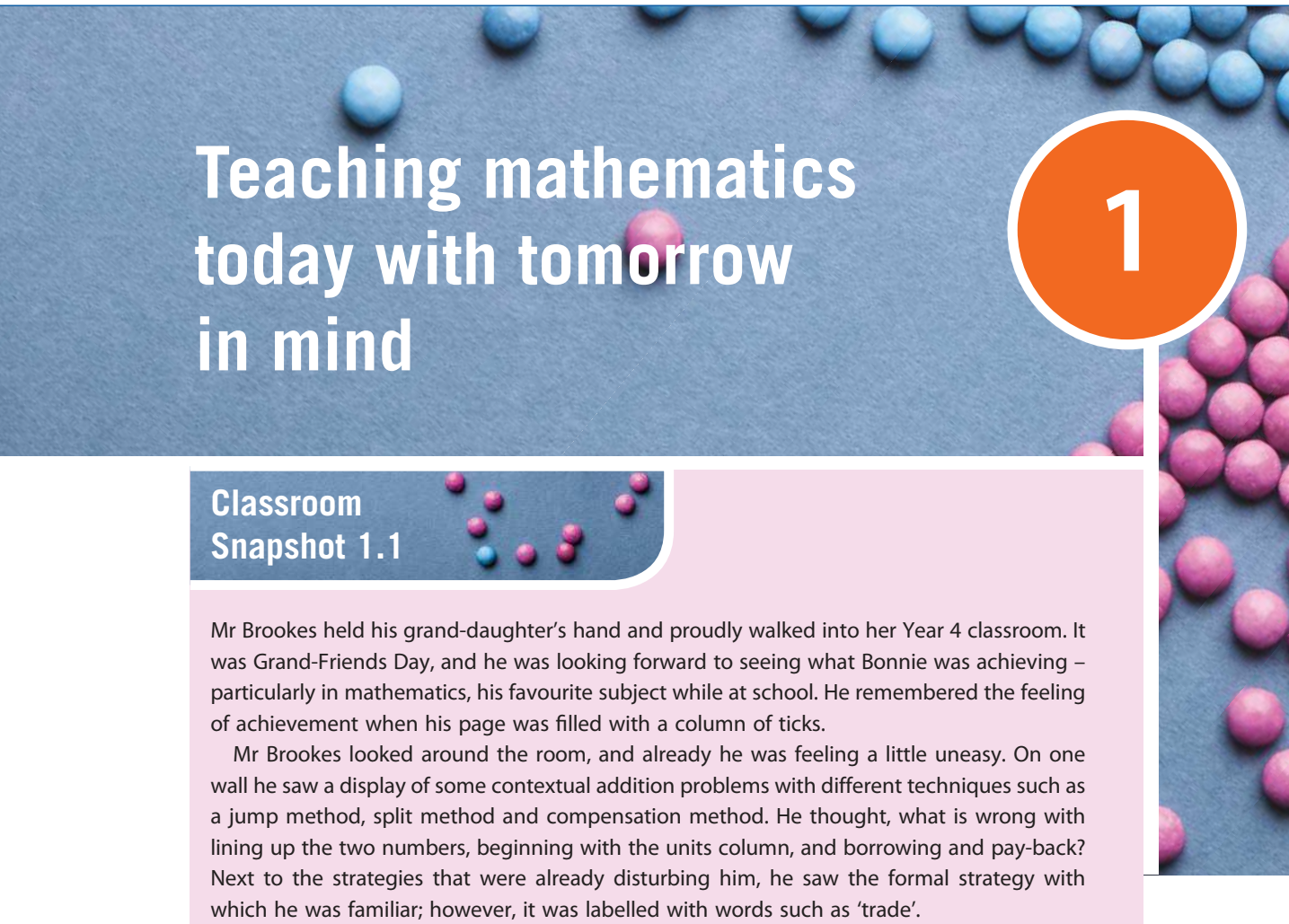

1 Teaching mathematics today with tomorrow in mind

Classroom Snapshot 1.1

Mr Brookes held his grand-daughter's hand and proudly walked into her Year 4 classroom. It was Grand-Friends Day, and he was looking forward to seeing what Bonnie was achieving – particularly in mathematics, his favourite subject while at school. He remembered the feeling of achievement when his page was filled with a column of ticks.

