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More Information

Chapter 1
Algebra

- Understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities.
- Divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero).
- Use the factor theorem and the remainder theorem.

1.1 The modulus function

WORKED EXAMPLE 1.1

Solve:

a $2|3x - 1| = \left|\frac{1}{2}x - 1\right|$

b $x^2 - 5|x| + 4 = 0$

Answer

a $2|3x - 1| = \left|\frac{1}{2}x - 1\right|$ — Split the equation into two parts.

$2(1 - 3x) = 1 - \frac{1}{2}x$ ---------- (1)

$2(1 - 3x) = -\left(1 - \frac{1}{2}x\right)$ ---------- (2)

$2 - 6x = 1 - \frac{1}{2}x$ — Using equation (1), expand brackets.

$-5\frac{1}{2}x = -1$ — Solve.

$x = \frac{2}{11}$

$2 - 6x = -1 + \frac{1}{2}x$ — Solve using equation (2).

$-6\frac{1}{2}x = -3$

$x = \frac{6}{13}$

The solution is: $x = \frac{2}{11}$ or $x = \frac{6}{13}$.

b $x^2 - 5|x| + 4 = 0$ — Subtract $x^2 + 4$ from both sides.

$-5|x| = -x^2 - 4$ — Divide both sides by -5.

$|x| = \frac{1}{5}(x^2 + 4)$ — Split the equation into two parts.

$x = \frac{1}{5}(x^2 + 4)$ ---------- (1)

$x = \frac{1}{5}(-x^2 - 4)$ ---------- (2)

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$5x = x^2 + 4$ — Using equation (1), rearrange.

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$ — Factorise.

$x = 4, 1$

$5x = -x^2 - 4$ — Using equation (2), rearrange.

$x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$ — Factorise.

$x = -4, -1$

The solutions are $x = \pm 1, \pm 4$

$1^2 - 5|1| + 4 = 0$ — Check.

$4^2 - 5|4| + 4 = 0$

$(-1)^2 - 5|-1| + 4 = 0$

$(-4)^2 - 5|-4| + 4 = 0$

EXERCISE 1A

TIP

Remember:
$|a| = |b| \Leftrightarrow a^2 = b^2$

1 Solve:

a $|x + 2| = 5$ b $|x - 1| = 7$

c $|2x - 3| = 3$ d $|3x + 1| = 10$

e $|x + 1| = |2x - 3|$ f $|x - 3| = |3x + 1|$

g $|2x + 1| = |3x + 9|$ h $|5x + 1| = |11 - 2x|$

2 Solve these equations.

a $|x + 1| + |1 - x| = 2$ b $|x + 1| - |1 - x| = 2$

c $-|x + 1| + |1 - x| = 2$

3 Solve these equations.

a $|x| = |1 - x| + 1$ b $|x - 1| = |x| + 1$ c $|x - 1| + |x| = 1$

4 Solve these equations.

a $x + |2x - 1| = 3$ b $3 + |2x - 1| = x$

5 Solve:

a $|x^2 - 4| = 12$ b $|6 + x^2| = 5x$ c $|x^2 + 3x| = x + 1$

d $|x^2 - 4| = 4x + 1$ e $|3x^2 - 2x| = 1 - x$ f $|x^2 - 3x + 6| = 4 + 2x$

6 Solve the simultaneous equations.

a $x + 2y = 4$
$|x + 2| + y = 5$

b $3x + y = 0$
$y = |x^2 - 2x|$

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Excerpt
More Information

7 Solve the equation $2|x-1|^2 + 3|x-1| - 2 = 0$.

8 a Solve the equation $x^2 - 5|x| + 6 = 0$.

b Sketch the graph of $y = x^2 - 5|x| + 6$.

c Write down the equation of the line of symmetry of the curve.

9 Solve the equation $|2x+1| + |2x-1| = 3$.

PS 10 Solve the equation $|3x - 2y + 10| + 2\sqrt{7 + 3x - 3y} = 0$.

1.2 Graphs of $y = |f(x)|$ where f(x) is linear

WORKED EXAMPLE 1.2

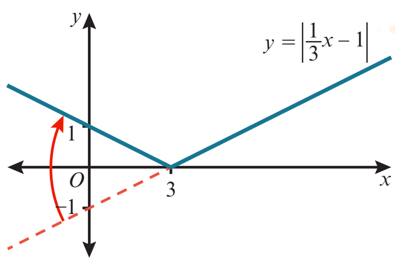
a Sketch the graph of $y = \left|\frac{1}{3}x - 1\right|$, showing the points where the graph meets the axes.

