

## Physical Principles of Sedimentary Basin Analysis

Presenting a rigorous treatment of the physical and mechanical basis for the modeling of sedimentary basins, this book supplies geoscientists with practical tools for creating their own models. It begins with an introduction to the properties of porous media, linear elasticity, continuum mechanics and rock compressibility – providing a thorough grounding for their use later in the text. A chapter on the modeling of burial histories is then followed by a series of chapters on heat flow, subsidence, rheology, flexure and gravity, which consider sedimentary basins in the broader context of the Earth's lithosphere. Later chapters then cover the topics of pore space cementation, compaction and fluid flow.

This volume introduces basic, state-of-the-art models and demonstrates how results can be easily reproduced with simple tools such as MATLAB and Octave (codes are available online at [www.cambridge.org/9780521761253](http://www.cambridge.org/9780521761253)). Throughout the book the main equations are derived from first principles, and their basic solutions are obtained and then applied. More technical details are supplied in notes, and the text is illustrated with real-world examples, applications and test exercises. This book is therefore a key resource for graduate students, academic researchers and oil industry professionals looking for an accessible introduction to quantitative modeling of sedimentary basins.

MAGNUS WANGEN has worked in the field of sedimentary basin-modeling since the late 1980s – conducting research on a wide range of topics. He obtained a Dr. Scient. degree in applied mathematics from the University of Oslo in 1993 with a thesis on the modeling of heat and fluid flow in sedimentary basins. Since the early 1990s he has developed two complementary basin simulators used by the oil industry. The first simulator deals with heat flow on a lithospheric scale, fluid flow, compaction and overpressure in sedimentary basins through the geohistory, while the second simulates hydrocarbon generation and migration. He is currently a research scientist at the Institute for Energy Technology in Norway. This book is based on a course in basin analysis that Dr. Wangen taught for a number of years while an assistant professor at UNIK (an affiliate of the University of Oslo at Kjeller).

PHYSICAL PRINCIPLES  
OF SEDIMENTARY BASIN  
ANALYSIS

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*To Ingeborg and Reidar*

We often fail to realize how little we know about a thing until we  
attempt to simulate it on a computer.  
*Donald Knuth, The Art of Computer Programming*  
(vol. 1, 3rd edn., p. 298)

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## Preface

*God is in the details.  
The devil is in the details.  
Ludwig Mies van der Rohe (among others)*

This book is based on lecture notes about the physical processes that govern sedimentary basins. The notes were the basis for a one-semester seminar named “Heat and fluid flow in sedimentary basins” offered by UNIK, an affiliate of the University of Oslo at Kjeller. As the title suggests, this book is about the physical principles of processes in sedimentary basins, for instance, heat and fluid flow. The subject is approached by deriving the basic equations from fundamental principles such as mass and energy conservation. The equations are then solved for simple problems that give insight into the processes.

It should be possible to reproduce most of the solutions, calculations and plots presented in the book with a modest effort and basic computer facilities. Reproduction of the results is the only way to ensure that the results are correct.

The book is written primarily for students who want to study heat and fluid flow in sedimentary basins from a physical point of view, and need to do their own modeling. The book requires some background in mathematics, and knowledge of continuum mechanics is an advantage. The reader should be familiar with calculus and linear algebra. It would be advantageous to be familiar with partial differential equations like the heat equation, Fourier series and complex numbers. As long as the reader is familiar with differentiation and integration, and has a sufficient interest in mathematics, she or he should be able to follow the derivations. Several linear (partial) differential equations are solved, but all details are provided. The aim has been to make the book as self-contained as possible by deriving all results that are presented. Details are necessary in this respect, in order to make the text complete and self-contained.

The book is meant as an introduction to and a primer for modeling, and it therefore covers the basic (state-of-the-art) models. It is not meant to cover the latest developments in the various fields. It does not attempt to cover the historical development of the various subjects either. This is reflected by the reference list, which easily could have been expanded 10 or 100 times. Each chapter has a last section with a few references that may serve as a starting point for further reading. A problem with writing such a book is to decide what

should be included and what should be left out. Important topics such as sedimentology, seismics, diagenesis and models for hydrocarbon generation and migration are left out.

Examples and applications of the models are shown. But geological processes are often very complex and specific examples often have several more aspects than those captured by the actual model. These other aspects are mentioned, but a discussion often leads far beyond this book and into special disciplines like for instance sedimentology, structural geology, geochemistry or petrology. The purpose of the examples is to show how the models work with real data, and the setting is therefore chosen to be as simple as possible.

This book can be used in different ways depending on the goals, the students' background and the amount of lecturing per week. It is, for example, possible to take two main routes through the book, one with respect to subsidence, rheology, flexure and gravity (Chapters 7, 8, 9 and 10) and another with respect to fluid flow (Chapters 11, 12, 13, 14 and 15). Common for both routes are the following chapters: properties of porous media, continuum mechanics, burial histories and heat flow (Chapters 2, 3, 5 and 6 respectively). Details of derivations are provided in notes that follow the sections, which may be left to the students to go through.

Inevitably, some errors will remain in this book, and in order to correct them I ask that any that are noticed are reported. It would also be great to know if there are alternative and simpler derivations than the ones presented or if there are better examples. Any suggestions that could improve the book will be greatly appreciated.

It is my hope that this book will be useful for anyone interested in a quantitative modeling of processes related to sedimentary basins and Earth science.

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