

## Chapter 1

# Sound propagation in a simplified sea

### Summary

Sound is a mechanical disturbance that travels through a fluid. The sound wave can be a short-duration pulse or a continuous wave oscillation (CW) that is usually, for simplicity, sinusoidal. Because most detectors of ocean sounds are pressure sensitive devices, the propagating disturbance is most often identified as a time-varying incremental pressure, i.e., an *acoustic pressure*. Sometimes the description is in terms of the incremental density, the incremental temperature, the material displacement from equilibrium, or the transient particle velocity of the sound.

In this chapter we assume that the medium is *homogeneous* (same physical properties at all points) and *isotropic* (same propagation properties in all directions). We also assume that there is no sound *absorption* (no sound energy conversion to heat) and no *dispersion* (no dependence of sound speed on sound frequency). And we assume that the acoustic pressure increment is very, very small compared to the ambient pressure (no finite amplitude, non-linear effects).

Several wave phenomena that occur in the sea will be discussed here. When sound encounters an obstacle, it is *scattered*; part of the scattered energy bends around the obstacle (this is called *diffraction*) and part is backscattered toward the source; when it is incident on a boundary surface where it meets a different density or different sound speed, some *reflection* occurs, accompanied by some *refraction* (i.e., transmission at an angle different from the incident angle). When a sound wave meets another sound wave, the two pressures may add constructively or destructively, and *interference* ensues.

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We seek to exploit the capabilities of underwater sound to discover more about the world's oceans and its inhabitants. This is done by interpreting the behavior of several descriptors of sound such as its pressure amplitude, particle velocity, density, intensity, radiated power, and propagation speed, all of which are introduced in this chapter. Also in this chapter, we study the effect of the ocean on sounds, including the basic behaviors such as: “reflection” from ocean surfaces; “refraction” of sound rays (the bending caused mostly by the spatial variation of the water temperature and salinity); “interference,” (the superposition of sounds from different sources, or sounds that have traveled different paths from the same source). These same phenomena may be known from one's study of optics, but the omnipresent effects are crucially important to understand and interpret sounds in the sea.

Various simplifying approximations are developed at this time to derive the sound propagation equations that describe the effects of the ocean surface, volume, and bottom. We discuss waves in plane, cylindrical, and spherical coordinate systems, which will be needed to express the propagation at sea simply and appropriately to the sources and the environment.

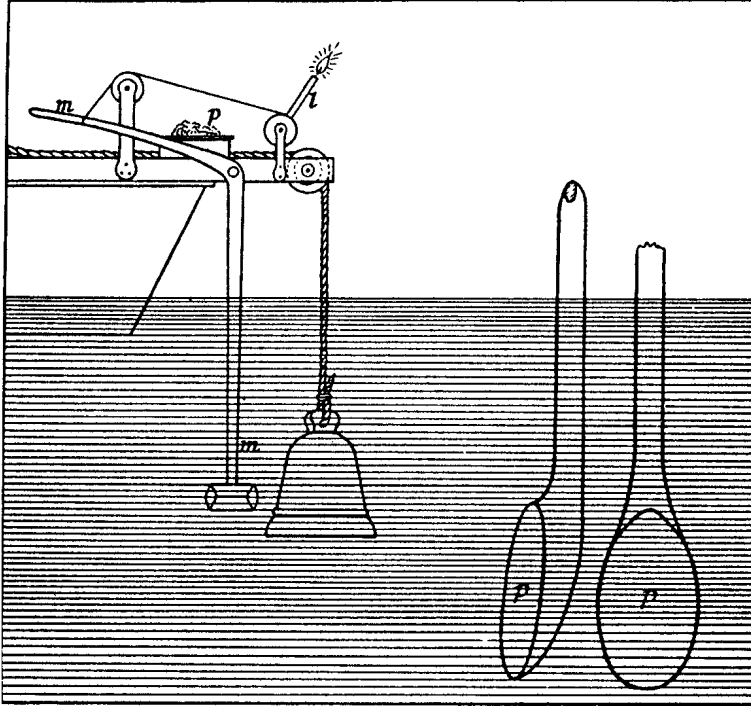
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### 1.1 Sound speed in water

Knowing the sound speed in water is critical to ocean communication and much of biological and geophysical ocean research. The earliest measurement was by Colladon and Sturm (1827) in the fresh water of Lake Geneva, Switzerland (Fig. 1.1). A value of 1435 m/s was found, but it was soon realized that the speed in saline water is somewhat greater than this and that, in general, the temperature of the water is an even more important parameter than salinity.

