

## Contents

<b>Preface</b>	<i>page</i>	ix
<b>1 Observed Markov Chains</b>		1
1.1 Introduction		1
1.2 Observed Markov chain models		1
1.3 Notation		4
1.4 Construction of Markov chains		6
1.5 The general Markov chain		12
1.6 Conclusion		17
1.7 Exercises		17
<b>2 Estimation of an Observed Markov Chain</b>		18
2.1 Introduction		18
2.2 Estimation based on one sample path		18
2.3 Estimation using $K$ sample paths of length $L$		23
2.4 Markov chains of order $M \geq 2$		25
2.5 Exercises		28
<b>3 Hidden Markov Models</b>		29
3.1 Definitions		29
3.2 Calculation of the likelihood		38
3.3 Exercises		46
<b>4 Filters and Smoothers</b>		47
4.1 Introduction		47
4.2 Decoding		50
4.3 Further remarks on filters and smoothers		51
4.4 Exercises		55
<b>5 The Viterbi Algorithm</b>		56
5.1 Introduction		56

vi	<i>Contents</i>	
	5.2 Viterbi decoding	58
	5.3 Estimation of the model	61
	5.4 Exercises	63
<b>6</b>	<b>The EM Algorithm</b>	64
	6.1 Introduction	64
	6.2 Steps of the EM algorithm	66
	6.3 Exercises	74
<b>7</b>	<b>A New Markov Chain Model</b>	75
	7.1 Introduction	75
	7.2 Construction of the model	76
	7.3 Filters	79
	7.4 Smoothers	82
	7.5 The Viterbi algorithm	88
	7.6 Parameter estimation by the EM algorithm	90
	7.7 Steps of the EM algorithm	91
	7.8 Exercises	97
<b>8</b>	<b>Semi-Markov Models</b>	98
	8.1 Introduction	98
	8.2 Semi-Markov models	98
	8.3 Transition probabilities for semi-Markov chains	100
	8.4 Exercises	109
<b>9</b>	<b>Hidden Semi-Markov Models</b>	110
	9.1 Introduction	110
	9.2 A semi-martingale representation for a semi-Markov chain	113
	9.3 Construction of the semi-Markov model	117
	9.4 Hidden semi-Markov models	117
	9.5 Exercises	124
<b>10</b>	<b>Filters for Hidden Semi-Markov Models</b>	125
	10.1 Introduction	125
	10.2 The Viterbi algorithm	126
	10.3 Smoothers	129
	10.4 EM algorithm for estimating a hidden semi-Markov model	139
	<i>Appendix A</i> Higher-Order Chains	147
	<i>Appendix B</i> An Example of a Second-Order Chain	151
	<i>Appendix C</i> A Conditional Bayes Theorem	156

	<i>Contents</i>	vii
<i>Appendix D</i>	<b>On Conditional Expectations</b>	158
<i>Appendix E</i>	<b>Some Molecular Biology</b>	161
<i>Appendix F</i>	<b>Earlier Applications of Hidden Markov Chain Models</b>	165
	<b>References</b>	169
	<b>Index</b>	173