Mr Brookes looked around the room, and already he was feeling a little uneasy. On one wall he saw a display of some contextual addition problems with different techniques such as a jump method, split method and compensation method. He thought, what is wrong with lining up the two numbers, beginning with the units column, and borrowing and pay-back? Next to the strategies that were already disturbing him, he saw the formal strategy with which he was familiar; however, it was labelled with words such as 'trade'.

On closer inspection of the displays, he observed that the students had made their own metre rulers, they had traced around their feet on grid paper and used string to find the perimeter, and they had built as many different rectangular prisms as they could with 24 small cubes. There were group results on the wall of throwing dice and tabulating the results, and another activity that involved strips of paper that were as long as the height of each student, which were used to find the average height of the class by breaking the strips into pieces until they were approximately the same length. In another display, he saw patterns being explored on the 100 chart. He couldn't believe his eyes when he saw times tables presented next to rows and columns of dots.

Mr Brookes was about to ask Bonnie whether he could look in her exercise book when he noticed an electronic board of some kind on the wall where he expected to see the chalkboard. On this board – which Bonnie called the interactive whiteboard – a group of students were predicting the shape that would be made (the cross-section) when they cut an object at particular positions. After choosing the shape, they cut the object with a virtual knife and checked their solution. On another table, two students were sharing an iPad, creating as many different four-sided figures as they could, and then exploring everything they could find out about the shapes. On the far side of the classroom, three students sat at computers and were entering the data about the area and perimeter of the student's feet in the class into a special program. When he turned to see what three other students were doing on computers at the back of the room, he noticed that they were completing a series of review questions against the clock.

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→ The changes that had occurred in the classroom since Mr Brookes had visited his own children's classroom were undeniable. He could see that the students had been doing mathematics, but he was astounded by the variety of the concepts covered, how the tasks were accessible to all students, the level of engagement in the class, and the use of concrete materials with computers and mobile technology. Bonnie proudly shared her achievements, but they weren't about the number of ticks in her maths book. Instead, she showed her mathematics discoveries and the interesting findings she had recorded electronically in her maths journal, stored on the school network.

Today's classroom

Teacher education, at all levels, requires a resource that will address issues surrounding the teaching of mathematics within the broad and ever-changing context of particular classroom settings. To meet this need, this book is grounded in empirically evidenced developmental models and linked closely to practical classroom practice. While many classrooms have been resourced with equipment such as base-10 materials, counters, shape kits, mobile devices, dice kits, drawing tools and interactive whiteboard (IWB) technology, extensive professional development is required to enable the range of classroom resources to be transformed into teaching tools. The difficulty faced by the teaching profession is in integrating a wide range of hands-on concrete materials with **information and communication technology (ICT)** to weave a pedagogically sound learning sequence. Technological change has occurred at an extremely fast pace, which means that many educators across the field without sufficient development of skills in this area have been forced into adopting teacher-centred techniques in an effort to use the varied range of educational resources available in our changing classrooms. This book provides the opportunity to meet this challenge and provide mathematics teachers and pre-service teachers with detailed teaching activities that are designed with developmental models as their basis. The aim is to provide you with a sensible and achievable integration of available educational tools, with research-based approaches to mathematical development that provide for the mathematical needs of all learners. As such, it is intended for primary pre-service teachers, and teachers looking for ways to enhance their teaching of primary mathematics, to assist them to design student tasks that are meaningful and to use educationally sound ways to improve their mathematics teaching.

Information and communication technology (ICT) the skills and knowledge required to use ICT effectively and appropriately to access, create and communicate information and ideas, solve problems and work collaboratively, and to make the most of the digital technologies available

In writing the book, we were guided by a moral imperative: we wanted students to be engaged in tasks designed to develop mathematical conceptual understanding in purposeful ways. Pre-service teachers are asked to consider the wealth of strategies and resources available to enhance students' learning. It is hoped that this book will encourage you to teach school children to become successful learners who are prepared for and responsive to the dynamic demands of the future.