\* This section contains some advanced analytical material.



**Fig. 1.1.** Colladon and Sturm's apparatus for measuring the speed of sound in water. A bell suspended from a boat was struck under water by means of a lever *m*, which at the same moment caused the candle *l* to ignite powder *p* and set off a flash of light. An observer in a second boat used a listening tube to measure the time elapsed between the flash of light and the sound of the bell. The excellent results were published in both the French and German technical literature. (*Annales de Chimie et de la Physique* 36, [2], 236 [1827] and Poggendorff's *Annalen der Physik und Chemie* **12**, 171 [1828].)

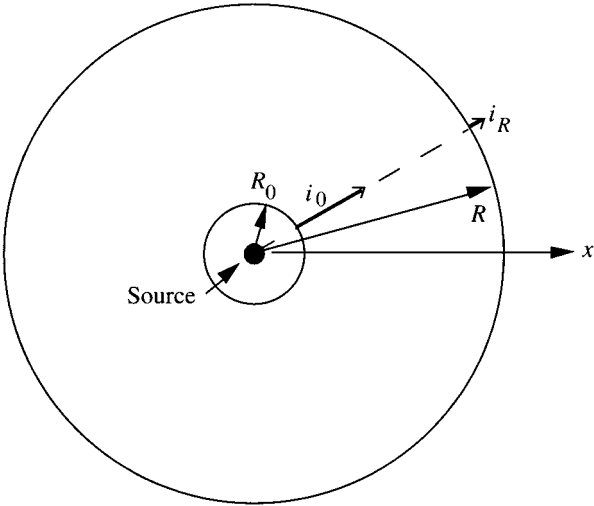
Numerous laboratory and field measurements have now shown that the sound speed increases in a complicated way with increasing temperature, hydrostatic pressure, and the amount of dissolved salts in the water. A very simple formula for the speed in m/s, accurate to 0.1 m/s, but good only to 1 kilometer depth, was given by Medwin (1975),

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 \\ + (1.34 - 0.0107T)(S - 35) + 0.016z \quad (1.1)$$

In (1.1), temperature *T* is in degrees centigrade, salinity *S* is in parts per thousand of dissolved weight of salts, and the depth *z* is in meters. A better, longer, but still simple expression (Mackenzie, 1981) is in Chapter 2. The best equation (Del Grosso, 1974) involves some 19 terms containing coefficients with 12 significant figures.

Portable sound “velocimeters,” which measure the time of travel of a megahertz pulse, have an accuracy of 0.1 m/s in non-bubbly water. But everpresent microbubbles, which are not considered in any of the sound speed equations above, can cause the actual speed of propagation at frequencies below about 100 kHz to be different from the velocimeter readings by tens of meters/s, particularly near the ocean surface, see Chapter 6.

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**Fig. 1.2.** Spherical spreading of a pulse wave front. The instantaneous intensity is  $i_0$  at the radius  $R_0$  and later is  $i_R$  at the radius  $R$ .

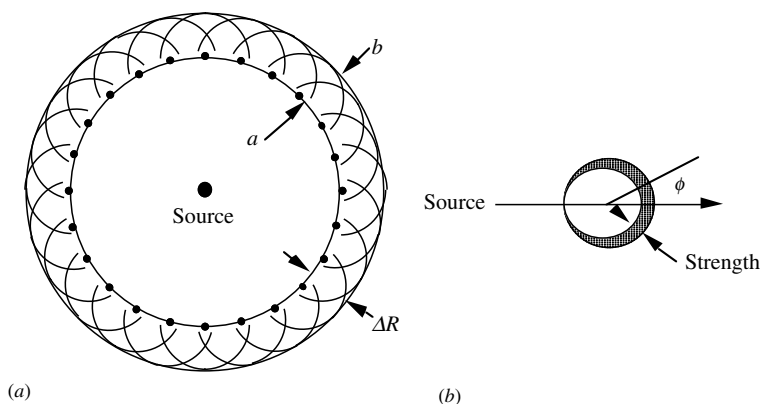
## 1.2 Pulse wave propagation

### 1.2.1 Intensity of a diverging compressional pulse

In a medium that is homogeneous and isotropic, a tiny sphere expands suddenly and uniformly and creates an adjacent region of slightly higher density and pressure. This higher density region is called a *condensation*. Assume that it has a thickness  $dr$ . The condensation “impulse” or “pulse,” will move outward as a spherical wave shell and will pass a reference point during time  $\delta t$ . It is called a *longitudinal wave* because the displacements in the medium are along the direction of wave propagation. As it propagates, the energy of the impulse is spread over new spherical shells of ever larger radius, at ever lower acoustic pressure. By conservation of energy, the energy in the expanding wave front is constant in a lossless medium.

