KANT'S MATHEMATICAL WORLD

Kant's Mathematical World aims to transform our understanding of Kant's philosophy of mathematics and his account of the mathematical character of the world. Daniel Sutherland reconstructs Kant's project of explaining both mathematical cognition and our cognition of the world in terms of our most basic cognitive capacities. He situates Kant in a long mathematical tradition with roots in Euclid's *Elements*, and thereby recovers the very different way of thinking about mathematics which existed prior to its "arithmetization" in the nineteenth century. He shows that Kant thought of mathematics as a science of magnitudes and their measurement, and all objects of experience as extensive magnitudes whose real properties have intensive magnitudes, thus tying mathematics directly to the world. His book will appeal to anyone interested in Kant's critical philosophy – his account of the world of experience, his philosophy of mathematics, and how the two inform each other.

DANIEL SUTHERLAND is Associate Professor of Philosophy at the University of Illinois at Chicago. He has published numerous articles on Kant's philosophy of mathematics and philosophy of science, including their relation to Euclid, Newton, Leibniz, Frege, and others.

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Mathematics, Cognition, and Experience

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PREFACE AND ACKNOWLEDGMENTS

I began this work far longer ago than I would like to admit, but at a fairly specific moment. After a restorative year away from graduate school, I moved to Santa Monica into a rent-controlled apartment the size of a postage stamp with a partial view of the Pacific. With fresh mind and heart, I embarked on a project to better understand Kant's philosophy of mathematics and science, and in particular his distinction between constitutive and regulative principles. I began with the constitutive mathematical principles of experience, and hence the Axioms of Intuition in the *Critique of Pure Reason*, but I fairly quickly realized that previous commentators had not properly understood this part of the *Critique*, nor appreciated its full significance. Kant's references to magnitude and to a homogeneous manifold in intuition seemed to me to indicate an unrecognized depth to his views with implications not just for the applicability of mathematics, but also for pure mathematical cognition and the mathematical character of the world of experience. Most importantly, it almost immediately suggested to me a new understanding of the role of intuition in mathematics in representing magnitudes, a role made necessary by Kant's understanding of the limits of conceptual representation. That was in 1992. These many years later, this book is an attempt to describe Kant's theory of magnitudes and its foundational role in Kant's account of mathematical cognition and our cognition of the world; I hope it will appeal to those interested in Kant's critical philosophy and its development as well as to scholars of Kant's philosophy of mathematics and to philosophers of mathematics interested in the history of their field. I aim to publish another work in the near future that is more narrowly focused on the implications of Kant's theory of magnitudes for his philosophy of geometry, arithmetic, algebra, and analysis and his philosophy of natural science.

My investigation of Kant's views of magnitude did not blaze an entirely new path, but significantly broadens it. Reading of "Kant's Theory of Geometry" by Michael Friedman and "Kant's Philosophy of Arithmetic" by Charles Parsons first inspired me to work on Kant's philosophy of mathematics and science, and Friedman's *Kant and the Exact Sciences* provided a crucial springboard. To my knowledge, Friedman was the first to draw a connection between Kant's theory of magnitudes and the Eudoxian theory of magnitudes found

in Euclid, and Friedman's interpretation of *quanta* and *quantitas* and Parsons' "Arithmetic and the Categories" were the starting point for my own investigations. This book is deeply indebted to Michael's and Charles' work, and to their help, support, and advice over the years, and especially to Michael for a postdoc in the History and Philosophy of Science program at Indiana University, and to Charles for regular conversations on two visits to Harvard, one as a graduate student and one as a visiting professor. Sometime later, Bill Tait's work and many pleasant conversations with him in Chicago, often marked by stimulating disagreement, have also been immensely valuable. But the generous help I received from these philosophers would have been for naught without the patient guidance and support of those at UCLA, especially Robert W. Adams, Tyler Burge, John Carriero, and Calvin Normore. Bob provided continual constructive feedback on drafts and had faith in me when I was struggling to find my way and lacked faith in myself; John was a terrific critical advisor, and pushed me to share my work with Friedman and Parsons; memorable conversations with Tyler helped me sharpen my core arguments; and Calvin proved a remarkable resource, imparting crucial historical insights even while racing to the airport to catch a flight.