As the expectation of accreditation increases, teachers around the world are being asked to consider issues such as student diversity, behaviour management and assessment for learning techniques within the key learning areas. Another contentious – but very real – issue is the impact of external assessment 'of' learning, in the form of national testing. Mathematics teachers are asking, 'How can we use this information to enhance the mathematics learning in our classroom?'

In many countries, we have reached a crisis point in relation to staffing schools with mathematics teachers in rural and remote areas. The problem is not being addressed, and 'out of field' teachers of mathematics – such as primary-trained teachers working in the secondary setting – need a tool kit to assist them to design student-centred mathematics activities. As in any teaching area, if the appropriate knowledge base is not strong, teachers resort to teaching in the manner in which they were taught in an effort to survive the situation. Australia and the wider Pacific region are not in a position to fill these vacancies with qualified teachers of mathematics. We need to accept this situation and to consider ways of supporting 'out of field' teachers while the workforce in this area grows.

Throughout the remaining 17 chapters of this book, the contents of each chapter are informed by evidenced-based research into how students develop mathematical concepts and how to create teaching environments conducive to engagement in mathematical learning. The TPACK framework below provides a structure that supports the consideration of different components of teachers' knowledge and our aim to enhance this knowledge by integrating theory and practice.

The TPACK framework

Technological Pedagogical Content Knowledge (TPACK) is a framework that builds on Shulman's (1987) formulation of Pedagogical Content Knowledge (PCK); it describes how teachers' understanding of technology and pedagogical content knowledge interact with one another to produce effective teaching with technology (Koehler & Mishra 2008a, 2008b). As shown in Figure 1.1, there are seven components to the framework, which can be summarised as follows.

Technological knowledge (TK)

TK refers to knowledge about various technologies, so it is continually in a state of flux as technology constantly changes. A person with good technological knowledge would be able to:

- apply it broadly in their everyday life and at work
- recognise when it could be used to assist in the achievement of a goal
- continually adapt to changes in new technology.

Content knowledge (CK)

CK is knowledge about the actual subject-matter that is to be taught or learnt and includes knowledge of concepts, theories and ideas, and evidence and proof. In mathematics

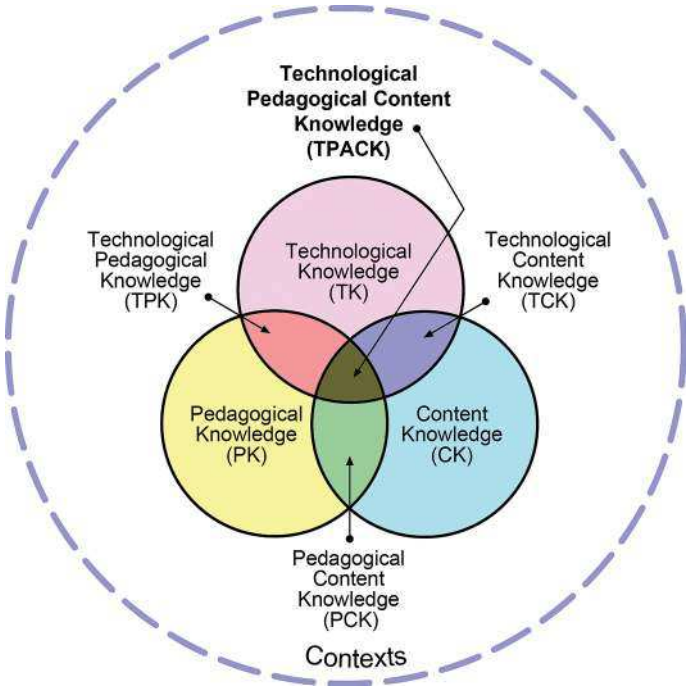

Figure 1.1 TPACK framework
Source: Reproduced by permission of the publisher, © 2012 by tpack.org