The *acoustic intensity* is the fluctuating energy per unit time that passes through a unit area. The total energy of the pulse is the integral of the intensity over time and over the spherical surface that it passes through. Figure 1.2 shows the expanding wave front at two radii. Applying the conservation of energy, the energy that passes through the sphere of radius  $R_0$  is the same as the energy passing through the sphere of radius  $R$ . Conservation of energy gives the sound intensity relationship, where  $i_0$  and  $i_R$  are the intensities at  $R_0$  and  $R$ ,

$$4\pi i_R R^2(\delta t) = 4\pi i_0 R_0^2(\delta t) \quad (1.2)$$



**Fig. 1.3.** Huygens wavelet construction for a pulse. (a) Points on a previous pulse wave front at “a” are the sources of wavelets whose envelope becomes the new wave front, “b.” The wave front thereby moves from a to b. (b) The dependence of wavelet strength on propagation direction, is shown by shadowing. The analytical description of the dependence on  $\phi$ , (1.4) is called the Stokes obliquity factor.

Solving for the intensity at  $R$ , one gets

$$i_R = \frac{i_0 R_0^2}{R^2} \quad (1.3)$$

The sound intensity decreases as  $1/R^2$  due to spherical spreading. Later, in Section 1.5.3 “Near field and far field approximations,” we will show that the sound intensity is proportional to the square of the sound pressure. Therefore, sound pressure decreases as  $1/R$  in a spherically diverging wave. We would have had the same result if the sphere at the origin had imploded instead of exploded. Then a *rarefaction* pulse, a propagating region of density less than the ambient value, would have been created.

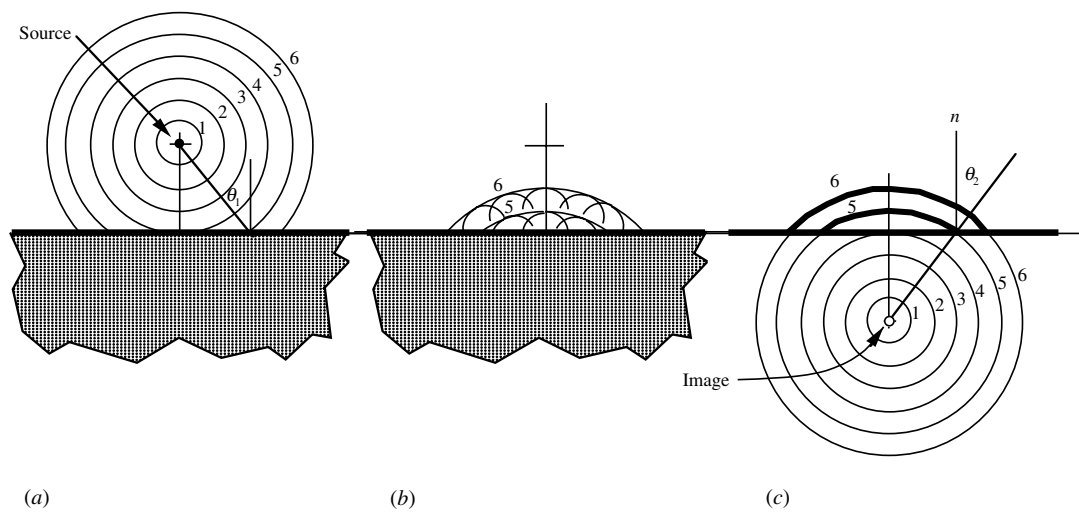
### 1.3 Pulse wave reflection, refraction, and diffraction

A useful qualitative description of wave propagation was first given by Christian Huygens, Dutch physicist–astronomer (1629–1695). Huygens proposed that each point on an advancing wave front can be considered as a source of secondary waves which move outward as spherical wavelets in a homogeneous, isotropic medium. The outer surface that envelops all these wavelets constitutes the new wave front (Fig. 1.3).