Use your graph to express $\left|\frac{1}{3}x - 1\right|$ in an alternative form.

b Use your answer to part **a** to solve graphically $\left|\frac{1}{3}x - 1\right| = 1$.

Answer

a



First sketch the graph of $y = \frac{1}{3}x - 1$.

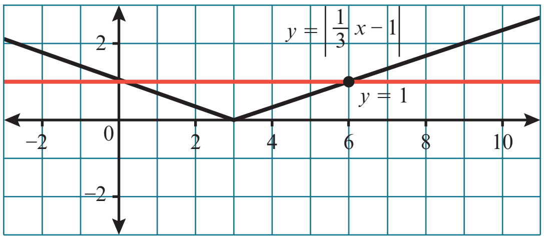
Reflect in the x-axis the part of the line that is below the x-axis.

The line has gradient $\frac{1}{3}$ and a y-intercept of -1.

The graph shows that $\left|\frac{1}{3}x - 1\right|$ can be written as:

$$\left|\frac{1}{3}x - 1\right| = \begin{cases} \frac{1}{3}x - 1 & \text{if } x \geqslant 3 \\ -\left(\frac{1}{3}x - 1\right) & \text{if } x < 3 \end{cases}$$

b



On the same axes, draw $y = 1$.

Find the points of intersection of the lines $y = \left|\frac{1}{3}x - 1\right|$ and $y = 1$.

There are two points of intersection of the lines $y = \left|\frac{1}{3}x - 1\right|$ and $y = 1$ so there are two roots.

The solutions to $\left|\frac{1}{3}x - 1\right| = 1$ are $x = 0$ and $x = 6$.

EXERCISE 1B

1 Sketch the graphs of each of the following functions showing the coordinates of the points where the graph meets the axes.

a $y = |x - 4|$ **b** $y = |5 - 2x|$ **c** $y = \left|3 - \frac{1}{4}x\right|$

2 Sketch the following graphs.

a $y = |x + 3|$ **b** $y = |3x - 1|$ **c** $y = |x - 5|$

d $y = |3 - 2x|$ **e** $y = 2|x + 1|$ **f** $y = 3|x - 2|$

g $y = -2|2x - 1|$ **h** $y = 3|2 - 3x|$ **i** $y = |x + 4| + |3 - x|$

j $y = |6 - x| + |1 + x|$ **k** $y = |x - 2| + |2x - 1|$ **l** $y = 2|x - 1| - |2x + 3|$

3 Describe fully the transformation (or combination of transformations) that maps the graph of $y = |x|$ onto each of these functions.

a $y = |x - 2| + 3$ **b** $y = |x + 3| - 2$ **c** $y = 1 - |x|$

d $y = |3x| + 1$ **e** $y = 2 - |x + 2|$ **f** $y = 1 - 3|x|$

4 Sketch each of the following sets of graphs.

a $y = x^2 - 2$ and $y = |x^2 - 2|$

b $y = \sin x$ and $y = |\sin x|$

c $y = (x - 1)(x - 2)(x - 3)$ and $y = |(x - 1)(x - 2)(x - 3)|$

d $y = \cos 2x$ and $y = |\cos 2x|$ and $y = \cos |2x|$

e $y = |x - 2|$ and $y = ||x| - 2|$

5 $f(x) = 3 - |2x - 3|$ for $-2 \leqslant x \leqslant 6$. Find the range of function f.

6 a Sketch the graph of $y = |2x - 3| + 1$ for $-2 < x < 6$, showing the coordinates of the vertex and the y-intercept.

b On the same diagram, sketch the graph of $y = 5 - x$.

c Use your graph to solve the equation $|2x - 3| + 1 = 5 - x$.

7 a Sketch the graph of $y = |2x - 1|$ for $-4 < x < 6$, showing the coordinates of the vertex and the y-intercept.

b On the same diagram, sketch the graph of $y = |3 - x|$.

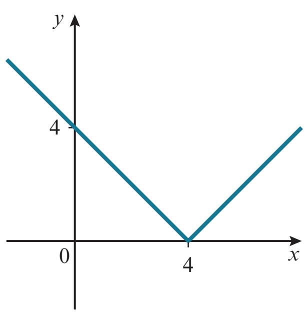
c Use your graph to solve the equation $|2x - 1| = |3 - x|$.

