves with constant deceleration of $3.6\,\text{m s}^{-2}$. It travels 350 m while its speed halves. Find the time it takes to do this.

9 A car reduces its speed from $18\,\text{m s}^{-1}$ to $9\,\text{m s}^{-1}$ while travelling 200 m. Assuming the car continues to move with the same uniform acceleration, how much further will it travel before it stops?

10 a A particle moves in a straight line with constant acceleration $a = -3.4\,\text{m s}^{-2}$. At $t = 0$ its velocity is $u = 6\,\text{m s}^{-1}$. Find its maximum displacement from the starting point.

b Explain why this is not the maximum distance from the starting point.

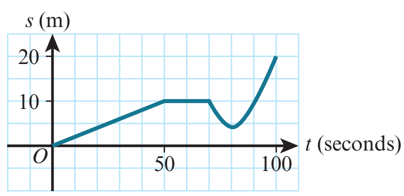
11 A car travelling at $10\,\text{m s}^{-1}$ is 25 metres from a pedestrian crossing when the traffic light changes from green to amber. The light remains at amber for 2 seconds before it changes to red. The driver has two choices: to accelerate so as to reach the crossing before the light changes to red, or to try to stop at the light. What is the least acceleration which would be necessary in the first case, and the least deceleration which would be necessary in the second?

12 A cheetah is pursuing an impala. The impala is running in a straight line at a constant speed of $16\,\text{m s}^{-1}$. The cheetah is 10 m behind the impala, running at $20\,\text{m s}^{-1}$ but tiring, so that it is decelerating at $1\,\text{m s}^{-2}$. Find an expression for the gap between the cheetah and the impala t seconds later. Will the impala get away?

1.4 Displacement–time graphs and multi-stage problems

WORKED EXAMPLE 1.4

The diagram shows the displacement–time graph for a cyclist moving in a straight line.



a Find the velocity of the cyclist over the first 50 seconds.

b Estimate the times when the velocity of the cyclist is $0\,\text{m s}^{-1}$.

c Find the greatest (positive) velocity over the 100 seconds.

Answer

a $\dfrac{10}{50} = 0.2\,\text{m s}^{-1}$ — The gradient of a displacement–time graph is equal to the velocity.

b $t = 50$ to 70 and at approximately 82 seconds — As the gradient represents the velocity, we look at when the gradient is zero.

c Velocity = gradient of graph

Greatest (positive) velocity is $t = 90$ to 100 — This is where the graph is steepest.

In this time s increases from 10 to 20

Velocity $= \dfrac{20 - 10}{100 - 90} = 1\,\text{m s}^{-1}$ — Gradient = increase in height/change in horizontal distance.

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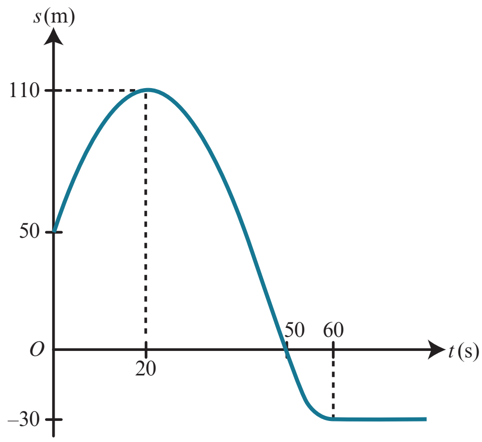
EXERCISE 1D

1 A particle moves in a straight line. Its displacement from point P is shown on the displacement–time graph.



a How far from P does the particle start?

b In the first 20 seconds, is the particle moving towards P or away from it?

c What happens when $t = 20$ seconds?

d What happens after 60 seconds?

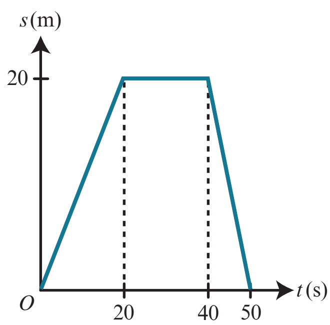
e At what time does the particle pass P?

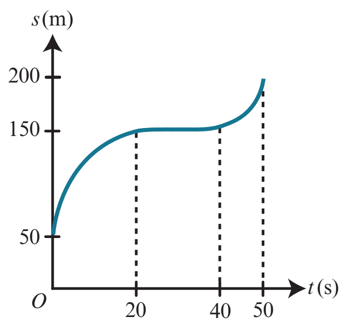
f Is the particle's speed increasing or decreasing during the first 20 seconds?

g Is the particle's speed increasing or decreasing between 50 and 60 seconds? What about its velocity?

h Find the total distance travelled by the particle in the first 60 seconds.

PS 2 For each displacement–time graph, draw the corresponding straight line velocity–time graph:

a 

b 

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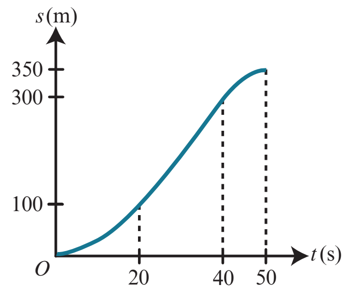
c



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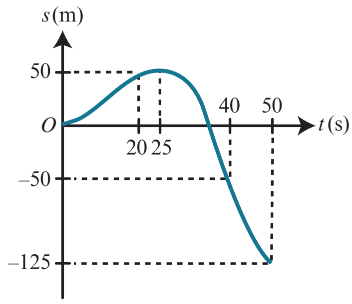
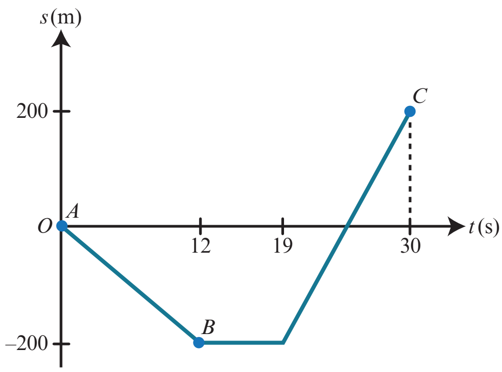


3 This displacement–time graph represents the motion of a particle moving in a straight line. The particle passes point A when $t = 0$.