The sources used in underwater sound measurements are sometimes condensation pulses, for example the shock wave from an explosion. The application of Huygens’ Principle to an idealized pulse wave front is particularly simple and physically direct.

Baker and Copson (1950) provided a secure mathematical basis for Huygens’ Principle. The concept is extensively used in optics, as well as acoustics. e.g., see A.D. Pierce (1981).

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**Fig. 1.4.** A spherical pulse from a point source and its reflection at a rigid, plane reflector. The usual penetration of the pulse into the lower half space behind the plane face of the reflector is not shown. (a) Successive positions of the incident pulse wave over a half space. (b) Huygens constructions of successive positions of the reflected pulse wave fronts. (c) The reflected pulse wave fronts appear to come from an image source in the lower half space. A homogeneous, isotropic medium is assumed. The geometry shows that  $\theta_2 = \theta_1$ .

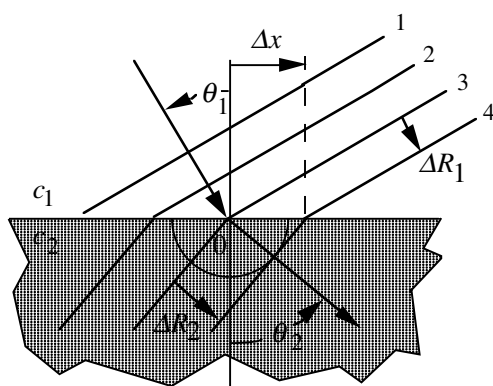
Stokes (1849) derived an obliquity factor which describes the pressure amplitude of the expanding wavelet with lesser side radiation and no back radiation, which agrees with observation. The shading in Fig. 1.3(b), the “strength,” follows the law

$$\text{amplitude} \sim \cos \frac{\varphi}{2} = \sqrt{\frac{1 + \cos \varphi}{2}} \tag{1.4}$$

In the short time  $\Delta t$  the disturbance from each of the secondary sources on the wave front travels a distance  $\Delta R$  (see Fig. 1.3). The outward surfaces of the wavelets coalesce to form the new wave front  $b$ . The strength of the wavelet is maximum in the direction away from the source and zero in the backward direction.

1.3.1 Reflection at a plane surface: law of reflection

The propagation of a pulse will be demonstrated graphically without the preceding details of the Huygens’ construction. See Fig. 1.4(a) where each successive position of the pulse at equal time intervals  $\Delta t$  is indicated by 1, 2, 3, etc. In a homogeneous, isotropic medium the wave front travels the same distance  $\Delta R$  during each interval. In the ray direction,



**Fig. 1.5.** Huygens construction for a compressional plane pulse at a sequence of times and positions in adjacent media showing Snell's Law of Refraction.

normal to the wave front for the isotropic medium, the distance of advance of the pulse is given by  $R = c\Delta t$  where  $c$  is the sound speed.

The Huygens construction of the interaction of a spherical pulse wave at a plane boundary, Fig. 1.4(b), suggests that the wave front of the reflection is expanding as if it has come from a source beneath the reflecting surface. The apparent source after reflection is called the *image*.

A way to treat the image and the real source is shown in Fig. 1.4(c). The image wave of the proper strength is initiated at the same time as the source, and when it moves into the real space it becomes the reflected wave.

The simple geometry shows that the angle of reflection  $\theta_2$  of the rays (perpendicular to the wave front) is equal to the angle of incidence  $\theta_1$ , and is in the same plane.

$$\theta_2 = \theta_1 \quad (1.5)$$

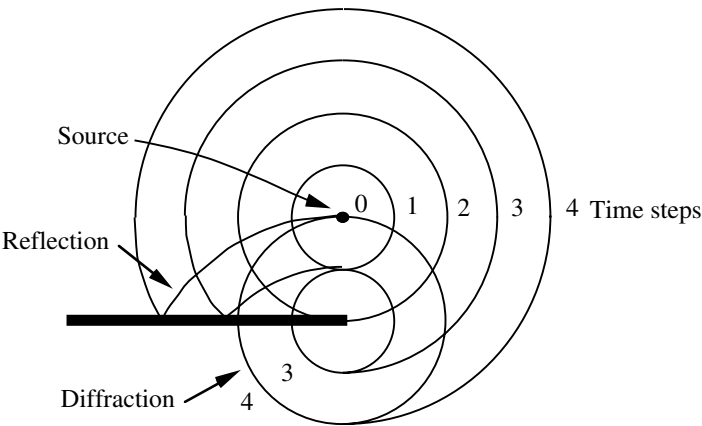
This is called the “Law of Reflection.” *Note:* Some people prefer to give the law of reflection in terms of its complement, the grazing angle that the ray makes with the surface, instead of the angle with the normal.