8 a Sketch the graph of $y = |2x + 1| + |1 - x|$.

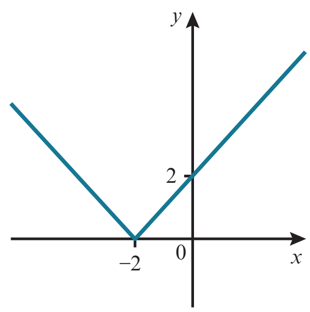
b Use your graph to solve the equation $|2x + 1| + |1 - x| = 3$.

9 Write the equation of each graph in the form $y = |ax + b|$.

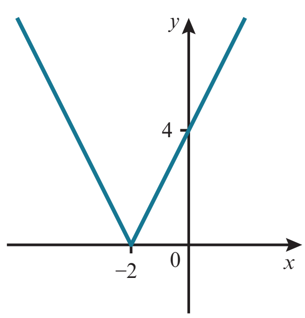
a i



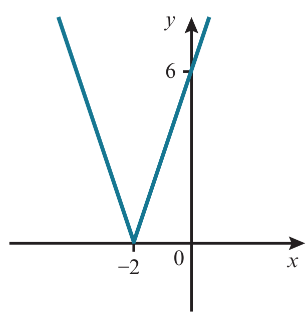
ii



b i



ii



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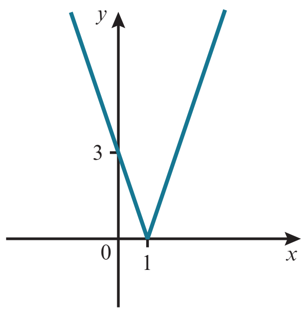
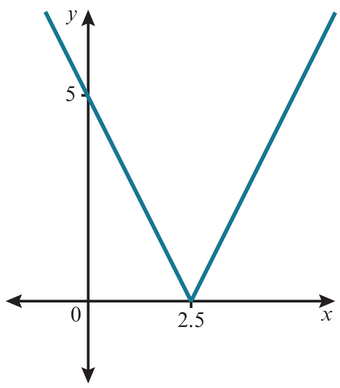
Excerpt
More Information

c **i**



ii



10 Sketch the graph of $y = x|x|$.

1.3 Solving modulus inequalities

WORKED EXAMPLE 1.3

Solve the inequality $|2x - 3| \leqslant |x - 2|$.

Answer

Method 1 — Use algebra.

$|2x - 3| \leqslant |x - 2|$ — Use $|a| \geqslant |b| \Leftrightarrow a^2 \geqslant b^2$.

$(2x - 3)^2 \leqslant (x - 2)^2$

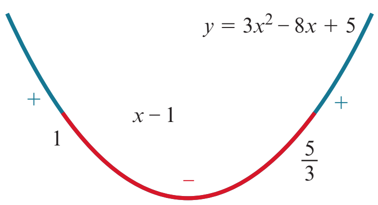
$4x^2 - 12x + 9 \leqslant x^2 - 4x + 4$

$3x^2 - 8x + 5 \leqslant 0$ — Factorise.

$(3x - 5)(x - 1) \leqslant 0$

Critical values are $\frac{5}{3}$ and 1.

Hence, $1 \leqslant x \leqslant \frac{5}{3}$.



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Excerpt
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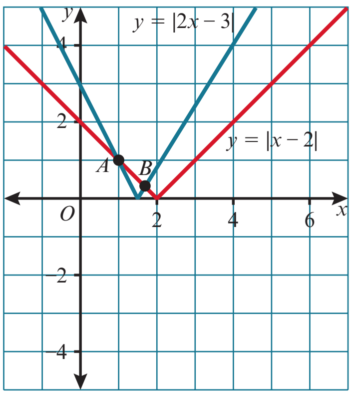
Method 2

Use a graph.



