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Partial Differential Equations in Fluid Mechanics

Edited by

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Preface

This volume is the result of a workshop, “PDEs in Fluid Mechanics”, which took place in the Mathematics Institute at the University of Warwick, September 26th–30th, 2016. This was the opening meeting of the Warwick EPSRC Symposium on *Partial Differential Equations and their Applications*, which continued throughout the 2016/17 academic year.

Several of the speakers agreed to write review papers related to their contributions to the workshop, while others have written more traditional research papers. We believe that this volume therefore provides an accessible summary of a wide range of active research topics, along with some exciting new results, and we hope that it will prove a useful resource both for graduate students new to the area and for more established researchers. We have also included a survey of the pioneering 1934 paper by Leray, rewritten in updated terminology and notation by Wojciech Ożański & Benjamin Pooley, which should make the many insights contained in Leray’s work more accessible to the modern reader.

We would like to express our gratitude to the following sponsors of the workshop and the writing of this volume of proceedings: the meeting itself was supported (as part of the 2016/17 Symposium) by the EPSRC, grant EP/N003039/1. JLR is currently supported by the European Research Council (ERC grant agreement no. 616797).

Finally, it is a pleasure to thank Hazel Higgins and Yvonne Collins of Warwick’s Mathematics Research Centre for all their help with the organisation of our workshop.

*Charles L. Fefferman
James C. Robinson
José L. Rodrigo*

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