### 1.3.2 Pulse refraction at a plane interface: Snell's Law

Now we assume that the pulse wave front has come from a very distant point source, that the curvature of the spherical wave front is negligible, and that it is effectively a plane wave front in our region of interest. Figure 1.5 shows that the wave is incident on the plane boundary between two media which have sound speeds  $c_1$  and  $c_2$ .

The figure is drawn for  $c_2 > c_1$  (which is the case for sound going from air to water); the reader can easily sketch the figure for the other case,  $c_1 > c_2$ . Successive positions of a plane pulse are shown as it moves

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**Fig. 1.6.** Huygens construction for diffraction of a pulse at a reflecting half plane. Incident, reflected, and diffracted pulse positions are shown for a sequence of times. The transmitted wave is omitted to simplify the sketch.

across the interface. In general, there will also be a reflected pulse but it is omitted here for simplicity. In the time  $\Delta t$  the pulse has moved a distance  $\Delta R_1$  in medium 1 and  $\Delta R_2$  in medium 2. In the same time, the contact of the pulse at the interface has moved a distance  $\Delta x$  along the horizontal  $x$  axis. The angles are measured between the rays and the normal to the interface, or the grazing angle between the pulse front and the interface. The propagation distances in the two media are

$$\Delta R_1 = \Delta x \sin \theta_1 \quad \Delta R_2 = \Delta x \sin \theta_2$$

The speeds are  $c_1 = \Delta R_1 / \Delta t$  and  $c_2 = \Delta R_2 / \Delta t$ . Therefore,

$$\sin \theta_1 / c_1 = \sin \theta_2 / c_2 \tag{1.6}$$

This is the well-known *Snell's Law of Refraction*. We use Snell's Law throughout the book.

Sometimes the law for refraction is given in terms of the grazing angle  $\phi = 90 - \theta$  in which case the sine  $\theta$  is replaced by the cosine  $\phi$ .

1.3.3 Diffraction at the edge of a plane

Assume that the pulse source is above a semi-infinite plane that permits part of the pulse wave to be transmitted (not shown), part to be reflected, and part to be diffracted. The situation is in Fig. 1.6, where the diffracting plane (heavy line) extends from the boundary edge, infinitely to the left, and infinitely in front and behind the page.

The impulse wave front spreads from the source and interacts with the plane. The interactions at the plane become sources of Huygens

wavelets. The envelopes of the wavelets coming only from the plane become the reflected waves. The outgoing wave beyond the edge of the plane continues, unaffected. The envelope of the Huygens wavelets originating at the edge form a wave front which appears to spread from the edge. That wave is called the *diffracted* wave. (The transmitted wave is omitted for simplicity.)

The diffracted wave is a separate arrival. It is strongest in the direction of propagation but it is more easily detected in any other direction because of its later arrival. The diffracted wave exists because there is a reflecting plane to the left of the edge and none to the right.

In general, “scattering” is a redirection of sound when it interacts with a body. Scattered sound in a fluid is made up of three components: the transmitted, reflected, and diffracted waves. Pulse sounds are helpful in analyzing scattering problems because they have distinctive arrival times that depend on their path lengths. Scattered and diffracted waves are particularly important because they are often used to identify the invisible bodies that cause them. They are discussed in quantitative detail in later chapters.