I can't possibly list all those who have helped me in the decades since this project began; moreover, the assistance of some will be apparent only in a subsequent book focused on some of the details of Kant's philosophy of mathematics. While on a postdoc at the History and Philosophy of Science Department at Indiana University in 1998–9, regular talks with Michael Friedman, Andrew Janiak, and Konstantin Pollock helped me further work out my views. In the early 2000s, Emily Carson, Lisa Shabel, and I, sometimes joined by Ofra Rechter, gathered to share and discuss work; I hope the valuable feedback and moral support I received was in some measure returned. Bill Hart was always willing to discuss the philosophy of mathematics with me, and also generously helped me learn to better communicate my ideas. I have also benefited from the work of, and in-depth conversations with, Lanier Anderson, Vincenzo De Risi, Katherine Dunlop, and Jeremy Heis on Kant's philosophy of mathematics. My debts include the innumerable insightful comments I received during many presentations, all of which I cannot list here, but I would like to single out those who provided me the opportunity to present my views over multiple talks, which proved particularly helpful: Matt Boyle at Harvard and at the University of Chicago, Jim Conant at the University of Chicago, Vincenzo De Risi at the Max Planck Institute in Berlin, the sorely missed Mic Detlefsen at Notre Dame and in France, Michael Friedman at Stanford, Jeremy Heis at UC Irvine, and above all Ofra Rechter and Carl Posy, whose conferences in Tel Aviv and Jerusalem were unique opportunities to advance our collective understanding of Kant's philosophy of mathematics. To them and the participants at these and the many other talks, I am grateful for helpful comments.

It is impossible for me to now recollect all those who gave me particularly helpful one-on-one feedback over the years, but they include all those mentioned above as well as Mahrad Almotahari, Andrew Chignell, Kevin Davey, Lisa Downing, Katherine Dunlop, Walter Edelberg, Stephen Engstrom, Sam Fleischacker, Marcus Giaquinto, Aidan Gray, Sean Greenberg, W. D. Hart, Richard Heck, Robert Howell, Peter Hylton, Vèronique Izard, Anja Jauernig, Michael Kremer, Tony Laden, Allison Laywine, Ian Mueller, Tyke Nunez, Marco Panza, Robert Pippin, Andrew Pitel, Erich Reck, Tom Ricketts, Vincenzo di Risi, Sally Sedgwick, Lisa Shabel, Karl Schafer, Houston Smit, Daniel Smyth, Clinton Tolley, Daniel Warren, and Eric Watkins. I'd also like to thank the graduate students at University of Chicago, Harvard, Stanford, and especially UIC for aid in clarifying and correcting my views over the last twenty years.

I owe special thanks to several for their help with specific chapters in this book: Matt Boyle and Tyler Burge for helpful discussions and correspondence and Thomas Land for detailed comments on Chapter 4; Matt Boyle for prompting me to write Chapter 5; Tyke Nunez, Thomas Land, and Daniel Smyth for very helpful responses on a penultimate version; Chen Liang for conversations that helped me develop my thoughts; and Matt Boyle and Tyler Burge for crucial comments that changed my views. I hope I have been able to incorporate and respond to their criticisms; they bear no responsibility for anything that might be problematic. Marco Panza and Ken Saito provided useful comments and correspondence on the foundations of Euclidean geometry for Chapter 6; Ian Mueller and Vincenzo De Risi had an outsized influence on my understanding of Euclid and the Euclidean tradition in Chapters 6 and 9; while Kevin Davey kindly read the same chapters and helped me avoid a few mistakes and unclear statements. Andrew Pitel provided helpful feedback on Chapter 8.

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Although most of this book thoroughly reworks and supersedes my earlier published views, some of it still draws from those articles. An earlier version of

Chapter 2 appeared as "The Point of the Axioms of Intuition," *Pacific Philosophical Quarterly* 86 (2005): 135–59. I am grateful to the editors for permission to borrow from this article. Chapter 3 and parts of Chapter 4 are based on "The Role of Magnitudes in Kant's Critical Philosophy," *Canadian Journal of Philosophy* 34(4) (2004): 411–42. Thank you to the editors of the *Canadian Journal of Philosophy* for permission to draw from this article. The core of Chapter 7 is found in "Kant's Philosophy of Mathematics and the Greek Mathematical Tradition," *Philosophical Review* 113 (2) (2004): 157–201; I am grateful to the editors of *Philosophical Review* for permission to publish the present version. Finally, Chapter 8 is based on part of "Philosophy and Geometrical Practice in Leibniz, Wolff, and the Early Kant," in *Discourse on a New Method: Reinvigorating the Marriage of History and Philosophy of Science*, edited by Michael Dickson and Mary Domski (Chicago: Open Court, 2010). I would like to thank Open Court for permission to use it.

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