The graphs of $y = |2x - 3|$ and $y = |x - 2|$ intersect at the points A and B.

$$|2x - 3| = \begin{cases} 2x - 3 & \text{if } x \geqslant \dfrac{3}{2} \\ -(2x - 3) & \text{if } x < \dfrac{3}{2} \end{cases}$$

Find the points of intersection.

$$|x - 2| = \begin{cases} x - 2 & \text{if } x \geqslant 2 \\ -(x - 2) & \text{if } x < 2 \end{cases}$$

At A, the line $y = -(x - 2)$ intersects the line $y = -(2x - 3)$.

$$-x + 2 = -2x + 3$$
$$x = 1$$

At B, the line $y = 2x - 3$ intersects the line $y = -(x - 2)$.

$$2x - 3 = -x + 2$$
$$3x = 5$$
$$x = \frac{5}{3}$$

To solve the inequality $|2x - 3| \leqslant |x - 2|$ find where the graph of the function $y = |2x - 3|$ is below the graph of $y = |x - 2|$.

Hence, $1 \leqslant x \leqslant \dfrac{5}{3}$.

EXERCISE 1C

1 Solve. (You may use either an algebraic method or a graphical method.)

a $|x + 2| < 1$ b $|x - 3| > 5$
c $|2x + 7| \leqslant 3$ d $|3x + 2| \geqslant 8$
e $|x + 2| < |3x + 1|$ f $|2x + 5| > |x + 2|$
g $|x| > |2x - 3|$ h $|4x + 1| \leqslant |4x - 1|$

2 a Sketch the graphs of $y = |x|$ and $y = 2|2x - 3|$ on the same axes.

b Solve the inequality $|x| > 2|2x - 3|$.

3 a On the same axes sketch the graphs of $y = 1 - |x - 2|$ and $y = |2x - 3|$.

b Solve the inequality $1 - |x - 2| > |2x - 3|$.

4 Rewrite the function $k(x)$ defined by $k(x) = |x + 3| + |4 - x|$ for the following three cases, without using the modulus in your answer.

a $x > 4$ b $-3 \leqslant x \leqslant 4$ c $x < -3$

TIP

When solving modulus inequalities use
$|a| \leqslant b \Leftrightarrow -b \leqslant a \leqslant b$
and $|a| \geqslant b \Leftrightarrow a \leqslant -b$ or $b \leqslant a$

1.4 Division of polynomials

WORKED EXAMPLE 1.4

Find the remainder when $x^3 - 3x + 4$ is divided by $x + 3$.

Answer

There is no x^2 term in $x^3 - 3x + 4$ so we write it as $x^3 + 0x^2 - 3x + 4$.

$$\begin{array}{r} x^2 \\ x+3\,\overline{)\,x^3+0x^2-3x+4} \\ \underline{x^3+3x^2} \\ -3x^2-3x \end{array}$$

Divide the first term of the polynomial by x : $x^3 \div x = x^2$

Multiply $(x + 3)$ by x^2 : $x^2(x + 3) = x^3 + 3x^2$

Subtract: $(x^3 + 0x^2) - (x^3 + 3x^2) = -3x^2$ and bring down the $-3x$ from the next column.

$$\begin{array}{r} x^2-3x \\ x+3\,\overline{)\,x^3+0x^2-3x+4} \\ \underline{x^3+3x^2} \\ -3x^2-3x \\ \underline{-3x^2-9x} \\ 6x+4 \end{array}$$

Repeat the process.

Divide $-3x^2$ by x : $-3x^2 \div x = -3x$

Multiply $(x + 3)$ by $-3x$: $-3x(x + 3) = -3x^2 - 9x$

Subtract: $(-3x^2 - 3x) - (-3x^2 - 9x) = 6x$ and bring down the 4 from the next column.

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Excerpt
More Information

$$
\begin{array}{r}
x^2 - 3x + 6 \\
x + 3\,\overline{)\,x^3 + 0x^2 - 3x + 4} \\
\underline{x^2 + 3x^2} \quad\quad\quad \\
-3x^2 - 3x \quad \\
\underline{-3x^2 - 9x} \quad \\
6x + 4 \\
\underline{6x + 18} \\
-14
\end{array}
$$

Repeat the process.

Divide $6x$ by x : $6x \div x = 6$

Multiply $(x + 3)$ by 6 : $6(x + 3) = 6x + 18$

Subtract: $(6x + 4) - (6x + 18) = -14$

The remainder is -14.