## 1.4 Sinusoidal, spherical waves in space and time

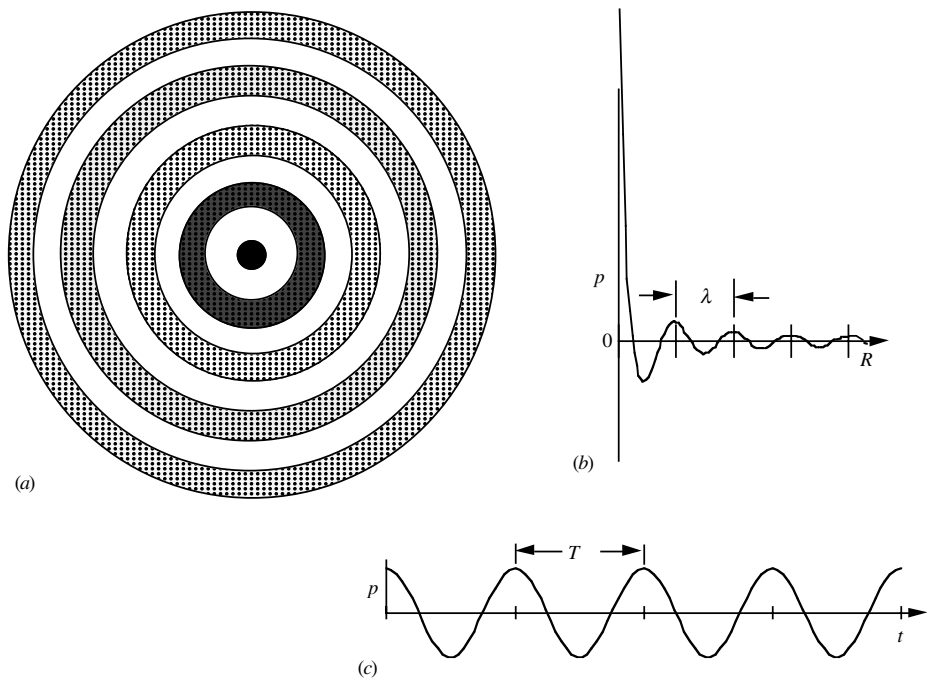
When a sinusoidally excited “point” source expands and contracts periodically it produces a continuous spherical wave. The resulting *condensations* (density and pressure above the ambient) and *rarefactions* (below ambient) in the medium move away from the source at the sound speed  $c$ , in the same manner as the disturbance from a pulse source. A representation of the sinusoidal fluctuations at some later instant would resemble the cartoon in Fig. 1.7(a). The distance between adjacent condensations (or adjacent rarefactions) along the direction of travel is the *wavelength*,  $\lambda$ .

The disturbances sketched in Fig. 1.7(a) radiate outward from a point source which is small compared to  $\lambda$ . As a condensation moves outward, the acoustic energy is spread over larger and larger spheres. Correspondingly, the *pressure amplitude* (the acoustic pressure at the peak of the sinusoid) decreases. Later we prove that, in spherical radiation, the pressure amplitude decreases as  $1/R$ , where  $R$  is the distance from the source. The distance between adjacent crests continues to be  $\lambda$ .

The simplest functions that repeat periodically are sines and cosines. The spatial dependence of the instantaneous sound pressure at large ranges may be written as, for example,

$$p = \frac{P_0 R_0}{R} \sin \frac{2\pi R}{\lambda} \quad \text{or} \quad p = \frac{P_0 R_0}{R} \cos \frac{2\pi R}{\lambda} \quad (1.7)$$

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**Fig. 1.7.** Radiation from a very small periodically pulsating source. (a) Pressure field at an instant of time. The dark condensations are lightened at increasing range to show the decreasing acoustic pressure. (b) Graph of range-dependent pressure at an instant of time. (c) Time-dependent pressure signal at a point in space.

where  $P_0$  is the amplitude of the pressure oscillation at reference range  $R_0$ . Equation (1.7) includes the decrease of pressure with increasing  $R$ . The amplitude at  $R$  is  $P(R) = P_0 R_0 / R$ .

The time between adjacent crests passing any fixed point is the *period*,  $T$  (Fig. 1.7(c)). The temporal dependence of the pressure oscillation at  $R$  is, for example,

$$P = P(R) \sin 2\pi ft \quad \text{or} \quad P = P(R) \cos 2\pi ft \tag{1.8}$$

where  $f$  is the *frequency* of the oscillation, measured in cycles per second or hertz (Hz).

The simplest functions that repeat periodically are sines and cosines which repeat themselves for every increment of  $2\pi$ . For example,  $\sin(\theta + 2n\pi) = \sin \theta$  where  $n$  is an integer. For two adjacent times,  $t_1$  and  $t_2 = t_1 + T$ , the functions repeat so that  $2\pi ft_2 = 2\pi ft_1 + 2\pi$ . Therefore,  $2\pi fT = 2\pi$ , and the period,  $T$ , is

$$T = 1/f \tag{1.9}$$