education, concerns are often raised about the level of pre-service and primary teachers' knowledge of mathematics, and there is an ongoing debate about what level of mathematics is required to teach effectively in these areas. The American Council on Education states that 'a thorough grounding in college-level subject-matter and professional competence in professional practice are necessary for good teaching . . . students learn more mathematics when their teachers report having taken more mathematics' (cited in Mewborn 2001, p. 28). However, it appears that simply having studied mathematics at a higher or even advanced level before undertaking teacher training does not necessarily equate with having effective content knowledge. In their wide-scale study into effective teaching of numeracy, Askew and colleagues (1997) found that there was a lack of evidence to support a positive association between formal mathematical qualifications and pupil gains, and that even teachers with high-level mathematics qualifications displayed knowledge that was compartmentalised and framed in terms of standard procedures, without underpinning conceptual links. Content knowledge is important as 'you cannot teach what you do not know' (Rowland et al. 2010, p. 22), but considering other forms of teacher knowledge is also relevant.

Pedagogical knowledge (PK)

PK is knowledge about the processes and methods of teaching and is a generic form of knowledge that applies to student learning, classroom management, lesson plan development and implementation, and student evaluation. PK requires an understanding of

cognitive, social and developmental theories of learning and how they apply to students in the classroom (Koehler & Mishra 2008b).

Pedagogical content knowledge (PCK)

Shulman (1987, p. 8) defines PCK as 'an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction'. A teacher with good PCK in mathematics would be aware of likely student misconceptions, the importance of recognising and catering for students' prior knowledge and how to make connections between different topics within mathematics.

Technological content knowledge (TCK)

TCK can be defined as an understanding of how technology and content influence and constrain one another. Teachers need to have a good understanding of which specific technologies can be used to create new representations and how the content can dictate or even change the ways in which these technologies are used. Technology can also be used to make the content accessible to students – perhaps in more ways than traditional methods allowed. TinkerPlots software (Konold & Miller 2005), for example, allows younger students to access many graphing concepts that traditionally were introduced in secondary school. Similarly, graphics calculators have allowed students to access higher level mathematical concepts that traditionally were the domain of upper secondary studies.

Technological pedagogical knowledge (TPK)

TPK represents an understanding of the way in which teaching and learning change when particular technologies are used. An important part of TPK is developing creative flexibility with available tools in order to use them for specific pedagogical purposes. Chapter 2 shows how a teacher used the IWB to effectively scaffold students' learning about bridging 10 using 10-frames. Tools such as the IWB can be used to change how teachers teach, and TPK requires an understanding of the use of technology – not for its own sake, but for that of advancing student learning and understanding.

Technological pedagogical content knowledge (TPACK)

TPACK is an understanding that emerges from an interaction of technology, content and pedagogy knowledge, and is different from knowing all three concepts individually. According to Koehler and Mishra (2008b, pp. 17–18), TPACK represents:

- an understanding of the representation of concepts using technologies
- pedagogical techniques that use technologies in constructive ways to teach content
- knowledge of what makes concepts difficult or easy to learn and how technology can redress some of the problems that students face
- knowledge of students' prior knowledge and theories of epistemology
- knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones.

While this book is relevant to the teaching of mathematics in many nations, particular reference is made to the Australian curriculum documents. However, the content can be related to most primary mathematics curriculum structures. The Australian Curriculum: Mathematics has three content strands: Number and Algebra; Measurement and Geometry; and Statistics and Probability. The syllabus includes stage outcomes, stage statements, content outcomes, content, background information and language sections. The curriculum documents include support materials such as assessment and teaching mathematics for children with special needs. Readers are encouraged to relate the ideas explored in this book to their own context and curriculum structure.

The next section provides the reader with an advance organiser that presents a summary of the key objectives of the remaining 17 chapters in the book.

Summary of chapters

Chapter 2: Exploring early mathematical development examines the key ideas associated with learning early number concepts. Readers will become familiar with counting principles and how they underpin number understanding. The capacity of learning frameworks and curriculum documents to guide teaching early number concepts is explored. The effective use of technology to develop an understanding of early number concepts is examined through active engagement in a number of different activities designed to help you think about and evaluate the role of technology as a teaching tool.