The calculation can be written as:

$$(x^3 - 3x + 4) = (x + 3) \times (x^2 - 3x + 6) - 14$$

↑ dividend ↑ divisor ↑ quotient ↑ remainder

EXERCISE 1D

1 Find the quotient and the remainder when the first polynomial is divided by the second.

a $x^3 + 2x^2 - 3x + 1,\quad x + 2$

b $x^3 - 3x^2 + 5x - 4,\quad x - 5$

c $2x^3 + 4x - 5,\quad x + 3$

d $5x^3 - 3x + 7,\quad x - 4$

e $2x^3 - x^2 - 3x - 7,\quad 2x + 1$

f $6x^3 + 17x^2 - 17x + 5,\quad 3x - 2$

2 Use polynomial division to simplify the following expressions.

a **i** $\dfrac{x^3 - 3x^2 - 13x - 30}{x^2 + 3x + 5}$ **ii** $\dfrac{x^3 + 5x^2 - 5x + 63}{x^2 - 2x + 9}$

b **i** $\dfrac{x^4 + 7x^3 + 13x^2 + 2x - 2}{x^2 + 3x - 1}$ **ii** $\dfrac{x^4 + 3x^3 - 32x^2 - 17x + 3}{x^2 - 4x - 3}$

3 **a** Use algebraic division to show that $x - 3$ is a factor of $2x^3 - 5x^2 - x - 6$.

b Hence show that there is only one real root for the equation $2x^3 - 5x^2 - x - 6 = 0$

4 Use algebraic division to show that $x + 2$ is a factor of $x^3 + 8$.

5 Use algebraic division to find the remainder when $(1 + x)^4$ is divided by $x + 2$.

1.5 The factor theorem

WORKED EXAMPLE 1.5

a Factorise $f(x) = 2x^3 - 7x^2 + 4x - 3$.

b Hence, solve $2x^3 - 7x^2 + 4x - 3 = 0$, stating the number of real roots of the equation.

Answer

a Let $f(x) = 2x^3 - 7x^2 + 4x - 3$

$f(1) = 2(1)^3 - 7(1)^2 + 4(1) - 3 = -4$,
so $x - 1$ is not a factor of $f(x)$.

If $x - c$ is a factor, then c can only be $\pm1, \pm3$ because the positive and negative factors of 3 are $\pm1, \pm3$.

$f(-1) = 2(-1)^3 - 7(-1)^2 + 4(-1) - 3 = -16$,
so $x + 1$ is not a factor of $f(x)$.

$f(3) = 2(3)^3 - 7(3)^2 + 4(3) - 3 = 0$,
so $x - 3$ is a factor of $f(x)$.

The other factors of $f(x)$ can be found by any of the following methods.

Method 1 Substitution.

Substitute the other factors of 3 into $f(x) = 2x^3 - 7x^2 + 4x - 3$.

This (in other questions) could lead to a long method and may not yield further results.

Method 2 Long division (generally a shorter method).

$$\begin{array}{r} 2x^2 - x + 1 \\ x - 3 \overline{)\, 2x^3 - 7x^2 + 4x - 3} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ -x^2 + 4x \qquad \\ \underline{-x^2 + 3x} \qquad \\ x - 3 \\ \underline{x - 3} \\ 0 \end{array}$$

Hence, $f(x) = (x - 3)(2x^2 - x + 1)$

$f(x) \equiv (x - 3)(2x^2 - x + 1)$

$(2x^2 - x + 1)$ will not factorise further.

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Excerpt
More Information

Method 3 — Equate coefficients.

Since $x - 3$ is a factor,
$2x^3 - 7x^2 + 4x - 3$ can be written as
$2x^3 - 7x^2 + 4x - 3$
$= (x - 3)(ax^2 + bx + c)$

$2x^3 - 7x^2 + 4x - 3$
$= (x - 3)(2x^2 + bx + 1)$ — Coefficient of x^3 is 2, so $a = 2$

Constant term is -3, so $c = 1$ since $-3 \times 1 = -3$.

$-7 = b - 6$, so $b = -1$ — Equate the coefficients of x^2.

$4 = 1 - 3b$, so $b = -1$. — Equate the coefficients of x.

Hence, $f(x) \equiv (x - 3)(2x^2 - x + 1)$

b $2x^3 - 7x^2 + 4x - 3 = 0$ — Factorise.
$(x - 3)(2x^2 - x + 1) = 0$

Either $(x - 3) = 0$ — Solve.
$x = 3$

Or $2x^2 - x + 1 = 0$ — Use the quadratic formula $a = 2$, $b = -1$, $c = 1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$$

No solutions.

