Primary Mathematics: Integrating Theory with Practice provides a comprehensive introduction to teaching and learning mathematics in today’s classrooms. Closely aligned with the Australian Curriculum: Mathematics, this text covers the core learning areas of measurement, space and geometry, early number concepts, data and statistics, chance and probability, and patterns and algebra. The text also addresses key considerations for teachers, such as planning, assessment, diversity and teaching outside traditional contexts.

This third edition has been thoroughly revised and features three new chapters focusing on the general capabilities and cross-curricular priorities, implementing STEM strategies in the primary setting, and transition to practice. Each chapter highlights how the theory of teaching mathematics can be put into practice effectively and includes new guided reflective questions and student tasks. Learning is also supported through key term definitions, snapshot case studies and reflection points, while activities throughout each chapter inspire readers to put their knowledge into practice.

Written by an expert author team, Primary Mathematics remains an essential resource that will prepare and excite pre-service teachers for their future as mathematics educators.

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Primary Mathematics
Integrating Theory with Practice

3rd edition

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How to use HOTmaths with this book

Once you have registered your HOTmaths access code, found on the inside front cover of this book, for subsequent visits the below navigation instructions provide a general overview of the main HOTmaths features used within this textbook.

Log in to your account via www.hotmaths.com.au.

Upon logging in you will automatically arrive at your Dashboard. This screen offers you access to **FUNdamentals** (colourful maths games and activities for Foundation to Year 2 students), **Games** and the HOTmaths **Dictionary**. The Dashboard can also be accessed via the icon on the right-hand side of the toolbar at the top of any HOTmaths lesson page.
Different HOTmaths streams can be accessed via the Course list dropdown. You can change the Course list and Course (year level) using the dropdown on the left-hand side of the toolbar.

You can then select a Topic, and finally a Lesson.

Most lessons contain a number of interactive and printable activities, which can be accessed via the links on the right-hand side of the orange toolbar. These include: Resources, Walkthroughs, Scorcher and Questions.
The Resources tab within lessons contain **widgets** (animations) and **HOTsheets** (activities). By clicking on the ‘Number bars’ link, you will access the widget below. Clicking the ‘Addition bingo’ link will give you access to the HOTsheet below.

**Number bars**

**HOTsheet**

**ADDITION BINGO**

Aim

Get 3 counters in a row—across, down or diagonally on the bingo board.

**Things you will need**

Each TEAM will need **two** packs of cards numbered **1 to 9**.

Each PLAYER will need:
- a bingo board
- some counters

**Rules**

- Play in teams of 4 or 5. One person is the **Caller**.
- The **Caller** has the two packs of number cards but does not need a bingo board.
- The others are the players. Each player has a bingo board and some counters.
The toolbar at the top of each lesson page is also the location for the search function, where you can enter the name of any widget or HOTsheet for quick access. The results page will automatically display videos based on the keywords searched, indicated by the videos tab being highlighted in blue. If you are looking for a widget or a HOTsheet, simply click onto the required tab and the results will appear. Using the above widget as an example, searching ‘Number bars’ and clicking on the widget tab will provide you a link to the Number Bars widget. By clicking locations next to the widget name you can see what HOTmaths lessons use the resource. You can also narrow your search results to a specific course, topic or lesson using the dropdown menus.

Throughout this textbook you will find numerous references to resources from HOTmaths. Please note that given its nature HOTmaths is constantly being updated. All pathways and references are correct as of May 2019 and every effort has been made to provide you with an accurate picture of the functions within HOTmaths.
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