Chapter 3: Exploring measurement presents the measurement sequence used to introduce and develop an understanding of each measurement attribute – that is, length, area, volume and capacity; angle, mass, time, temperature; and money and value. The principles of measuring with units will be explored and the importance of developing estimation skills and meaningful benchmarks emphasised. Strategies for utilising technology to support, develop and extend students' measurement experiences are addressed.

In *Chapter 4: Exploring geometry*, we consider the breadth of concepts included in the geometry section of curriculum documents generally, and in the Australian Curriculum: Mathematics in particular. The chapter presents a theoretical framework, known as the van Hiele theory, as a lens through which to view students' geometrical thinking and a pedagogical framework that is useful for designing sequential student tasks to assist students to grow in their understandings of geometrical concepts. The important role of language and maintaining 'student ownership' of the geometrical ideas is explored, as is the use of technological tools to enhance our teaching of geometrical concepts for the e-generation.

The difference between additive and multiplicative thinking and the appropriate use of drill and practice activities are both explained in *Chapter 5: Exploring whole number computation*. This chapter explores various representations that teachers can use to illustrate different ways of thinking and the effective use of the range of technological tools available to explore whole number computation.

Chapter 6: Part-whole numbers and proportional reasoning investigates the importance and representation of part-whole numbers such as fractions, decimals and percentages.

The importance of understanding proportional reasoning concepts in daily life is explored, together with strategies for using technology effectively in this domain.

Underpinning *Chapter 7: Exploring patterns and algebra* is an understanding of the central importance of patterns in early childhood and primary school mathematics, and the importance of mathematical structure and its relevance to children's learning of mathematics. The chapter explores the process of using sequences effectively to find and justify rules and to explain phenomena. Strategies for representing and resolving number sentences, equivalence and equations are presented. Effective ways to use technology to explore algebraic situations where students are encouraged to describe relationships between variables are investigated.

Chapter 8: Exploring data and statistics examines suitable statistics questions for investigation by children of different ages, using a cycle of problem, plan, data, analysis and conclusion (PPDAC) (Wild & Pfannkuch 1999). The importance of variation in data and different types of variables and the difference between a population and a sample are investigated. Readers will explore different ways of displaying data to 'tell a story'. The importance of drawing inferences from data and the uncertainty associated with these inferences are discussed. Readers will engage in activities that use technology to support the development of statistical understanding.

Chapter 9: Exploring chance and probability begins with a consideration of the difference between objective and subjective views of probability. A range of tools for investigating probability are explored, and applications of probability in daily life are provided. Strategies for using technology effectively to develop ideas about uncertainty in the primary classroom are presented.

Chapter 10: Capitalising on assessment for, of and as learning focuses on the notion of assessment 'for', 'of' and 'as' learning, and how these forms of assessment work together in the mathematics classroom. A developmental framework to assist in designing an assessment item and assessing the quality of a student's response – the Structure of the Observed Learning Outcome (SOLO) model (Biggs & Collis 1982) – is presented. Issues surrounding national testing data are raised in the light of positive ways to support growth in mathematical understanding. Readers will engage with various ICT tools that can assist in creating valid assessment items.

Chapter 11: Planning for mathematics teaching in the 21st century classroom considers the role played by digital technologies in today's classrooms and how ICT impacts upon mathematics teaching. The chapter explores the ways in which ICT is described and integrated in curriculum documents and strategies to support students in their use of ICT. Activities in this chapter demonstrate the way technology can be incorporated into classroom routines to enhance learning experiences for students.

Chapter 12: Diversity in the primary mathematics classroom raises issues concerning the complexity of primary mathematics classrooms and a range of potential barriers to learning mathematics experienced by many students. Strategies for planning for diversity in the mathematics classroom are explored; these utilise the potential of technology to meet all learners' mathematical needs.

Chapter 13: General capabilities and cross-curriculum priorities describes the seven general capabilities and three cross-curriculum priorities that are named in the Australian Curriculum and suggests ways in which they can be addressed through appropriate mathematics teaching. A range of contexts is presented that demonstrate how the general capabilities and cross-curriculum priorities can be accessed through mathematics, and vice versa.