There is only one real solution to the equation, which is $x = 3$.

EXERCISE 1E

1 Decide whether each of the following expressions is a factor of $2x^3 - 3x^2 - 3x + 2$.

a	**i** $x - 1$	**ii**	$x + 1$
b	**i** $x - 2$	**ii**	$x + 2$
c	**i** $x - \frac{1}{2}$	**ii**	$x + \frac{1}{2}$
d	**i** $2x - 1$	**ii**	$2x + 1$
e	**i** $3x - 1$	**ii**	$3x + 2$

2 Fully factorise the following expressions.

a **i** $x^3 + 2x^2 - x - 2$ **ii** $x^3 + x^2 - 4x - 4$

b **i** $x^3 - 7x^2 + 16x - 12$ **ii** $x^3 + 6x^2 + 12x + 8$

c **i** $x^3 - 3x^2 + 12x - 10$ **ii** $x^3 - 2x^2 + 2x - 15$

d **i** $6x^3 - 11x^2 + 6x - 1$ **ii** $12x^3 + 13x^2 - 37x - 30$

3 Solve the following equations.

a **i** $x^3 + 12 = 2x^2 + 11x$ **ii** $x^3 - x^2 - 17x = 15$

b **i** $x^3 - 5x^2 + 7x - 2 = 0$ **ii** $x^3 - 6x^2 + 7x - 2 = 0$

4 Find the roots of the following equations.

a **i** $x^3 - 6x^2 + 11x = 6$ **ii** $x^3 - 2x^2 + 6 = 5x$

b **i** $x^3 + x^2 - x - 1 = 0$ **ii** $x^3 - 3x^2 - 10x + 24 = 0$

5 **a** Show that $(x - 2)$ is a factor of $p(x) = x^3 - 3x^2 - 10x + 24$.

b Hence express $p(x)$ as the product of three linear factors and solve $p(x) = 0$.

P **6** **a** Show that $(x - 3)$ is a factor of $p(x) = x^3 - x^2 - 2x - 12$.

b Hence show that $p(x) = 0$ only has one real root.

7 $x^3 + 7x^2 + cx + d$ has factors $(x + 1)$ and $(x + 2)$. Find the values of c and d.

8 $f(x) = x^3 - ax^2 - bx + 168$ has factors $(x - 7)$ and $(x - 3)$.

a Find a and b.

b Find the remaining factor of $f(x)$

9 The polynomial $x^2 + kx - 8k$ has a factor $(x - k)$. Find the possible values of k.

10 The polynomial $x^2 - (k + 1)x - 3$ has a factor $(x - k + 1)$. Find k.

PS **11** The polynomial $x^2 - 5x + 6$ is a factor of $2x^3 - 15x^2 + ax + b$. Find the values of a and b.

12 Use the factor theorem to factorise the following quartic polynomials $p(x)$. In each case, write down the real roots of the equation $p(x) = 0$.

a $x^4 - x^3 - 7x^2 + x + 6$ **b** $x^4 + 4x^3 - x^2 - 16x - 12$

c $2x^4 - 3x^3 - 12x^2 + 7x + 6$ **d** $6x^4 + x^3 - 17x^2 - 16x - 4$

e $x^4 - 2x^3 + 2x - 1$ **f** $4x^4 - 12x^3 + x^2 + 12x + 4$

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Excerpt
More Information

1.6 The remainder theorem

WORKED EXAMPLE 1.6

Use the remainder theorem to find the remainder when $3x^3 - 4x^2 + 3x - 1$ is divided by $2x - 1$.

Answer

Let $f(x) = 3x^3 - 4x^2 + 3x - 1$

$$\text{Remainder} = f\left(\frac{1}{2}\right)$$
$$= 3\left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1$$
$$= \frac{3}{8} - 1 + \frac{3}{2} - 1$$
$$= -\frac{1}{8}$$

TIP

If a polynomial $P(x)$ is divided by $x - c$, the remainder is $P(c)$.

If a polynomial $P(x)$ is divided by $ax - b$, the remainder is $P\left(\frac{b}{a}\right)$.

EXERCISE 1F

1 When $3x^3 - 2x^2 + ax + b$ is divided by $x - 1$, the remainder is 3. When divided by $x + 1$ the remainder is -13. Find the values of a and b.