The particle is at point B when $t = 12$ and at point C when $t = 30$.

a Describe what happens between $t = 12$ and $t = 19$.

b Write down the displacement of C from A. Hence find the average velocity of the particle during the 30 seconds.

c Find the average speed of the particle during the 30 seconds.

PS **4** A cyclist is free-wheeling down a long straight hill. The times between passing successive kilometre posts are 100 seconds and 80 seconds. Assuming his acceleration is constant, find this acceleration.

TIP

Each individual km can be considered, or the combined motion for 2 km can be considered.

5 A train is slowing down with constant deceleration. It passes a signal at A, and after successive intervals of 40 seconds it passes points B and C, where $AB = 1800\,\text{m}$ and $BC = 1400\,\text{m}$.

a How fast is the train moving when it passes A?

b How far from A does it come to a stop?

PS **6** A particle is moving along a straight line with constant acceleration. In an interval of T seconds it moves D metres; in the next interval of $3T$ seconds it moves $9D$ metres.

How far does it move in a further interval of T seconds?

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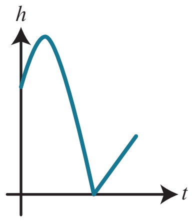
7 A ball is thrown vertically upwards, travels up and then down; when the ball hits the ground it bounces up again.

The following graph shows the height of the ball above the ground against time.



Sketch a graph to show the distance that the ball has travelled against time.

8 Amir and Sofia start side by side on the starting line of a 100 m track. Amir runs the 100 metres at a constant speed of v m s^{-1}. Sofia starts from rest 1 second after Amir and accelerates at a constant 0.5 m s^{-2}.

a When $v = 4$, how far has Amir run when Sofia overtakes him?

b What happens when $v = 5$?

9 A particle starts from rest and moves with constant acceleration.

a Sketch the displacement–time graph.

In the first 4 seconds the particle moves 16 metres.

b Find how far the particle travels in the next 4 seconds.

PS

10 A motorbike and a car are waiting side by side at traffic lights. When the lights turn to green, the motorbike accelerates at 2.5 m s^{-2} up to a top speed of 20 m s^{-1}, and the car accelerates at 1.5 m s^{-2} up to a top speed of 30 m s^{-1}. Both then continue to move at constant speed.

a Using the same axes, sketch the displacement–time graphs.

b After what time will the motorbike and the car again be side by side?

c What is the greatest distance that the motorbike is in front of the car?

M

11 The displacement, in metres, of a particle is plotted against time, in seconds. The resulting displacement–time graph is modelled as a quadratic equation that passes through (0, 20), (10, 5) and (12, 20).

At some time the displacement is x metres and 4 seconds later the displacement is again x metres. Find the value of x.

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1.5 Velocity–time graphs and multi-stage problems

WORKED EXAMPLE 1.5

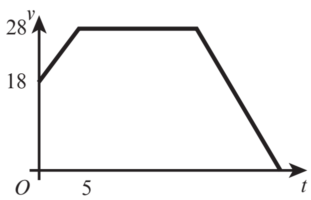
A train moves along a straight track. The train passes a signal at time $t = 0$ seconds and is moving with a velocity of $18\,\mathrm{m\,s^{-1}}$. The train accelerates at $2\,\mathrm{m\,s^{-2}}$ for 5 seconds, travels at a constant velocity for some time, and then decelerates at $1.4\,\mathrm{m\,s^{-2}}$ until it stops at a station. The signal is 1000 m from the station. Find the time at which the train reaches the station.

Answer

When $t = 0$, $v = 18$ — State the initial values.

When $t = 5$, $v = 18 + 2 \times 5 = 28$ — Use $v = u + at$ to find the velocity after 5 seconds.

The velocity–time graph looks like this: — Although we don't have all the information for a velocity–time graph, drawing a sketch of what we do know may help to answer the question.



While accelerating, the train travels $0.5(18 + 28) \times 5 = 115$ metres — Use $s = \frac{1}{2}(u + v)t$ to find the distance travelled during the first section of the motion.

The deceleration phase takes $\frac{0 - 28}{-1.4} = 20$ seconds — Use $\frac{v - u}{a} = t$ to find the time taken to deceleration to rest.

The distance travelled while decelerating is $0.5(28 + 0) \times 20 = 280$ metres — Use $s = \frac{1}{2}(u + v)t$ to find the distance travelled during the final section of the motion.

So the distance travelled at $28\,\mathrm{m\,s^{-1}}$ is $1000 - 115 - 280 = 605$ metres — Use the total distance to find the distance travelled during the constant speed section of motion.

This takes $605 \div 28 = 21.6$ seconds — Use time $= \frac{\text{distance}}{\text{speed}}$.

$5 + 21.6 + 20 = 46.6$ — Finally add the time for each of the three sections.

The train reaches the station at time $t = 46.6$ seconds