Chapter 14: STEM in the primary setting explores the nature of STEM in the primary context and considers issues and debates pertinent to the implementation of a STEM strategy. Teaching strategies such as coding and robotics are presented, alongside practical real-life mathematical scenarios that link STEM-related careers and primary students' life experiences.

In *Chapter 15: Surviving as an 'out of field' teacher of mathematics* we identify issues pertinent to 'out of field' teachers of mathematics. Strategies are offered to assist 'out of field' teachers of mathematics while considering the invaluable knowledge a primary-trained teacher can bring to the secondary mathematics classroom. Examples are presented of teachers' critical pedagogical content knowledge that has a high impact on students' growth and development across the main strands of the curriculum.

Chapter 16: Teaching mathematics beyond the urban areas delves into the challenges and rewards of working in remote areas of countries such as Australia and New Zealand, and small Pacific nations. Teaching strategies are presented to assist in maintaining a positive learning environment in remote and small Pacific-nation classrooms. The importance of the relationships among and between parents, students, teachers and other community members is explored, along with practical suggestions for making the most of the available resources. The chapter demonstrates that, even with limited or no internet access, ICT can still be utilised as a tool to enhance and extend upon more traditional classroom materials; it is about making the most of the tools and resources that are available.

Chapter 17: Considerations for implementing ICT in the mathematics classroom reacquaints readers with the TPACK framework and the different types of teacher knowledge required for effective teaching for numeracy. It provides opportunities to develop strategies for evaluating and reflecting upon teachers' own use of technology. The notion of a community of practice is explored, as are ways to establish one within your school. Strategies are presented for evaluating the use of online resources, and insights are provided into the change process and how school-wide approaches to teaching mathematics with ICT can be sustained.

Chapter 18: Becoming a teacher of mathematics addresses the challenges associated with moving from being a pre-service teacher into being a member of the teaching profession. It explores ideas associated with testing and the pressures this can place on students and teachers; collecting evidence of teaching effectiveness for both accreditation and personal development; keeping a professional portfolio; and becoming a full member of a professional learning community. At all times mathematics, and its teaching and learning, is at the centre of the discussion.

How to use this book

Each of the chapters includes common key components that provide different ways to consider the material.

- *Learning outcomes* describe the learning that is expected as a result of engaging with the different components of the chapter.
- *Key terms* identify and define the common terms that are used throughout the chapter.
- *Classroom snapshots* are short vignettes of real classroom situations that exemplify some pedagogical principles, including technology use.
- *Pause and reflect* sections are points where the reader is asked to think deeply about some specific issue or dilemma.
- *Activities* present a task or short investigation to enable the reader or students to become familiar with a tool or way of presenting mathematics in a technological environment.
- *Guided student tasks* present practical tasks at the end of the chapter to further explore mathematics education concepts and issues.
- *Further reading* sections at the end of each chapter provide some of the research background, as well as places to find other ideas and material.

As with all texts, the more engagement there is with the ideas and activities presented, the better the learning is likely to be. Educational resources are changing so fast that this book can present only what is available at a single point in time. Teachers starting their careers will find that what is new and cutting edge when they first enter the classroom is obsolete before they are ready to retire. Bonnie's grandfather probably used a slate and chalk when he started school. Bonnie's parents used a ballpoint pen and saw the introduction of whiteboards instead of chalkboards. In three generations, we have changed teaching from literally 'chalk and talk' to a dynamic and creative endeavour, in which the teacher and students are co-learners and partners. It is difficult to predict what skills and knowledge school students will need in the future, but it is hoped that this book will provide teachers with ways in which they can apply educational research to inform teaching practices that inspire mathematical development.

Companion website

Visit the book's companion website, www.cambridge.edu.au/academic/primarymathematics, to find links to a range of useful resources. This margin icon is used throughout the book to indicate that a weblink relating to the content under discussion or websites for exploration are available on the companion website. The weblink number can be used to help you easily identify the link on the companion website.