2 When $x^3 + ax^2 + bx + 5$ is divided by $x - 2$, the remainder is 23. When divided by $x + 1$ the remainder is 11. Find the values of a and b.

3 When $x^3 + ax^2 + bx - 5$ is divided by $x - 1$, the remainder is -1. When divided by $x + 1$ the remainder is -5. Find the values of a and b.

4 When $2x^3 - x^2 + ax + b$ is divided by $x - 2$, the remainder is 25. When divided by $x + 1$ the remainder is -5. Find the values of a and b.

5 Find the values of a and b if $ax^4 + bx^3 - 8x^2 + 6$ has a remainder $2x + 1$ when divided by $x^2 - 1$.

6 The expression $px^4 + qx^3 + 3x^2 - 2x + 3$ has a remainder $x + 1$ when divided by $x^2 - 3x + 2$. Find the values of p and q.

7 The expression $ax^2 + bx + c$ is divisible by $x - 1$. It leaves a remainder 2 when divided by $x + 1$, and has a remainder 8 when divided by $x - 2$. Find the values of a, b and c.

P 8 The cubic polynomial $x^3 + x^2 + Ax + B$, where A and B are constants, is denoted by $f(x)$. When $f(x)$ is divided by $x - 1$ the remainder is 4, and when $f(x)$ is divided by $x + 2$ the remainder is 10. Prove that $x + 3$ is a factor of $f(x)$.

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END-OF-CHAPTER REVIEW EXERCISE 1

1 Given that $k > 0$, find in terms of k the solution of the inequality $|x - k| \leqslant |2x - k|$.

PS 2 Solve the equation $|x + k| = |x| + k$, where $k > 0$.

3 a State the sequence of three transformations that transform the graph of $y = |x|$ to the graph of $y = 5 - 3|x|$. Hence sketch the graph of $y = 5 - 3|x|$.

b Solve the equation $|2x - 1| = 5 - 3|x|$.

c Write down the solution of the inequality $|2x - 1| \leqslant 5 - 3|x|$.

4 Solve the equation $x|x| = x^2$.

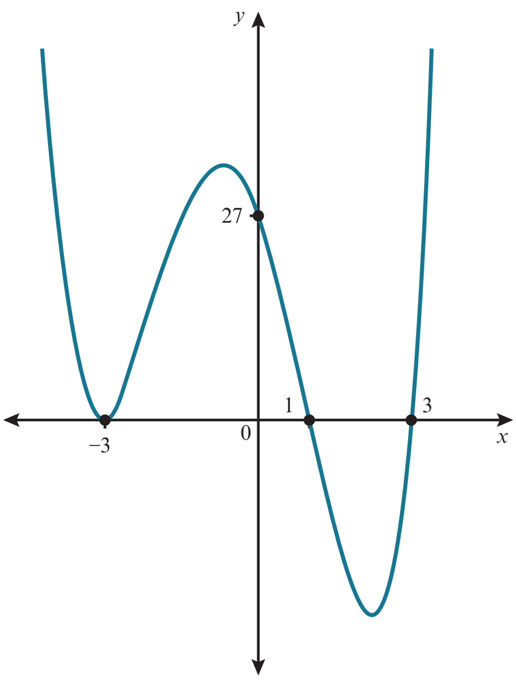
5 Sketch the graph of $y = x + |x|$.

6 Show that $\dfrac{x^3 + 2x^2 - 3x - 6}{x + 2} = x^2 + bx + c$ where b and c are integers to be found.

PS 7 The polynomial $x^2 - 4x + 3$ is a factor of the polynomial $x^3 + ax^2 + 27x + b$. Find the values of a and b.

P 8 The cubic polynomial $x^3 + x^2 + Ax + B$, where A and B are constants, is denoted by $f(x)$. When $f(x)$ is divided by $x - 1$ the remainder is 4, and when $f(x)$ is divided by $x + 2$ the remainder is 10. Prove that $x + 3$ is a factor of $f(x)$.

PS 9 The diagram shows the graph with equation $y = ax^4 + bx^3 + cx^2 + dx + e$. Find the values of a, b, c, d and e.



PS 10 Given $f(x) = 8x^3 - x^2 + 7$, the remainder when $f(x)$ is divided by $x - a$ is eight times the remainder when $f(x)$ is divided by $2x - a$. Find the two possible values of the constant a.

